THE GROVND OF ARTES:

Teaching the worke and practiſe of
Arithmeticke, both in whole numbres
and Fractions, after a more eaſyer
and exacter ſorte, then anye lyke
hath hytherto beene
ſet forth: with di-
uers new ad-
ditions.

Made by M. ROBERTE
RECORDE
Doctor of Phyſike.

The Bookes Verdicte.

To pleaſe or diſpleaſe ſure I am,
But not of one ſorte to euery man.
To pleaſe the beſte ſorte woulde I fayne.
The frowarde diſpleaſe ſhall I certayne.
Yet wiſhe I wyll, though not with hope,
All eares and mouthes to pleaſe or ſtoppe.

TO THE MOSTE MIGHTY PRINCE EDVVARD THE ſyxte, by the grace of God king of Englande, Fraunce and Ireland. &c.

THE EXCELlency of mās nature beeing ſuche, as it is by goddes diuine fauour (moſte mightye prince) not only exalted in highnes of degree farre aboue all other corporal thinges, but by perfection of reaſon and ſearche of wytte much approching toward the diuine nature of God, as not only the holy ſcriptures do teſtify, but alſo thoſe natural Philoſophers, whiche exactly didde conſidre the nature of man, and namely the farre reache and infinite compaſſe of the workes of the wytte, were inforced to confeſſe that man rightly was able to know him ſelf. And if he didde dulye pondre the nature of him ſelf, he ſhoulde fynde it ſo ſtraunge, that it might ſeme vnto hym a very myracle. And therof ſprang this ſaying: *Magnum miraculum eſt homo, maximum miraculum ſapiens homo.* For vndoubtedly a man is one of the greateſt miracles that God of God wrought, ſo a wiſe man is plainly the greateſt.

And therefore was it, that ſome dyd accompt the head of a man to bee the greateſt miracle in the world, bicauſe not only of the ſtrange workmanſhip that is in it, but much more of the efficacye of reaſon, wit, memory, imagination, and ſuch other powers and works of the mynd, which can more eaſilye conceaue any thing in a maner, then vnderſtand it ſelf. And amongſt all the creatures of God it fyndeth none more difficult to bee perceaued, then theſe ſame powers of it ſelf, whereby it doth conceaue & iudge, as it maye be wel coniectured by the diuerſity of opinions, that the wiſeſt philoſophers did vtter, touching the ſpirit of man & the ſubſtāce of it, whereof at this preſent I intende to make no reherſal, but who ſo lyketh to read therof, may fynd it largely ſet foorth not only in Ariſtotle his bokes *de Anima*, but alſo in Galen his boke called *Hiſtoria Philoſophica*, and again in Plutarch his work, *de Philoſophorū placitis*, whoſe words ar alſo repeted of Euſebius in his boke, τῆς εὐαγγελικῆς προπαρασκευῆς, vnto whō I remit them that haue deſire to vnderſtād the intricate difficulty of knowyng our owne ſelues, as touching our beſt part, and that parte whereby we deſerue to beare the name of men.

This matter ſemed ſo obſcure & difficulte in knowledg, that Galen, who for his excellent wiſedome & iudgement in naturall workes, is called of many men A miracle in nature, yet in ſerching the nature & ſubſtaunce of the ſpirite of man, he not only confeſſeth him ſelfe ignorant, but counteth plain temeritie to attempte to fynd it: ſo far aboue the hope of mans knowledg is that part whereby man doth know and iudg of thinges. And although the ignorant ſort (whiche hate all thinges that thei know not) do lyttle eſteme the profoundneſſe of mans ſpirit and of reaſon, the chief power and facultie of it: Yet as there is a kynd of feare and obedience of all vnreaſonable beaſtes vnto man, by the workynge power of God, ſo is there in thoſe ſmall reaſoned perſons a certain kind of reuerence toward wiſdome and reaſon, which they doo ſhewe oftentimes, and by power of perſuaſion ar inforced to obey reaſon, wyll they nyll they.

And herby came it to paſſe, that the rudeneſſe of the fyrſt age of man was brought vnto ſomme more ciuile trade, as it is well declared by Cicero in the beginning of hys fyrſte boke *De inuentione Rhetorica*, where he ſayth thus.

Nam fuit quoddam tempus, cum in agris homines paſſim beſtiarum more uagabantur, & ſibi victu fereo vitam propagabant, nec ratione

animi quicquam, ſed pleraq; viribus corporis adminiſtrabant. Non dum diuinæ religionis, non humani officij ratio colebatur. Nemo legitimas viderat nuptias, non certos quiſquam inſpexerat liberos, non ius æquabile quid vtilitatis haberet, acceperat. Ita propter errorem atq; inſcitiam cæca ac temeraria dominatrix animi Cupiditas ad ſe explendam viribus corporis abutebatur perniciosiſſimis ſatellitibus. Quo tempore quidam magnus videlicet vir & ſapiens cognouit, quæ materia eſſet, & quanta ad maximas res opportunitas animis ineſſet hominum, ſi quis eam poſſet elicere, & præcipiendo meliorem reddere. Qui diſperſos homines in agris, & in tectis ſylueſtribus abditos, ratione quadam compulit in vnum locum, & congregauit: & eos in vnam quanque rem inducens vtilem atq; honeſtam, primo propter inſolentiam reclamantes, deinde propter rationem atq; orationem ſtudioſius audientes, ex feris & immanibus mites reddidit & manſuetos.

This long repetition of Tullies wordes wyll ſeme tedious to them which loueth but lytle, & careth muche leſſe for the knowledge of reaſon, but vnto your maieſtie I dare ſay it is a delectable remembraunce: and vnto me it ſemed ſo pleaſaunt, that I coulde ſcarſe kepe my penne from writinge all that myne eyes dyd ſo greedilye reade.

This ſentence of Cicero am I lothe to tranſlate into engliſhe, partly for that vnto your maieſty it needeth no tranſlation; but eſpecially knowing howe farre the grace of Tullies eloquence doth excell any engliſhe mans tonge, and muche more exceedeth the baſeneſſe of my barbarous ſtyle, yet for the frute of the ſentence I had rather vnto my mere engliſh contrey men vtter the rudeneſſe of my tranſlation, then to defraud them the benefyte of ſo good a leſſon, truſtynge they wyll ſo learne to loue reaſon, that they wyl alſo gladly and greatly imbraſe al good ſciences that may helpe to the iuſt furniture of the ſame, when they conſidre that informed reaſon was the onlye inſtrument, or at leaſt the chiefe meane to bringe men vnto ciuyle regiment, from barbarous maners & beaſtly conditions.

For the time was (ſayth Tully) that men wandred abroad in the fieldes vp and doune lyke beaſtes, and vſed no better ordre in feedyng then they, ſo that by reaſons rule they wrought nothing, but moſte of their doyng dyd they atcheue by force of ſtrength. At this tyme there was no iuſte regarde of religion towarde God, nor of duety towarde man: no man had ſeene righte vſe of mariage, nother dyd any man knowe their owne children from other, nor no man had felt the commoditie of iuſt lawes. So that through

Errour and Ignoraunce. Whylſt luſt lyke a blynde & heady ruler abuſed bodily ſtrength as his moſt mortall miniſter, for the ſatiſfying of hys deſyre. At that tyme was there one, whiche not only in power, but alſo in wiſedome was great, and he conſidered how that in the myndes of men was bothe apt inſtrumentes and great occaſiō to the due accompliſhment of moſte weightye affayres, yf a man coulde applye them to vſe, and by teachyng of rules frame them to better trade. Thys man with perſuaſion of reaſon gathered into one place the people that were wanderynge aboute the fyeldes, and laye lurkyng in wilde cotages and woodes, and bryngyng theym into one common ſocietie, hee trade them to all ſuche thinges as eyther were profitable or honeſt, although not without repinyng at the firſt, by reaſon that they had not bene ſo accuſtomed before: Yet at length through reaſon and perſuaſion of wordes, they obeyed him more diligentlye: and ſo of a wyld and cruell people, he made them curteous and gentle.

Thus hath Tullye ſet foorth the efficacie of Reaſon and Perſuaſiō, how it was able to converte wylde people to a myldnes, and to chaunge their furious crueltie into gentill curteſy. Were it not now a great reproche in thys oure tyme (when knowledge raigneth ſo large) that men ſhould ſhewe them ſelfe leſſe obſequious to [illegible] maye it maye be thought, that nowe euery man hauing ſufficient knowledge of him ſelf, [illegible] not to harken to the perſuaſion of others.

In dede he that thinketh him ſelfe wiſe, wyll not eſteeme the reaſon of any other, be he neuer ſo wiſe. So that of ſuch one it may well be ſayd: He that thinketh him ſelf wyſer then he is, maye iuſtly be counted a double foole. Wherefore ſuche men are not to be permitted in opē audience to talke, but muſt bee put to ſilence, and made to giue eare to Reaſon: which reaſon conſiſteth not in a multitude of woordes heaped raſhely togyther, and applied for one purpoſe: but Reaſon is the expreſſyng of a iuſte matter with wittye perſuaſions, furniſhed with learned knowledge, ſuche knowledge hadde Moyſes, beinge experte in all learnyng of the Egyptians, as the ſcriptures declare, and therfore was able to perſuade the ſtubburne people of the Iewes, although not without great peine. Such knowledg and ſuch reſons did Dryius ſhewe, which was the firſt lawmaker of all the weſt part of Europe. Like reaſon and wiſdome did Zamolxis vſe amongeſt the Gothes: Lycurgus, vnto the Lacedemonians: Zaleucus to the Locrians: Solon to the Athenienſes: and Dunwallo Molmutius 2000 yeares paſt, amongeſt the old Brytaynes of this realm. And therby com-it to paſſe, that theſe lawes continued long, tyll more perfecte reaſon altered manny of theym, and wylfull power oppreſſed moſte of theym.

At the begynnynge when theſe wiſe men perceued how hard it was to bryng the rude people to vnderſtād reaſon, they iudged the beſte meanes to attain this honeſt purpoſe, to depende of learnyng in euery kynde, for by learnyng as Ouid ſayeth,

Pectora molleſcunt, aſperitasq; fugit.

Stoute ſtomackes doo waxe mylde,
And ſharpe fierceneſſe is exilde.

Therfore as Beroſus doth teſtifye, Sarron, that was the thirde kinge ouer all this weſt parte of Europe, for to bryng the people from beaſtly rage to manly reaſon, dyd erecte ſchooles of liberall artes, whiche toke ſo good ſucceſſe, that his name continued in that ſorte famous aboute 2000 yeares after: for Diodorus Siculus, whiche was in the tyme of Julius Ceſar, maketh mention of the learned men of the Celtes, and nameth theym Sarronides, that is to ſaye, Sarron his ſcholars and folowers.

Among theſe artes that then wer taught, ſome dyd informe the tonge, and make men hable both to vtter aptlye theyr mynde, and alſo to perſwade, as Grammar, Logike and Rhetorike, although not ſo curiouſly as in this time. Some other did appertain to the iuſte ordre of partition of landes: the true vſyng of weightes, meaſures and reckenynges in all ſortes of bargaines, and for order of buylding, & ſundry other vſes: thoſe were Arithmetike & Geometrye. Againe to incourage men to the honour of God, they taught Aſtronomy: whereby the wonderfull works of God were ſo manifeſtly ſet forth, that no mans tongue nor penne can in lyke ſort expreſſe his infinite power, his vnſpekable wyſdome, and his exceedynge goodneſſe toward man: whereby he doth bountifully prouide for man all neceſſaries, not onlye to lyue, but alſo to liue pleaſantly. And ſo was their confidence in gods prouidence ſtrongly ſtayed, knowing his goodnes to bee ſuche that he woulde helpe man, as he coulde: and his power to be ſo great, that he coulde doo what he woulde: and thyrdly his wiſedome to bee ſo pure, that he would doo nothyng but that that was beſte. Beſyde theſe ſciences, they taught alſo Muſike, whiche moſte commonly they dydde apply partly to religious ſeruices, to draw men to delyte therin, and partly to ſonges made of the manners of men in praiſe of vertue, and diſcōmendation of vice. Whereby it came to paſſe that no man would diſpleaſe them, nor doo any thyng euill that mighte come to theyr hearinge: for theyr ſonges dyd make euill men more abhorred in that tyme, then any

Excommunication dooth in this tyme. The poſteritie of theſe Muſitians contynue yet bothe in Wales and Irelande, called Bardes vnto this daye, by the auncient name of Bardus, theyr fyrſt founder.

And as theſe ſciences dyd increaſe, ſo dyd vertu encreaſe therby. Again, as theſe ſciēces did decay, ſo vertu loſt hir eſtimation, & conſequently was lytle in vſe. Wherof to make a full declaration, were a thyng meete for a Prince to heare, but it would require a peculiar treatiſe: wherefore at this preſente I counte it ſufficient lightlye to haue touched this matter in generall woordes, and to ſay no more of the particularities thereof, but only touching one of thoſe ſciences, that is Arithmetike, by which not only iuſte partition of landes was made, but alſo touching byinge and ſellynge all aſſiſes, weightes, and meaſures were diuiſed, and all rechoninges and accomptes dryuen: yea by proportion of it, were the true ordres of Iuſtyce lymytted, as Ariſtotle in his Ethikes dooth declare: and the degrees of eſtates in the common wealthe eſtablyſhed, althoughe that proportion bee called Geometricall and not Arithmeticall, yet dooth that proportion appertaine to the arte of Arithmetike: and in Arithmetike is taught the progreſſion of ſuche proportions, and all thynges therto belõging. Wherfore I may well ſay, that ſeing Arithmetik is ſo many wayes needfull vnto the firſt plantyng of a cōmon welthe, it muſt needes be as much required to the preſeruatiō of it alſo. for by the ſame meanes is any common welthe continued, by whyche it was erected and eſtablyſhed. And if I ſhal in ſmal matters in apperaunce, but in deede very weyghty, put one exaumple or two, what ſhall wee ſaye for the ſtatutes of this Realme, whyche bee the onlye ſtay of good ordre in maner nowe? As touchyng the meaſuring of ground by lengthe and bredthe, there is a good and an auncient ſtatut made by arte of Arithmetik, and now it ſhalbe to lytle vſe, if by the ſame arte it be not practiſed and tried: for the aſſiſe of bred and drinke, the two moſt common and moſt neceſſarye thinges for ſuſtentation of man, there was a goodly ordinaunce in the lawe made, whiche by Ignoraunce hath ſo growen out of knowledge and vſe, that fewe men doo vnderſtande it: and therefore the ſtatute bookes wonderfully corrupted, and the commons cruelly oppreſſed. not withſtandinge ſome menne haue written, that it is to doubtful a matter to execute thoſe aſſiſes by thoſe ſtatutes, by reaſon they depend of the ſtanderd of the coyne, whiche is muche changed frome the ſtate of that tyme, whanne thoſe ſtatutes were made. Thus ſhall euery man reade (that lyſteth) in the

Abridgmente of ſtatutes, in the tytle of Weightes and Meaſurs, in the ſeuenth nubre of the Engliſhe booke, where he ſhould haue tranſlated a good ordinaunce, whiche is ſette forth in the French boke: but no merueyle yf the Abridgmente dooth omytte it, ſeyng the great boke of ſtatutes doth omytte the ſame ſtatute, as it hath done dyuers other very good lawes. And this is the frute of Ignoraunce, to reiecte and condemne all that it vnderſtandeth not, althoughe they vſe ſome clokes for it: but ſuche clokes as being alowed, might ſerue to repell al good lawes, whiche God forbyd.

Again, ther is an auncient ordre for aſſiſe of fyrewood, and coales, whiche was renewed not many yeares paſte, and nowe howe Auarice and Ignoraunce doth canuas that ſtatute, it is to pitifull to talke of, and more myſerable to feele.

Farthermore for the ſtatute of coynage, and the ſtanderd thereof, if the people vnderſtoode rightly the ſtatute, they ſhuld not nor wolde not (as they often do) gather an excuſe for their folly therby, but as I ſayd, theſe ſtatutes by wiſedome and good knowledg of Arithmetike were made, and by the ſame muſt they be continued. And let Ignorance no more meddle with the vſe of them, then it dyd wyth the making of them.

Oh in how miſerable caſe is that realme

where the miniſters and interpreters of the lawes, are deſtitute of all good ſciences, whiche bee the keyes of the lawes? Howe can they either make good lawes, or mayntayne them, that lacke that true knowledg, whereby to iudge them? And happye maye that realm be accompted, where the Prince hym ſelfe is ſtudious of learnyng, and deſireth to vnderſtand equitie in all lawes. Therefore moſt happye are we the louynge ſubiectes of your maieſtie, whiche may ſee in your highneſſe not onlye ſuche towardneſſe, but alſo ſuche knowledge of dyuers artes as ſeldome hath beene ſeene in anye prince of ſuche yeares: whereby wee are inforced to conceaue this hope: Certainlye that he which in thoſe yeares ſeeketh knowledge when knowledg is leaſt eſteemed, and at ſuche an age can diſcerne them to be enemies bothe to his royall perſon and to his realmes, whiche labour to withdrawe hym from knowledge to exceſſiue paſtyme, and from reſonable ſtudy to ydle or noyſom pleaſures, he muſte needes when he commeth to more mature yeares bee a moſte prudent prince, a moſte iuſte gouernour, and a righte iudge, not only of his ſubiectes commonly, but alſo of the miniſters of his lawes, yea and of the lawes them ſelf. And to be able to conceaue the true equitie and exacte vnderſtandynge of all his lawes and ſtatutes, to the comforte of his good ſubiectes, and the confuſion and reproche of them whiche labour to obſcure or peruert the equitie of the ſame lawes and ſtatutes. Now ſom of thoſe ſtatutes maye bee applyed to vſe as well in this our tyme as in any other tyme, I haue peculiarly declared in this booke, and ſome other I haue omitted for iuſt conſideratiõs till I may offer them fyrſt vnto your Maieſtie, to way them as to your highneſſe ſhall ſeeme good: for many thinges in theym are not to be publyſhed without your highneſſe knowledge and approbation, namely bycauſe in them is declared all the rates of alloies for all ſtanderdes frome one vnce vpwarde, with other myſteries of mynte matters, and alſo moſte part of the varieties of coynes that haue been currant in this your Maieſties realme, by the ſpace almoſt of vj. hundred yeares laſt paſt, and many of them that were currant in the tyme that the Romaines ruled here.

Al which wyth the ancient deſcription of England and Irelande, and my ſimple cenſure of the ſame I haue almoſt completed, to bee exhibited to youre highneſſe. In the meane ſeaſon moſt humbly beſeeching your Maieſty to accepte this ſimple treatiſe, not worthy to be preſented to ſo high a Prince, but that my lowly requeſt to your Maieſtie is, that this amongeſt other of my bookes, maye

may paſſe vnder the protectiō of your highneſſe, whome I beſeeke God moſt earneſtlye and dayly, accordyng to my dutye, to aduaunce in all honoure and princely regalitie, and to increaſe in all knowledge, iuſtice & godlye pollycye.
Amen.

Your Maieſties moſte obedient ſubiect and ſeruaunt,

Roberte Recorde.

TO THE LOVINGE READERS.

SORE OFTENtimes haue I lamented with myſelfe the infortunate conditiõ of England, ſeeyng ſo many greate clerkes to aryſe in ſundrye other partes of the worlde, and ſo few to apear in this our nation: where as for pregnancy of naturall witte (I thinke) fewe nations do excell engliſhemen. But I can not impute the cauſe to any other thynge, than to the contempt or miſregarde of learnynge. For as Engliſhmen are inferiour to no men in mother witte, ſo they paſſe al men in vain pleaſures to whiche they may attaine without great peyne or laboure: and are as ſlacke to any neuer ſo great cõmoditie, if they hange of it any peynfull ſtudy or traueilſom labor.

Howbeit, yet all men ar not of that ſort, though the moſte parte bee, the more pitye it is: but of them that are ſo glad not onely with paynfull ſtudy and ſtudyous payne to attaine lernyng, but alſo with as great ſtudy and payne to cõmunicate theyr learnyng to other, & make all England (yf it mought bee) partakers of the ſame, the moſte parte are ſuche, that vnneth they canne ſupporte their owne neceſſarye charges, ſo that they are not able to beare any charges in doynge of that good, that els they deſire to do.

But a greater cauſe of lamentacion is this, that when learned men haue taken paynes to doo thynges for the ayde of the vnlearned, ſcarſe they ſhall bee allowed for their weldoyng, but derided and ſcorned, and ſo vtterly diſcouraged to take in hand any like enterpriſe againe. So that if any be founde (as there are ſome) that do fauour learning and learned wittes, and can bee contente to further knowledge, yea onely wyth theyr woord, ſuch perſons thoughe they be rare, yet ſhall they encourage learned menne to enterpryſe ſome thynges, at the leaſte, that England may reioyce of. And I haue good hope that England will (after ſhe hath takē ſome ſure taſte of learning) not only bringe forth more fauourers of it, but alſo ſuche lerned men, that ſhe ſhall be able to compare wyth anye Realme in the worlde. But in the meane ſeaſon, where ſo fewe regarders of learnyng are, howe greatly they are to bee eſteemed that doo fauour and further it, my penne wyll not ſuffiſe at full to declare.

Therefore gentle reader, where as I do vpon moſte iuſte occaſion iudge, yea and

knowe

know aſſuredly, that ther be ſum mē in this Realme whiche bothe loue, and alſo muche deſyre to further good learnynge, and yet am not well able to write their condigne praiſe for the ſame, I thinke it better with ſylence to ouerpaſſe it, than other to ſaye to lyttle of it, or to prouoke againſt theym the malice of ſuche other, whiche doo nothinge them ſelfe that is praiſe worthy, and therefore can not abyde to heare the prayſe of any other mannes good deede. And conſydering theyr great fauour vnto learnynge though I my ſelfe bee not worthy to bee reckened in the noumbre of greate learned menne, yet am I bolde to put my ſelfe in preaſſe wyth ſuche abilitye as God hathe lente me, thoughe not wyth ſo great cunnyng as many men, yet wyth as great affection as anye man to healpe my countreye men, and wyll not ceaſſe daylye (as muche as my ſmall abilitie will ſuffer me) to endyte ſome ſuche thynge that ſhall bee to the inſtruction, thoughe not of learned menne, yet at the leaſte of the vulgar ſorte. whoſe argument alwaies ſhall be ſuch, that it ſhall delite all learned wittes, though they do not learne anye greate thynges out of it.

But to ſpeake of this preſente booke of Arithmetike, I dare not nor wyll not ſette it forthe wyth anye woordes, but commytte it to the iudgemente of all gentell readers, and namely ſuch as loue good learnyng, beſechyng them ſo to eſteeme it, as it dooth ſeeme worthy. And ſo, other to accept the thing for it ſelfe, other at the leaſt to alowe my good endeuour. But I perceaue I nede not vſe anye perſuaſions vnto theym, whoſe gentle nature and fauourable mynde is ready to receyue thankfullye, and interprete to the beſt all ſuch enterpriſes attempted for ſo good an ende, thoughe the thynge doo not alwaies ſatiſfye mens expectation. This conſydered, dyd bolden me to publyſh abroad this lyttle boke of the Arte of numbryng, whiche if you ſhall receyue fauourably, you ſhall encourage me to gratifye you herafter with ſome greater thyng.

And as I iudge ſome men of ſo louinge a mynd to their natiue contry, that they wold much reioyce to ſe it to proſper in good lernyng and witty artes, ſo I hope well of all the reſt of Englyſhmen, that thei wil not be vnmyndefull of his due praiſe, by whoſe meanes they are healped and furthered in any thynge. Neither oughte they to eſteeme this thinge of ſo lyttle valewe as manye men of lyttle diſcretion oftentymes doo: for who ſo ſetteth ſmall price by the wittye deuiſe and knowledge of numberyng, he little conſidereth it to be the chiefe poynt (in maner) wherby men differ from all brute beaſtes, for as in all other thinges (almoſte) beaſtes are partakers with vs, ſo in nũbꝛing we differ clean from them, and in maner peculiarly, ſythe that in many thinges they excell vs againe:

The Foxe in craftye witte exceedeth moſte men,
A Dogge in ſmellyng hath no man his peere.
To foreſyght of wether if you looke then,
Many beaſtes excell man, this is cleere.
The wittineſſe of Elephantes doth letters attain
But what cunning doth there in the Bee remaine?
The Emmet foreſeeyng the hardneſſe of winter,
Prouideth vitailes in tyme of ſommer.
The nightingale, the lynet, the thruſh, the larke,
In Muſicall harmonie paſſe many a clerke.
The hedgehogge of aſtronomy ſeemeth to know,
And ſtoppeth his caue where the wind wil blow.
The ſpyder in weauinge ſuche arte dooth ſhowe,
No man can him mend, nor folowe I trowe.
When a houſe wyll fall, the myſe righte quicke,
Flee thence before, can man do the lyke?

Many thynges els of the wittineſſe of beaſtes and byꝛdes myght I here ſaye, ſaue that an other tyme of them I intende to wꝛite, wherin they excell in maner all men, as it is dayly ſeene: but in nũbꝛe was there neuer beaſt founde ſo cunnynge, that coulde knowe oꝛ diſcerne one thyng from many: as by daylye experience you maye well conſyder. whan a byche hath many whelpes, or a henne manye chyckins: and lykewaies of other what ſo euer they be, take from them all theyr yonge, ſauyng onely one, and you ſhall perceaue plainly that they miſſe none, though they wyll reſiſt you in takyng them awaye, and wyll ſeeke them agayne yf they maye knowe where they bee: but elles they wyll neuer myſſe theym truely: but take awaye that one that is lefte, and then wyll they crie and complaine: and reſtore to them that one, then are they pleaſed againe: ſo that of numbre this maye I iuſtly ſay, It is the onely thynge (almoſte) that ſeparateth manne from beaſtes. He therfore that ſhall contempne numbre, hee declareth hym ſelfe as brutyſhe as a beaſte, and vnworthy to be counted in the felowſhip of menne. But I truſte there is no manne ſo foule ouerſeene, though many right ſmally do it regarde.

Therfore will I now ſtay to write agaynſt ſuche, and returne againe to thys my boke, whiche I haue wrytten in the fourme of a Dialogue, bicauſe I iudge that to be the eaſyeſt waye of inſtruction, whan the ſcholar maye aſke euery doubt orderly, and the maſter maye anſwere to his queſtion plainely.

How be it, I thynk not the contrary, but as it is eaſier to blame an other mannes worke than to make the lyke, ſo there will be bee ſomme that wyll fynde faulte, bycauſe I wryte in a Dialogue: but as I coniectur, thoſe ſhall be ſuche, as doo not, can not, either wyll not perceiue the reaſon of righte teaching, and therfore are vnmeet to be anſwered vnto, for ſuche men with no reaſon wyll be ſatiſfied.

And if any man obiecte that other bokes haue bene written of Arithmetike alreadye ſo ſufficiently, that I needed not now to put pen to the booke, excepte I will condemne other mens writtinges: to them I anſwer. That as I condemn no mannes diligence, ſo I knowe that no one manne can ſatiſfye euery man: and therfore lyke as many wiſe eſteeme greatly other bookes, ſo I doubte not, but ſome will lyke this my boke aboue anye other Engliſhe Arithmetike hytherto written, and namely ſuche as ſhall lacke inſtructers, for whoſe ſake I haue ſo plainly ſet forth the Examples, as no booke that I haue ſeen, hath done hytherto: which thing ſhalbe great eaſe to the rude readers.

Therfore gentle reader, though this boke can be but ſmal ayde to the learned ſort, yet vnto the ſimple ignorāt (which nedeth moſt healpe) it maye bee a good furtherance and meane vnto knowledge.

And though vnto the Kinges his maieſtie priuatelye I do it dedicate, yet I doubt not (ſuche is his clemencye) but that hee can bee content, yea and muche deſirous that all his louing ſubiectes ſhall take the vſe of it, and employ the ſame to their moſt profit: which thynge if I perceaue that they thankefullye doo, and receyue wyth as good wyll as it was writte, then wyl I ſhortly with no leſſe kindneſſe ſette forth ſuch introductions into Geometrye and Coſmographye, as I haue at tymes promiſed, and as hytherto in Englyſh hath not been enterpriſed, wherewith I dare ſay, all honeſt heartes will bee pleaſed, and all ſtudious wittes greatly helped.

I will ſaye no more, but lette euerye man iudge as he ſhall ſe cauſe. And thus for this tyme I wyll ſtaye my penne, committing you all to that treewe fountaine of perfecte numbre, which wroughte the whole world by numbre & meaſure: he is Trinitie in Unity, and Unitie in Trinitie; to whome bee all praiſe, honor, and glorye.

AMEN.

The Contentes of the fyrſt Dialogue.

The declaration of the profite of Arithmetike.
Numeration with an eaſye and large table.
Addition,
Subtraction,
Multiplication,
Diuiſion.
with dyuers examples, and ſome newe fourmes of workynge.
Reduction with diuers declarations of coynes, weightes, and meaſures, of ſundry formes, nowe newly added to theſe other rules.
Progreſſion bothe Arithmeticall and Geometricall, wyth certaine queſtions touchyng the ſame.
The Golden rule, & the Backar rule, with dyuers queſtions thereto belonging.
The double rule of proportion.
The rule of felowſhip, bothe with tyme, and without tyme.
Unto all theſe is added theyr proofe.

In the ſeconde Dialogue.

The fyrſt v. kyndes of Arithmetike wrought by counters.
The common kyndes of caſting accomptes after marchantes faſhion and auditors alſo.
Numbring by the hand newely added.

The

The contentes of the thyrde Dialogue touching Fractions.

What a Fraction is.
Numeration in Fractions.
The order of worke in Fractions.
Multiplication.
Diuiſion.
Reduction
- Of diuers fractions into one denomination in 3 varieties.
- Of fractions of Fractions.
- Of improper fractions.
- Of Fractions to the ſmalleſt denomination.
- Of Fractions into other partes of thynges.

Agayne of Multiplication.
Duplation.
Agayne of Diuiſion.
Mediation.
Addition.
Subtraction.
The Golden rule with dyuers queſtions.
The proofe of the Golden rule.
The Backer rule.
A queſtion of lone.
The ſtatute of Aſſiſe of bread and ale, recognised with newe tables thereto annexed.
The ſtatute of meaſuryng of ground, with a table and queſtions.

Que-

Queſtions of ſocietie, with the reaſon of the rule, and proofes of their workes.
To finde three numbres in any proportion.
The rule of Alligation with dyuers queſtions, and the proofe of their woorkes, and many variations of ſuche ſolutions.
The rule of Falſhode or falſe poſition, with dyuers queſtions, and theyr proofe.

Before the Introduction of Arithmetike theſe figures muſte be learned.

Fygures of numbre.

i.	one.
ii.	two.
iii.	thre.
iiii.	foure.
v.	fiue.
vi.	ſixe.
vii.	ſeuen.
viii.	eyght.
ix.	nyne.
x.	ten.
xi.	aleuen. &c.
xvi.	ſyxtene
	&c.
xix.	nynetene.
xx.	twenty.
	&c.
xl.	fortye.
l.	fyftye.
lx.	ſyxtie.
	&c.
xc.	nynetye.
C.	a hundred.
C. C.	two hundred. &c.
D.	fyue hundred.
	&c.
D. C.	ſyxe hundred.
	&c.
M.	a thouſande.

Fygures of monney.

c.a cee, the xvi.
q,a kewe,the viii. } parte of a penny.
q,a farthing the iiii
ob, an halfepenny.
1 d,a penny.
1 s,a ſhyllynge.
1 li,a pounde.

A DIALOGVE BETWENE THE MAISTER and the Scholar, teaching the arte and vſe of Arithmetike with the penne.

The Scholar ſpeaketh.

SYr, ſuche is your auctoritie in mine eſtimation, that I am content to conſent to your ſaying, and to receiue it as truth, though I ſe none other reaſō that doth leade me thereunto: wher as els in mine owne conceite it appeareth but vayn, to beſtow any tyme priuatly in learning of that thinge, that euery chylde maye and doothe learn at all times and hours, when he dothe any thing him ſelfe alone, and muche more whan he talketh or reaſoneth with other.

Maſt. Lo, this is the faſhion and chance of all them that ſeke to defend their blinde ignorance: that when they thynke thei haue made ſtrong reaſon for them ſelfe, thā haue they proued the quite contrary. For if numbryng bee ſo common (as you graunt it to be) that no man can doo any thinge alone, and muche leſſe talke or bargain with other, but he ſhall ſtill haue to doo with Numbre: this proueth not numbre to be contemptible and vile, but rather right excellent and of hygh reputation:ſyth it is the grounde of al mens affayres,ſo that without it no tale can be tolde, no communication without it can be longe continued,no bargaynyng without it can duely be ended, nor no buſyneſſe, that man hath,iuſtly completed. Theſe commodities(if there were none other) are ſufficient to approue the woorthyneſſe of numbre: But there are other innumerable far paſſyng all theſe,which declare Numbre to excede all praiſe.Wherfore,in all great woorkes,are Clerkes ſo much deſired?Wherfore are Auditours ſo rychely feed ? What cauſeth Geometricians ſo highly enhãced? Why ar Aſtronomers ſo greatly aduaunced ? bycauſe that by numbre ſuche thinges they do fynde,whiche elles ſhulde farre excell mans mynde.Scholar.Verilie ſir if it bee ſo, that theſe men,by numbring their cunninge doo attayne,at whoſe great woorkes moſte men doo wonder,then I ſee well I was muche deceaued, and noumbringe is a more cunnyng thynge then I tooke it to be.

M.If nũbre wer ſo vile a thing as you dyd eſteme it,then nede it not to be vſed ſo much in mens communication. Exclude noumbre and anſwere me to this queſtion. How many yeares olde are you?

Scholar. Mum.

M. Howe many daies in a weke? how many wekes in a yeare? What landes hath your father? How many men doth he kepe? Howe longe is it ſyth you came from hym to me?

Scholar. Mum.

Maſter. So that if numbre wante, you anſwere all by Mummes: How many myle to London?

Scholar. A poke full of plummes.

Maiſter. Why, thus may you ſee what rule Numbre beareth, and that if numbre be lackyng, it maketh men dombe, ſo that to moſte queſtions, they muſt anſwer Mum.

Schol. This is the cauſe ſyr, that I iudged it ſo vyle, bycauſe it is ſo common in talkyng euery while: For plenty is not deintie, as the cõmon ſayeng is.

Maſter. No, nor ſtore is no ſore, perceiue you this? The more common that a thynge is, being nedefully required, the better is the thinge, and the more to bee deſyred. But in numbring as ſome of it is lyght and plaine, ſo the moſte part is difficult, and not eaſy to atteyne. The eaſyer part ſerueth all men in commune, and the other part requyreth ſom learnynge. Wherfore as without numbring a man can doo almoſte nothyng, ſo with the helpe of it, you may attayne to all thyng.

Schollar. Yea ſyr? Why? then it were beſt to learne the arte of numbryng fyrſt of all other learninge, and then a man nede learne no more, yf all other come with it.

Maſter. Naye not ſo, but if it bee firſte learned, then ſhall a manne bee able (I meane) to learne, perceaue, and attayne to other ſciences, which without it, he ſhuld neuer gette.

Sco. I perceaue by youre former wordes that Aſtronomie and Geometrie depende muche of the helpe of numbringe, but that other ſciences, as muſike, phyſike, lawe and Grammar, and ſuche like, haue anye helpe of Arithmetike, I perceaue not.

Ma. I may perceaue your great clerklynes by the ordring of your ſciences, but I will let that paſſe nowe, becauſe it toucheth not the matter yͭ I entend, and I wyl ſhew you howe Arithmetike doth profite in all theſe, ſomewhat groſſely, according to your ſmall vnderſtanding, omittyng other reaſons more ſubſtantiall.

ike.

Fyrſt (as you recken them) Muſike hathe not onely great healpe of Arithmetike, but is made and hathe his perfectnes of it, for all muſike ſtandeth by numbre and proportion.

icke.

And in phyſik, beſide the calculation of Criticall dayes, with other thynges whiche I omytte: How can any man iudge the pulſe rightly that is ignorant of the proportion of numbres?

The

And as for the lawe it is plaine, that the man that is ignoraunt of Arythmetike, is neither meet to be a Iudge neither a Proctour. For howe canne he well underſtande an other mans cauſe appertaining to diſtribution of goodes, or other dettes or ſummes of monye, if hee bee ignorant of Arythmetike? This oftentimes cauſeth right to be hyndred, when the Iudge either delyteth not to heare of a matter that he perceaueth not, either can not iudge it for lacke of underſtandinge: This commeth by the ignorance of Arithmetike.

Gram

Now as for grãmar, me thinketh you ſhuld not doubt in what it nedeth numbre, ſithe you haue learned that Nounes of al ſortes, pronounes, verbs & participles, are diſtinct diuerſly by numbres: byſides ẏ varietie of nounes of numbre, & aduerbes. And if yow take away numbre from grammar, then is all ẏ quantitie of ſyllables loſt. And many other waies dothe numbre helpe grammer. Wherby were all kyndes of meeters found and made? Was it not by numbre?

Phy

But how nedeful Arithmetik is to al partes of Phyloſophy, they may ſone ſe, ẏ readeth either Ariſtotle, Plato, or any other Phyloſophers writing. For all theyr examples almoſte and their probations depende of Arithmetike. It is the ſayinge of Ariſtotle, that hee that is ignorante of Arithmetik, is mete for no ſcience. And Plato his maſter wrote a like ſentence ouer his ſchoolehouſe doore, Let none enter in hither (quoth he) that is ignorant of geometrie. Seeyng he wold haue all his ſcholers expert in Geometry, much rather he wold the ſame in Arithmetik, without whiche, Geometrye can not ſtande.

nitye. And howe needfull Arithmetik is to diuinitye, it appeareth, ſeeyng ſo many doctors gather ſo great miſteries out of numbre, and ſo muche dooe write of it. And yf I ſhuld go about to wryte al the commodities of Arithmetike in ciuylle actes, as in gouernaunce of common weales in time of peace, and in due prouiſion and ordre of armies in tyme of warre: For numbring of the hoſte, ſummynge of their wages, prouiſions of vitayles, vewinge of artillerye, with other armour: Beſyde the cunnyngeſt poynte of all, for caſtinge of groundes for encampynge of men, with ſuche other like. And how manye wayes alſo Arithmetike is conducible for all priuat weales, of Lordes & al poſſeſſioners, of merchauntes and all other occupyers, & generally, for all eſtates of men, beſides auditours, treaſorers, receyuers, ſtewardes, baylyffes, and ſuch lyke, whoſe offyces without Arithmetyke is nothynge. If I ſhoulde (I ſay) particularly repete all ſuche commodities,

dities of thys noble ſcience of Arithmetyke, yt were ynough to make a very great boke.

S. No, no ſyr: you ſhal not nede: for I doubt not, but this that you haue ſayde, were enough to perſwade any man to thynke this art to be ryght excellent and good: and ſo neceſſary for man, that (as I thynke nowe) ſo muche as a man lacketh of it, ſo muche hee lacketh of hys ſence and wytte.

Maſter. What? are you ſo farre chaunged ſince, by hearing the fewe commodities in generall? By likelyhode you wolde be farre chaunged, if you knewe all the commodities particular.

S. I beſeeche you ſyr, reſerue thoſe commodities that reſte yet behinde, vnto their place more conuenient. And if ye wyll be ſo good as to vtter at this time this excellente treaſure, ſo that I may be ſomewhat enriched therby, & if euer I ſhalbe able, I wyll requite your payne.

Maſter. I am very glad of your requeſt, & I wyll doo it ſpedily, ſith that to learne it you be ſo ready.

Sc. And I to your authoritie my wyt do ſubdue, what ſo euer you ſaye, I take it for trewe.

Ma. That is to muche, and meete for no man to be beleued in all thinges, without ſhewyng of reaſon. Though I myght of my Scholar ſome credence require, yet except

cept

cept I ſhewe reaſon, I do it not deſyre. But nowe ſith you are ſo earneſtly ſet this arte to attayne, beſt it is to omitt no tyme, leaſt ſome other paſſion coole this great heate, and then you leaue of before you ſee thende.

uerbe
dye.

St. Though many there be ſo vncōſtant of mynde, that flitter and turne with euery winde, whiche often begyn and neuer come to the ende, I am none of their ſorte, as I truſt you partly know. For by my good wil what I ones begin, till I haue it fullye ended, I wolde neuer blynne.

Maſter. So haue I founde you hytherto in deede, and I truſt you wyll increaſe rather then go backe. For better it were neuer to aſſaye, than to ſhrinke and flee in the middle waye. But I truſte you wyll not ſo do, therefore tell me briefely, what call you the ſcience that you deſire ſo greatly?

Sco. Why ſyr, you knowe.

M. That maketh no matter, I would heare whether you knowe, and therefore I aſke you. For great rebuke it were, to haue ſtudied a ſcience, and yet can not tell how it is named.

Sc. Some call it Arſmetrik, and ſome Augriſme.

Ma. And what doeth thoſe names betoken?

Sco. That yf it pleaſe you, of you woulde I lerne.

Maſter. Bothe names are corruptely

wryten, Arſmetrike for Arithmetike, as the Greekes call it, and Augrim for Algoriſme as Arabians ſoundeit, whiche bothe betoken yͤ ſcience of numbringe. For Arithmos in Greke is called numbre, & of it cometh Arithmetike, the arte of nūbring. So that Arithmetike is a ſcience or art teaching the maner and vſe of numbring. This arte may be wrought dyuerſly, wyth penne or wyth counters. But I will firſt ſhewe you the working with the penne, and then the other in ordre.

Ariſm
tike.
Ariſm

Sco. This I will remembre. But howe many thinges are to bee learned, to attaine this arte fully?

Ma. There are reckened commonly, vij. partes or workes of it.

Numeration, Addition, Subtraction, Multiplication, Diuiſion, Progreſſion, and Extraction of rootes: to theſe, ſome men adde Duplation, Triplation, and Mediation. But as for theſe laſte thre, they are contayned vnder the other ſeuen. For Duplatiō and Triplacion, are contayned vnder Multiplication, as it ſhal appeare in their place. And Mediation is contayned vnder Diuiſion, as I will declare in his place alſo.

Sc. Yet than there remaine the firſt ſeuen kindes of numbryng.

Ma. So there dothe: Howe bee it, If I ſhall ſpeake exactly of partes of numbring, I muſte make but fyue of them: For Progreſſion is a compounde operation of Addition, Multiplication and Diuiſion. And ſo is the extraction of Rootes. But it is no harme to name them as kyndes ſeuerall, ſeing they appeare to haue ſome ſeueral workyng. For it forceth not ſo much to contend for the numbre of them, as for the due knowledge and practiſyng of them.

S. Than you will that I ſhall name them as ſeuen kyndes diſtinct. But now I deſyre you to inſtruct me in the vſe of eche of thẽ.

Ma. So wyll I, but it muſte be doone in ordre, for you maye not learne the laſte as ſoone as the firſt, but you muſt learne them in that ordre as I dyd rehearſe them, if you wyll learne them ſpedily and well.

Sc. Euen as you pleaſe. Than to begyn, Numeration is the firſt in ordre, what ſhall I do with it?

M. Firſt you muſt know what the thyng is, and than after learne the vſe of the ſame.

NVMERATION.

Numeration is the arte to expreſſe and read all ſummes propoſed: and is of two ſorts: for either it gathereth the value of a ſumme propoſed, either elſe it expreſſeth a ſumme cōceued by figures and places due.

Sc. Why? than me thinketh you putte a difference betwen the value and the fygures.

M. Yea ſo do I: For the value is one thyng, and the figures are an other thinge: and that commeth partly by the diuerſity of fygures, but chiefly of the places, wherein they be ſet.

S. Than I muſt knowe here thre things: the Value, the Figure, and the Place.

M. Euen ſo; but yet adde Order to them as the fourth. And fyrſte marke, that there are but ten figures, that are vſed in Arithmetike, and of thoſe ten, one dooth ſignifye nothinge, which is made lyke an o, and is called priuatly a Cyphar, though all the other ſomtyme be likewiſe named. The other nyne are called Signifying figures, and be thus figured.

A Cyphar.

1 2 3 4 5 6 7 8 9

Figu

And this is their valewe.

ij. iij. iiij. v. vj. vij. viij. ix.

But here muſt you marke, that euerye fygure hath two values: One allwaies certain that it ſigifieth properly, whyche it hathe of his forme: and the other vncertaine, whiche he taketh of his place.

lace.

A place is called the ſeat or roume that a fygure ſtandeth in. And looke how many figures are written in one ſumme, ſo manye places hath that whole nũbre. And the firſt place muſt be called that that is next to the right hande, and ſo reckeninge by order towarde the lefte hande: ſo that that place is laſt, that is nexte to the lefte hand. As for example: If there ſtode before you ſix men in a rowe, ſide by ſide, and you ſhoulde tell them as thei ſtand in order, beginning with the man that were next to your right hand, than he that were next him, ſhould bee called the ſecond, and ſo forthe to the fartheſt from your right hand, which is the firſt and the laſte.

S. Syr I perceaue you wel. ſo mighte I recken letters or any other thynge. As if I ſhould wryte, viii. letters after this order, a, b, c, d, e, f, g, h. nowe muſte I ſaye. h. is the firſte, g, the ii, f the iii. e the iiii. d the v. c the vi, b the ſeuenth, and a the eight.

M. That is well done. And after the ſame ſorte vſe hereafter, that what I declare by an other. one example, doo you expreſſe by an other. and ſo I ſhal perceaue whether you vnderſtande ſtand it or no. And ſo paſſe ouer nothinge, tyll you perceaue it well, and bee experte therin.

St. Syr, I pray you howe many of theſe places be there in all?

M. There is no certaine nũbre of theym, but they are ſomtimes more and ſomtimes fewer, accordinge to the ſum that is expreſſed For ſo many as the figures are, ſo many ar the places: and the laſt places ſo called, not bicauſe it is laſt of al other, but it is the laſt of that preſent ſumme, and it maye be the myddle place in an other ſumme.

Scō. Me ſeemeth I perceaue this verye well, as touching the ordre of reckeninge of the places: But as for the numbre of theim, you ſay there is no certainty. Nowe there reſteth to declare the value of the fygures by diuerſity of places, which you called, The valew vncertayne.

Value v
certai

Ma. But firſte let me heare whether you knowe perfitely the certyne valewe.

S. Yes ſyr as you wrote them, ſo I marked them.

M. Howe write you them v?

S. By this figure. 5.

M. And howe vj?

Scolar. Thus. 6.

M. Write theſe three numbres eche by it ſelfe as I ſpeake them. vij, iiij, iij.

Scolar. 7. 4. 3.

M.

Maſt. Howe write you theſe four other, ij, i, ix, viij?

S. Thus (I trowe) 2, 1, 6, 8.

M. Nay, there you miſſe: Looke on mine example againe.

Sco. Syr, troth it is, I was to blame, I tooke 6 for 9, but I wyll be warer herafter.

M. Now then take heede, theſe certayne values euery figure repreſenteth, when it is alone written without other figures ioyned to him. And alſo when it is in the firſt place though many other doo folow: as for example. This figure 9 is ix. ſtanding now alone.

Sc. How, is he alone and ſtandeth in the mydle of ſo many letters?

M. The letters are none of his felowes. And if you were in Fraunce in the middle of M. Frenche men, if there were none Engliſh man with you, you wolde recken your ſelfe to bee alone.

Scho. So it is. Then 9 without more figures of Arythmetike, betokeneth ix, whatſoeuer other letters be about it.

M. Euen ſo. and ſo dothe it, if it be in the fyrſt place ioyned wit other, how manye ſo euer doo folowe, as in this example. 3679. you ſee 9 in the firſt place, and doth betoken nyne, as if he were alone.

Sc. I perceaue that. And dooth not 7 that ſtandeth in the ſeconde place, betoken vij? And ſo and vj in the third place betoken vj? And ſo

3 .In the fourth place, betoken three?

M: Their places be as you haue ſayd, but their values are not ſo. For as in the firſt place euery figure betokeneth his own valu certain only, ſo in ẏ ſecõd place euery figure betokeneth his own value certain x.times; as in ẏ exãple, 7 in ẏ ſecõd place is vij.times x. that is, lxx. And in ẏ third place, euery figure betokeneth his own valew a hundreth tymes, ſo that 6 in ẏ place betokeneth vj.C. And in the .iiij. place euery fygure betokeneth his owne valewe a.M. tymes, as in ẏ foreſaide number: 3 in the fourth place ſtandeth for iij.M. And in the fifth place, euery fygure ſtandeth for his owne valewe x.M. times. And in the vj.place, a.C.M.times. And in the.vij.place, a. M.M.tymes: And in the viij.place, x.M.M.tymes: ſo ẏ euery place exceedeth the former ten tymes.

Sco. As thus, if I make this number at all aduentures, 91359684, here are.viij. places. In the firſt place is 4, & betokeneth but foure: in the ſeconde place, is 8, and betokeneth x.times eight, that is, lxxx. In the thyrde place is 6, and betokeneth vj. C. In the fourth place 9, is ix M. And 5 in the v.place, is x.M.tymes 5, that is l.M. So 3 in the ſixt place, is a C M. tymes 3. that is CCC.M. Then one in the vij. place, is M.M. And 9 in ẏ viij place, x.M.M.times 9, that is xc.M.M. But nowe I canne not eaſily nor quickly read it in ordre.

M. That ſhal you practiſe by this meanes. Fyrſt put a pricke ouer the fourth fygure, and ſo ouer the vij. And if you had ſo many ouer the x, xiij. xvj, and ſo forth, ſtill leauing two fygurs betwene eche two pricks. And thoſe roomes betwene the prickes are called Ternaries.

narye
rie.

Then begin at the laſt pricke, and ſe how many figures are betwene him and the end. whiche can not paſſe thre, reckening hym ſelfe for one, then pronounce them as if they were written alone from the reſt, and adde at the ende of their valewe ſo manye times thouſande as your numbre hath prickes.

After that come to the next thre figures, and ſound them as if they were apart from the reſt, and adde to their valewe ſo manye tymes thouſandes, as there are prickes betwene them & the fyrſt place of your whole nūbre. And ſo do by euery other iij. figures folowing, if you haue mo. As in example, 9 1 3 5 9 6 8 4, thys was your numbre. Put a pricke ouer 9 in the fourth place, and ouer one in the vij. place, and then no more, (for your places come not to tenne) as thus

$9\dot{1}35\dot{9}684.$

Now go to the laſt pricke ouer 1, & take it, and the fygure 9 that foloweth it, and valewe them alone.

Scholar. 9 1 that is xci.

M.

M. So is it: but than adde for the numbre of your prickes twyſe M.

Scholer. That is xii. thouſand thouſand.

M. So is it. Then take the thre other figures from to the next pricke, and valewe them.

Scholer. 359, that is CCC.lix.

M. Nowe adde for the one pricke that is betwene them and the firſt place, M.

Schol. CCClix, thouſande.

M. Then come to the other thre fygures that remaine.

Schol. 684, that is vi C.lxxxiiii.

Maſter. Nowe haue you valewed all. And at the ende of this laſte nũbre you ſhall adde nothing, bycauſe there remayneth no prick nor numbre after it: yet proue in an other numbre, as thus. 230864089105340.

Scholer. 230864089105340. I haue pricked them as you taughte me, but I am in doubt, whether I haue doon wel or no, becauſe of the cyphers: For I remembre, you tolde me that they do ſignifye nothyng, and therfore I doubt whether I ſhuld recken the for a figure in ſetting of the prickes: and againe I knowe not wherfore they ſerue.

M. That will I tell you now. In dede they are of no valewe theym ſelfe, but they ſerue to make vp numbre of places, and ſo maketh the fygure folowing them to bee in a further place, and therfore to ſignifie the more value: as in this example, 90, ŷ cypher is of no value, but yet he occupieth the fyrſte place, and cauſeth 9 to be in the ſecōd place, and ſo to ſignyfie 9. tymes 10, ŷ ys, 90. ſo that two cyphers thruſteth the fygure folowing them into the thirde place, and ſo forth.

S. Than I perceaue in the example aboue, I haue pricked wel inough: for though that cyphar that is pricked ſignifye nothing, yet muſt he haue the pricke, bicauſe he came in the xij. place. Than wyll I proue to numbre that ſumme. Fyrſt there is 230 M, M, M, M, & then foloweth 864 M, M, M. And what ſhall I nowe do? There is a cyphar in the thirde place, and no fygure after hym, but they that I haue reckened.

M. He dyd ſerue for them that you haue all ready reckened, to make them in a place further then thei ſhuld be, if he were away: and therfore now you ſhall let hym go. And ſo do alwaies when he occupieth the place nexte before any pricke, which is the laſte of that Ternary: and a cyphar in ŷ laſte place doth nothinge.

S. Thē I ſhal ſaye but 89 thouſand M.

M. No, but go foorth.

S. 105 thouſand. Now are all my pricks ſpent, and yet remaine 340. ſo that I muſte beleue them CCCxl. only.

M. Now can you reckē after this ſorte.

And remēbꝛe that euery ſuche roume ſo parted, is called a Ternary oꝛ Trinitie.

Som do part ſuche great numbꝛes with letters, after this manner:

c b a c b a c b a c b a c b a
2 3 0 8 6 4 0 8 9 1 0 5 3 4 0. In which example ye may ſe that a ſupplieth the roume of your pricke. And ſome doo parte the nūbꝛes with lynes after this foꝛme,

2 3 0 |8 6 4|0 8 9| 1 0 5 |3 4 0. Where you ſee as many lines as you made prickes, & to one intēt, ſaue yͭ the lines doth moꝛe plainly parte euery thꝛe figures, accoꝛdyng as they ſhould be valewed vnder one Denominaciō.

S. Yea ſyꝛ, but if you ſhould ſhewe me a numbꝛe ſo parted, I ſhould take it foꝛ many numbꝛes, and not foꝛ one.

M. So myght you doo, not knowing my meaning. But what if I did ſet foꝛth the nūbꝛe withoute lynes, and your ſelfe (foꝛ the eaſe of reckening)dyd ſo part it with lynes, wold you foꝛget wherfoꝛe ye did it, & then take them foꝛ many numbꝛes?

S. No I trow not, but yet I doubte.

M. Then vſe that that you lyke beſt, foꝛ al yͤ thꝛe waies are to one entent, ſaue (as I ſayd) that the lynes doth moꝛe plainlye diſtinct the Denominacions.

S. What call you Denominations?

M. It is the laſte valewr oꝛ name added to any ſumme. As when I ſaye: C. Thou.

poundes: pounds is the denomination. And lyke wayes in ſaying: 15 men, men is the denomination, & ſo of other. But in this place (that I ſpake of before) the laſte numbre of euery Ternary, is the denomination of it. As of the firſt ternary the denomination is unites: and of the ſecond ternary, the denomination is thouſandes: and of the thyrde ternary, thouſande thouſandes, or millions: of the iiii; thouſand thouſand thouſands, or thouſand millions: and ſo forth.

S. And what ſhall I call the value of the iii figures that may be pronounced before the denominators? as in figures 203000000, that is, CCiii milliõs, I perceaue by your wordes that myllions is the denomination, but what ſhall I call the CCiii ioyned before the millions?

nerator M. That is called the numerator or valewer, and the whole ſumme that reſulteth of them bothe, is called the Summe, value or numbre.

mm . S. Now is there any thing elſe to be learned in Numeration? or elſe haue I learned it fully?

Maſt. I might here ſhewe you who were the firſt inuentors of this arte, & the reaſons of all theſe things that I haue taught you, but that will I reſerue, tyll yee haue learned ouer all the practiſe of this arte, leaſte I ſhoulde trouble your witte with ouer ma

nye

ny thinges at the firſt.

But yet this muſt you marke, that there are thre kyndes of numbre: one called digets: an other articles: and the third myxte numbres. A diget is any numbre vnder 10, as this: 1, 2, 3, 4, 5, 6, 7, 8, 9. And 10 with all other that may bee diuided into 10 partes iuſte, and nothyng remayne, are called articles, ſuche are 10, 20, 30, 40, 50, &c. 100, 200, &c. 1000. &c. And that numbre is called myxt, that contayneth articles, or at the leaſt one article and a digette: as 12, 16, 19, 28, 38, 107, 1095. and ſo foorth. And for the more eaſe of vnderſtandinge and remembraunce, marke this: The diget numbre is neuer written with more then one figure, but the article and the myxt numbre is euer written wyth more then one figure: and thus they differ, y^t the article hath euermore thys cyphre 0, in hys fyrſte place: and the myxt numbre hath euer there ſome digette.

Thre
des o
bre.

Dig

Myx

Artic

Sco. By theſe laſt woordes, I perceaue it muche better than I dyd before, and now (I thinke) I will neuer miſſe to knowe thoſe three a ſunder.

M. If you remembre now all that I haue ſayde, you haue learned ſufficiently this firſt kinde of Arithmetike, called Numeration. How be it, I will yet exhort you now, to remembre both this that I haue ſayde, and all that I ſhall ſay, and to exerciſe your witte in the practiſe of it: for rules without practiſe are but a light knowledge: and practiſe it is, that maketh men perfect and prompte in all thinges.

And as you haue learned to gather the valewe of a ſumme propoſed, ſo muſt you practiſe to expreſſe any ſumme with their due figures: as for a profe. Now expreſſe you this ſumme: Fiue thouſand, twoo hundred, fiftie and ſeauen?

Sco. Thys troubleth me now, whether I ſhoulde begynne at the fyrſte fygure or at the laſte. For reaſon (me thynketh) ſhoulde cauſe me to begyn at the fyrſt: and yet yf I wryte it as you ſpake it, I muſt begyn at the laſte.

Ma. When you knowe your places perfectely, you may begynne wher you lyſt. But the more eaſe for your hande is to begynne with the laſte, that is to ſay, as I dyd ſpeake them. Yet for the more ſuretye, a whyle you may begynne with the fyrſte, repetinge my wordes backward, thus: Seuen, fyftye, two hundred, fyue thouſande: or elſe ſounding them all by theyr dygit or valower, as thus: Seuen, fyue, twoo, fyue: for that waye is eaſieſt. But than muſt you looke well, whether ther be any cypher in your ſumme, that he maye bee ſette in hys place. As if youre laſte valewer of your ſumme (as you ſpeake it) bee aboue 9, than is there a cyphar in the fyrſt place. And if it be a hundred or aboue, than is there twoo cyphars, one in the fyrſt place, and an other in the ſeconde, and ſo foorthe.

But bycauſe thys thynge is ſuche, that canne not bee ſette foorthe withoute manye woordes, I thynke beſt heere now at the ende of Numeration, to adde a table, eaſye and ready for the fyrſt perceiuyng of it.

Lo, this is that table.

The right syde or hande.

The names of the digettes, values certaine, or valowers.

The denominatours of the places, or valewe vncertaine.	Nyne.	Eyght.	Seuen.	Syxe.	Fyue.	Foure.	Three.	Twoo.	One.	cyphar.	The ordre of the places.
Unites.	9	8	7	6	5	4	3	2	1	0	Fyrste.
Tennes.	9	8	7	6	5	4	3	2	1	0	Seconde.
Hundredes.	9	8	7	6	5	4	3	2	1	0	Thyrde.
Thousandes.	9	8	7	6	5	4	3	2	1	0	Fourth.
x. thousandes.	9	8	7	6	5	4	3	2	1	0	Fyfte.
C. of thousandes.	9	8	7	6	5	4	3	2	1	0	Syxte.
Millions.	9	8	7	6	5	4	3	2	1	0	Seuenth.
x. of millions.	9	8	7	6	5	4	3	2	1	0	Eyghteth.
C. of millions.	9	8	7	6	5	4	3	2	1	0	Nynthe.
M. of millions.	9	8	7	6	5	4	3	2	1	0	Tenthe.
x. M. of milli.	9	8	7	6	5	4	3	2	1	0	eleuenth.

This table (as you maye see) hath aleuen places, and in eche of them are set all the digets, whose certaine valewe is written on the righte hande of the table, and the valew vncertaine on the lefte hande. [illegible] this thys table you maye learne bothe how to expreſſe any numbre that you lyſt, (yf that it excede not .xi. places) that is to ſaye, thre thouſand myllions, and ſo maye you by the helpe of it, valewe all ſummes propoſed vnder the ſayd numbre.

For example: Take the ſumme that I propoſed before, which was, fyue thouſand, two hundred, ſyxtye and ſeuen. And yf you wyll expreſſe it, take ỹ fyrſt nũbre (as I ſpeake it) which is fyue M. whoſe valower or certain valewe is v. and his vncertain valewe or denomination is M. Fyrſt you ſhall ſeke at the right hande the valower v. Than ſeke alonge vnder the title of denomination toward the lefte hande, tyl ye find thouſandes, and vnder it right at the foote of the table, is the nũbre of the place that is the fourth, wherein you muſte wryte your digitte or valower, fyue.

Afterward cometh the ſecond parte of the numbre two hundred, whoſe valower is .ij. and his denomination C. Seeke .ij. at the righte hande of the table, and go alonge vnder the denominations, towarde the lefte hande, tyll you come vnder C. than looke to the foote of the table, and there ſhall you ſee the numbre of the place, that is to ſaye, the thirde, wherein you muſt ſet your digitte ij. Than do ſo by your other two numbres that remaine, and you ſhall fynde them in the ſeconde place for your fyfty, and 7 in the fyrſte place for your ſeuen. And thus maye you do with other numbres.

S. Mayſter I thanke you hartely. I perceiue you ſeeke to inſtructe me moſte plainly & briefly, and not to hyde your knowledg ẘ ſubtile wordes as many do. For this rule is ſo playne, that I canne deſire it no plainer. And though it ſeeme ſomwhat longe, yet I perceaue it to be a ſure waye.

M. So is it. and though it be longe, yet it is neyther to longe, neyther to playne for yonge learners that lacke practyſe: for thys table is in ſteede of a teacher, to theym that lacke one. But nowe I truſte I haue ſayde ynough of Numeration: whyche after you haue well practyſed, then maye you learne forther.

y num
r pla
ckes

S. Yet I praye you in one thynge to tell me your iudgment. Why do men recken the ordre of the places backward, frō the righte hande to the lefte?

M. In that thinge all men do agree, that the Chaldays, whiche fyrſte inuented thys arte, did ſet theſe figures as thei ſet all their letters. for they wryte backwarde as you tearme it, and ſo doo they reade. And that may appeare in all Hebrewe, Chaldaye and Arabike bookes. for they be not only wrytten from the ryght hande to the lefte, and ſo muſte bee readde, but alſo the ryght ende of the booke is the beginninge of it: where as the Greekes, Latines, and all nations of Europe, do wryte and reade from the lefte hand towarde the ryght. And al theyr bokes begyn at the lefte ſyde.

S. That reaſon doth ſatiſfie me.

M. It nother ſatiſfieth me, nother liketh me weil, bicauſe I ſee that the Caldays and Hebrues do not ſo vſe their owne numbres, as at an other tyme I wyll declare. But thys playne reaſon maye beſt ſatiſfye you preſentlye: That ſeeyng in pronouncinge of numbres wee keepe the ordre of our owne readynge, from the lefte hande to the ryght: And againe, we doo euer name the greater numbres before the ſmaller: it was reaſon, that the leſſer places conteynynge the leſſer numbres, ſhould be ſette on the right hand, and the greater places conteynyng the greater numbres, to procede towarde the lefte hande.

S. This reaſon to me is ſo playne, that it ſeemeth now agaynſt reaſõ to make a doubt of that ordre. So that now for Numeratiõ I am cleerly ſatiſfyed: ſo that only practiſe ſhall make me fully reddy and experte in it. And in the mean ceaſon, I deſyre to learne the other kyndes of Arithmetyke.

M. That is well ſayd: but what ſhoulde you next learne, can you tell?

S. I remembre, you ſayde that Addition

was nexte.

Maſter. Euen ſo: and what that is, muſt you fyrſte knowe.

ADDITION.

Addition is the reduction and bringing of two ſummes or more into one: as if I haue 160 bookes in the Latyne tongue, and 136 in the Greeke tong, and woulde knowe howe many they bee in all. I muſte write thoſe, ij, numbres one ouer an other, writyng the greateſt numbre higheſt, ſo that the fyrſt fygure of the one, bee vnder the firſt figure of the other: And the ſeconde vnder the ſeconde, and ſo foorth in ordre.

When you haue ſo done, draw vnder them a right lyne, then wyll they ſtande thus.

160
136

Now begyn at the firſt places towarde the righte hande alwayes, & put togyther the ij, firſt fygures of thoſe two ſummes, and loke what cometh of them, write vnder thē, right vnder the line, As in ſaiyng, 6 and 0 is 6: Wryte 6 vnder 6: as thus.

160
136
6

And then go to the ſeconde fygures, and do likewiſe: as in ſaiyng, 3 and 6 is 9, write 9 vnder 6 and 3, as here you ſee.

160
136
96

And lyke wayes do you with the figures that be in the thirde place, ſaying: 1 and 1 bee 2; write 2 under them, & then will your whole ſum appeare thus.

160
136
296

So that nowe you ſee, that 160, and 136, do make in all, 296.

S. What? this is very eaſy to do: me thinketh I can do it euen ſyth.

There came throughe Cheapes ſyde two droues of cattell, in the firſt was 848 ſheep, and in the ſeconde was 186 other beaſtes.

848
186

Thoſe two ſummes I muſt write as you taught me, thus.

Than if I putte the two fyrſt fygures togither, ſaying: 6 and 8 they make 14. That muſt I write under 6 and 8, thus.

848
186
14

M. Not ſo. and here are you twiſe deceaued. Fyrſt, in goyng about to adde togither two ſummes of ſundry thynges, whych you ought not to doo, except you ſeeke only the numbre of thē, and care not for the thinges. For the ſumme that ſhould reſult of that addition, ſhould bee a ſumme nother of ſheepe nor other beaſtes, but a confuſed ſumme of bothe.

Howe be it ſometymes ye ſhall haue ſummes of dyuers denominations to bee added, of whiche I wyll tell you anone; but fyrſte

I wyll ſhew you, where you were deceiued in an other pointe, and that was in writing 14, whiche came of 6 and 8, vnder 6 and 8, whiche is vnpoſſible: For howe can two fygures of two places be written vnder one fygure and one place?

Sc. Truthe it is: but yet I dyd ſo vnderſtande you.

M. I ſayde in deede that you ſhould write that vnder them, that dyd reſulte of theym both togyther: whyche ſayinge is alwayes trew, if that ſumme do not exceede a Diget. But if it be a Myxt numbre, then muſt you wryte the diget of it vnder your figures, as I haue ſayde before: but and if it be an Artycle, than wryte 0 vnder them, and in bothe ſortes you ſhall kepe the Article in your mynd. And therefore when you haue added your ſecond figures, which occupy the place of tennes, you ſhall put that 1 therto, which you kepte in your mynde. for though it were ten in deed, yet in that place it is but as one, bicauſe that euery 1 of that place is tenne, for it is the place of tennes. And in lyke maner if you haue in the ſecond place ſo great a numbre, that it amounteth aboue 9, than write the digites, and reſerue the article in your mynde, euer adding it to the next place followinge, and ſo of all other places, howe many ſo euer you haue. And if you haue a Myxte numbre, when you haue added your laſte figures, then write the digite vnder the laſte fygures, and the article in the nexte place beyonde them: ſo ſhall your numbre reſulting of Addition, haue one place more than the numbres whyche you ſhould adde togyther.

S. Now doo I perceaue you. and the reaſon of this is, (as I vnderſtand) bicauſe that no one place can containe aboue 9, whyche is the greateſt fygure that is, & then all tennes or articles muſt be put to the nexte place folowinge. for euery place (as I may ſee) excedeth the other place nexte before him, by x.

Now (if it pleaſe you) I wyll returne to my example of cattel. But I remembre you ſayd, I might not adde ſummes of ſundrye thinges togither, and that myght I ſee by reaſon.

M. Trouth it is, if you ſeke ẏ due ſū of any things. but if you only ſeke a bare ſum, and haue no reſpecte to ẏ thyng, thā wer it better to name ẏ ſūe only wout any thing, as in ſaing 848, wout naming ſheepe or any thing elſe. And likewaies 186, naming nothing.

Nowe lette me ſee: howe caune you adde thoſe twoo ſummes.

S. I muſt fyrſt ſet them ſo, that the twoo fyrſt fygures ſtande one ouer an other, and the other eche one ouer his fellowe of the ſame place. than ſhall I drawe a lyne vnder the both. And ſo lykewayes of other figures, ſettinge alwaies the greateſt numbre higheſt, thus.

8 4 8
6 8 6

Than muſt I adde 6 to 8, whiche maketh 14, that is a myxte numbre: therfore muſt I take the dyget which is 4, and write it under 6 and 8, keping the article 1 in my mynde, thus.

8 4 8
4 8 6
4

Next that do I come to the ſecond figures, adding them togyther, ſaying: 8 and 4 make 12, to whyche I put the one reſerued in my mynde, and that maketh 13, of whiche numbre I write the diget 3 under 8 & 4, and kepe the article in my mynde, thus.

8 4 8
1 8 6
3 4

Than come I to the third figures ſaying: 1 and 8 make 9, and 1 in my mynde maketh 10. Syr shal I wryte the cyphar under 1 and 8?

Maſter. Yea.

Sch. Than of 10 I write the cyphar under 1 and 8, & kepe the article in my mynd.

M. What needeth that, ſeeing there foloweth no more fygures?

S. Syr, I had forgotten: but I will remembre better herafter. Then ſeeyng I am come to the laſte fygures, I muſte write the cypher under them, and the article in a farther place after the cyphar, thus.

8 4 8
1 8 6
1 0 3 4

Ma. So now ye ſe that of 848, and 186 added togither, there amounteth 1034.

648
186

1034.

S. Nowe I thinke I am perfit in Addition.

M. That will I proue by this example.

There are twoo armies of ſouldiours, in the one are 106800, and in the other 9400: Howe many are there in bothe armies ſaye you?

S. Fyrſt I ſet them one ouer an other, beginning with the firſt nũbres at the right hand, thus.

```
106800
  9400
------
```

But the nether nũbre will not matche the ouer numbre.

M. That forceth not.

S. Than doo I adde 0 to 0, and there amounteth 0, that muſt I wryte vnder the fyrſte place, thus.

```
106800
  9400
------
     0
```

M. Well ſayde.

S. Then lykewayes in the ſeconde place I adde 0 to 0, & there aryſeth 0, whiche I wryte vnder the ſeconde place, thus.

```
106800
  9400
------
    00
```

Than I come to the thirde place, ſaying: 4 & 8 make 12, of whiche I wryte the digite 2, and kepe the article 1 in my mynde, thus.

```
106800
  9400
------
   200
```

Then adde I 9 to 6, whiche maketh 1[illegible] to that I adde the article 1 that was in my mynde, and it is 1[illegible] wryte 6 vnder 6 and 9, and kepe one in [illegible]

D.i.

thus.

106800
9400
6200

Ma. Why do you not wryte bothe fygures, ſeyng you are come to the laſte couple of numbres?

S. Naye, reaſon ſheweth me that I muſte adde that article that is in my mynde, vnto the nexte fygure of the ouer ſumme, though there bee no more in the nether ſumme.

M. That is well conſydered: then doo ſo.

S. Then ſaye I, 0 in the ouer ſumme and 1 in my mynde make 1, that I wryte vnder 0: Then foloweth there two in the ouer ſum whiche hath none to be added to it, for there is none in the nether ſumme, nor yet in my mynde, therefore I thinke I muſte wryte that euen as it is.

M. Yea.

Sch. Then doth my whole ſumme appeare thus.

106800
9400
116200

M. Yf you marke this, you haue learned perfitely the common addition of all ſummes whiche are of one denomination, ſo that ye obſerue this alſo, that in Addition you muſte haue two numbres at the leaſte, or elſe howe can you ſay that you do adde? And euer lette the greateſt numbre bee written hygheſt, for that is the beſte waye though it be not neceſſary.

And forget not this, that yf you haue many numbres to adde togither, you ſhall haue often oftentymes an Article of a greater valewe then 10: ſomtymes 20, ſometymes 30, ſome times more, yea peraduenture, 100. Therefore as you dyd with the article 10, ſo doo with them, reſeruyng them in your mynde, and adding to the numbre nexte folowing ſo manye, as their valewe or valewe certaine is: that is to ſaye, 2 for 20, 3 for 30, and ſo forth of other. But if the article bee 100, then muſte you not adde the article to the nexte fygures folowinge, but to the third figures from them, as I will ſhew you anon by example. And if it chaunce the numbre to be ſuche that it do comprehende two ſundry articles, (that is one of tennes, and another of hundredes) then muſte you reſerue them bothe in your mynde, and adde the article of tennes to the fygures that folowe nexte, and the article of hundredes, to the fygure of the thirde place from thence.

Nowe take this example for all. I wold adde theſe xij. ſummes in one, whiche I ſette after this manner. Then do I begyn and gather the ſumme of the firſte figures, whiche commeth to 107. For firſt I take 9 there x tymes and that is 90: then 9 and 8 is 17 that is in all 107. of whiche

4589
4599
5299
3699
1899
4090
1099
3199
299
699
899
499
889

ſum I write the 7 vnder the firſte figures, & than haue I an article of an hundred in my mynde, whyche other I muſte keepe in my mynde, tyll I come to the thyrde fygures, which are in the rowmes of hundreds, or els I maye for feare of forgettyng write one vnder the thyrde rowe of fygures, makinge two lynes as you ſee heere doone. And then muſte I wryte the digets vnder the loweſt lyne: and this is the ſureſt waye, when the ſumme is ſo great, that the addition of one rowe paſſeth 100.

$$\begin{array}{r}
4889\\
4599\\
2299\\
3699\\
4389\\
4999\\
1099\\
5198\\
299\\
699\\
899\\
499\\
389\\
\hline
11\\
1\\
\hline
57
\end{array}$$

When I haue ſo done, I muſt than come to the ſeconde rowe of fygures, and adde them togither, whiche doth make 115, of whiche ſumme I wryte the digite 5, vnder the ſame ſeconde rowe, and then I haue an artycle remayning of twoo fygures, of whiche the one (that ſtandeth for 10) muſte be added to the ſecond or nexte place after theym that I did laſte adde. And the other (that ſtandeth for 100) muſt bee added to the thyrde place from thence.

S. That is to ſaya, the fourth place frome the fyrſte lyne or rowe of fygures.

Ma. Euen ſo. And then wyll the ſumme appeare thus. Then adde the thyrd rowe of fygures, with the twoo vnites betwene the lynes, and the ſumme amounteth to 50: of whiche I write the cypher vnder the ſame third row, & the 5 vnder yͤ next fygures toward the lefte hande.

Then I adde the fygures of the fourth rowe, with the 1 and 5 that are vnder them betwene the twoo lynes, and they make 29: then write I the 9 (that is the diget) vnder the forth place, and the 2 that is the article beyond it, toward the lefte hande. So thoſe ſums do make 29057.

4889
4599
2299
3699
2399
4090
1099
5198
299
699
899
499
389
11
51
29057

Sco. This ſeemeth ſomewhat harde, by the reaſon of ſo many numbres togither. Howbe it I thinke if I do often proue euen with this ſame example, I ſhall be able to do ſo ſhortly with any other ſumme.

M. So ſhall you. For it is often practiſe that maketh a man quicke and rype in all thinges: But bycauſe of ſuch great ſummes in which there may chaunce to be ſome errour, I will teache you how you ſhal proue whether you haue doone well or no.

S. That were a great helpe and eaſe.

M. Begyn fyrſt with the higheſt numbre,

and than all the other orderly, and adde thē togither, not hauing regard to their places, but as though they were all vnites: And ſtill as your numbre encreaſeth aboue 9, caſt away 9. Then go forthe, euer caſting awaye 9, as often as it amounteth therto: And ſo doo till you haue gone ouer all the numbres that you entended firſt to adde, and what ſoeuer remayneth after ſuche addition and caſtyng away of 9, write it in ſome voyd place by the ende of a lyne, for the better remembrance, and then putte togither the fygures that reſulte of the addition, ſtyll caſtynge away 9 alſo. And then that that remayneth, wryte at the other ende of that line: and yf thoſe twoo figures bee like, then haue you well doone by lykelyhod: but if they be vnlike, then haue you miſſed. As for example in this preſent ſumme: The fyrſt fygure of the ouer lyne is 9, let him go: then 8 and 8 is 16, take awaye 9 and there remayneth 7, adde to it 4 that foloweth, and that maketh 11, from whiche if you take 9, there reſteth 2. Then come to the nexte rowe, whoſe firſt and ſeconde numbre are 9, therefore ouerpaſſe theym bothe, and take the 5 to the 2, whiche dyd remayne in the fyrſt rowe, that maketh 7, put thereto the 4 folowinge, that maketh 11, thence take 9, and there remayneth 2. Next that go to the third line, whoſe two firſt numbres you maye lette paſſe, bycauſe

cauſe they are nynes: then take the twoo 2, which with the other two that remayned in the ſeconde rowe, make 6. Then go to the fowerth rowe, whoſe two fyrſt numbres let go, and take the 6 to the 6 that remayned, & that maketh 12: take awaye 9, and there reſteth 3, whiche with the 3 that is nexte, maketh 6: And ſo go throughe all the other numbres, and you ſhall fynde that there remayneth 5, after you haue caſte awaye 9 as often as you fynde it: therefore wryte 5 at one ende of a lyne in a voyde place, thus.

5 ————

Then gather all the figures of the totall ſūe which is vnder the loweſt lyne, and caſte awaye 9 as often as you fynde it, as thus. 7 and 5 make 12, take away nine, and there reſteth 3: to that if you adde the 2 that is laſt (for you may let go the 9) thē dothe it make 5, whych you muſte write at the other ende of the lyne that you made in the voyd place, and it wyll be thus. 5 ———— 5.

And thā you ſe yͤ thoſe two fygures be like, wherby you maye know that you haue done well. and ſo may you proue in any other.

Scho. If it pleaſe you, I wyll proue in an other ſumme.

Maſter. With a good wyll.

Scho. Then wyll I take one of your former examples, which was thys.

Fyrſt in the higheſt lyne 8 and 6 make 14, than 9 taken away, ther remaine 5, to whiche I adde the 1 that foloweth, and that maketh 6. Then come I to the ſecond line, where I fynde firſte 4, whiche with 6, maketh 10, from that I take 9, and there reſteth 1; the next fygure is 9, & therfore I let him alone, ſo fynde I one remaining, whiche I ſet at the ende of a lyne thus, 1 ———

106800
9499
116299

Than I come to the total ſūm, and ther I finde that al the fygures put togither, make 10, from whiche I take 9, and there reſteth 1 alſo, whiche I put at the other end of the line thus, 1 ——— 1

And bycauſe they be lyke, I know that I haue well added.

Maſt. So you know now bothe how to adde two ſummes or more togither: And alſo how to proue whether you haue done well or no: whiche thynge alſo you may do by Subtraction. But becauſe you can not yet ſkyll of it, I wyll let that paſſe tyll anone, & wyll teache you now how to adde ſummes of dyuers denominations: whyche thynge can neuer be, but whē yͤ one denominatiō is ſuch, that it containeth the other certaine tymes. And yet you ſhall adde them to the other, not after thys ſorte as you dyd them that were of one denomination, but after ſuche a ſorte as I wyll nowe ſhewe you, that

dition
whien
dyuers
nomina
ns.

is to ſaye.

Yf you haue a ſumme of diuers denominations, then loke that ye ſet euery denomination by him ſelfe with ſome note or fygure of hys denomination, as they be wonte to bee written: Then write your other ſummes ſo vnder that fyrſt, that euery one be ſet vnder the other of the ſame denominations. As for example: Yf your denominations be poūds, ſhillinges, and pens, write poundes vnder poundes, ſhyllinges vnder ſhyllings, & pens vnder penys: and not ſhillinges vnder pens, nor pens vnder poundes.

Scholar. Now that you haue ſpoken it, me thinketh it needed not to warne me of it, for it were agaynſt reaſon ſo to confounde ſummes: but yet if you had not ſpoken of it, peraduenture I ſhould haue bene deceiued in it.

10 01

Maſter. If you doe ſay it is ſo playne, I wyll ſpeake no more of it, but with an example make the matter to appeare euidentlye. Fyrſte one man oweth me 22 li, 6 s. 8 d. An other oweth me 5 li. 16 s, 6 d. And another oweth me 4. li. 3 s. I wold know what this is all togither. Therefore muſt I fyrſte ſet downe my greateſt ſumme, and then the other, euery one vnder his denomination, agreynge to the greateſt ſumme, as here you ſee.

Then muſte I begyn at the ſmalleſt numbres, (which muſt alwaies bee ſet nexte the ryghte hande) and adde them togyther. And if the ſumme of them wyll make one of the nexte denomination, then muſte I keepe it in my minde tyll I come to that place, or elſe for more eaſyneſſe, wryte it vnder that place betwene the double lyne, & vnder that fyrſte place muſte I note the reſidue, if there remaine any of the ſame denomination. but if there remaine none, then need I to write vnder it nothinge. And this is all that you muſt marke in this Addition: for all other thynges are lyke to the other manner of addition before mentioned. Therfore the chiefeſt pointe of this Addition is, to knowe the valewes of cõmon coynes & rated ſummes. As how many ſhyllings be in a pound, how many pennies in a ſhillinge. Of whiche and of other lyke thinges, I will inſtructe you hereafter, in teachynge of Reduction. But nowe I may not diſturbe your witte frome the thynge that we are about.

li.	s.	d.
22	6	8
5	16	6
4	3	

Therfore let vs retourne to that former example, which I propoſed of three detters, whiche ſummes when I hadde ſet ordrely, they ſtoode thus, with a double lyne vnder them.

li.	s.	d.
22	6	8
5	16	6
4	3	
32	6	2

Then to adde them vnto one ſumme, I muſt begyn at the righte hande, where the ſmalleſt denomination is, and adde them togither firſt, ſaying: 6 and 8 make 14, nowe ſeeynge theſe 14 are pennies, and that 12 pence make one ſhyllyng, which is the nexte valewe, I take awaye 12 from 14, & there reſteth 2, whiche I write vnder the pennies and for the other 12, which make 1 ſhilling, I write 1 vnder the tytle of ſhyllinges, thus.

li.	s.	d.
22	6	8
5	16	6
4	3	
	1	
		2

Than doo I adde all the ſhyllynges togyther, and find them 25, to which I adde the 1 betwene the two lynes, and that maketh 26, but bicauſe that 20 ſhyllinges doo make 1 pounde, I take awaye 20 from 26, and for that 20, I write 1 vnder the poundes betwene the two lynes, & the other 6 that remayneth, I write vnder the ſhyllings, thus.

li.	s.	d.
22	6	8
5	16	6
4	3	
1	1	
	6	2

Than come I to the poundes addyng theim all togyther, and fynde them to bee 31: therto I adde the 1 betwene the ij lines, & that maketh 32, whiche ſum I write downe whole, bicauſe there reſteth no greater denominatiō & the my whole ſumme appeareth thus.

So is my totall ſumme 32 li. 6 s. 2 d. And this may you proue in an other lyke ſumme.

li.	s.	d.
22	6	8
5	16	6
4	3	
1	1	
32	6	2

S. Then wyll I caſte the whole charge of one monethes commons at Oxford with battelyng alſo.

Maſter. Go to, let me ſee how you can doo.

S. One wekes cõmons was 11 d. ob. q̃. and my battelyng that weke was 2 d. q̃. q. The ſeconde weekes cõmens was 12 d. and my batlyng 3 d. The third wekes cõmõs 10 d. ob. & my batling 2 d. q. c. The fourth wekes cõmõs 11 d. q̃. & my batling 1 d. ob. c. Theſe 8 ſũmes wold I adde into one whole ſumme, and therfore I wyll ſet them one ouer an other, thus. But I had forgotten, I ſhuld haue ſet the greateſt ſum higheſt.

d.		
11	ob.	q̃
2		q̃. q
12		
3		
10	ob.	
2		q. c.
11		q̃
1	ob.	c.
	4	[illegible]
	[illegible]	[illegible]

M. So is it cõmonly beſt, how be it here it forceth not: and in ſuche ſũmes as thys is, that go by ordre of wekes, daies, or yeares, it is better to keepe that ordre, then to alter them, and to ſet the greateſt nũbre higheſt, for that ſerueth for ſuche ſummes as go not by ordre.

Scholar.

Scholer. Then if I haue ſet them well ynough, I will begyn to adde them thus.

d.	ob.	q̄	q	c.
11	ob.	q̄		
2		q̄.	q	
12				
3				
10	ob.			
2			q.	c
11		q̄		
1	ob.			c.
			q	

Fyrſt of the ſmalleſt valewes at the right hand, whiche are called cees, I fynde 2, and ſeeyng that 2 cees, doo make one q, I wyll write nothing vnder the cees, but will write 1 q for 2 cees, vnder the kewes betwene the lynes, after this maner.

Than come I to the nexte valewes, where I fynde 2 q, and to them I adde the q, that is betwene the lynes, and ſo are they 3 q, but bicauſe 2 q, maketh one q̄, I write one q̄ vnder the farthings betwen the lines, & the q that remaineth, muſt I write beneth the nethermoſt line, vnder the kewes, thus.

d.	ob.	q̄	q	c.
11	ob	q̄		
2		q̄.	q.	
12				
3				
10	ob			
2			q.	c
11		q̄.		
1	ob			c
		q̄.	q.	
			q	

Then come I to the farthynges, where I fynde 3, and the other q̄ that is betweene the lynes, maketh 4 farthinges: And bicauſe 4 q̄ make iuſt 1 peny, I ſhall write nothing vnder the farthinges, but muſte write 1 vnder the pence, betwene the

the lynes.

Next that muſt I adde the halfepence togither, of whiche there are 3, but ſeeing that 2 ob. make 1 d, I muſt write 1 vnder ẏ pẽs betwene the lines: but how ſhall I do it, for there is 1 already?

M. Haue you forgotten how I did in addition of the great ſumme before? you muſte ſet it vnder the other, ſo ſhall they both ſtād for 2, for yf you ſhould ſet it before or behind the other, they ſhuld make 11.

Sco. I remembre it now, and I perceaue the reaſon. Then I will write 1 ob. vnder the halfepens, and for the other twoo halfepence, whiche make 1 d, I write 1 vnder the pens: Then come I to the pens, & finde that there are of them 52, thē put I to them the 1 betwene the lines, and that maketh 540 which amounteth to 4 s, 6 d: the 6 d. I muſt write vnder the pens, & the 4 s I muſte ſette (I ſuppoſe) farther towarde the lefte hande by theym ſelfe.

M. Euen ſo.

Schollar. Than appereth all my additiō, thus. And the ſumme is 4 s 6 d ob. q.

	d.		
	1	ob.	q.
	2		q.
	12		
	3		
	10	ob.	
	2		q. 6.
	11		q.
	1	ob.	c.
	1		q. q.
	2		
4. s	6	ob.	q.

M. Now haue you done this well. But tel me, why dyd you wryte kewe, cee, thus: q, c, & not rather thus qᶜ, as the faſhion is?

Scho. Becauſe I thought it was yͤ beſt way for due gathering of euery denomination by him ſelfe.

M. So was it in deede. Wel now, can you tell how to proue this addition, and ſuche other lyke of dyuers denominations, and to trie whether you haue doone well or no?

Sco. I would I coulde.

Ma. That ſhall you doo by thys meanes: Fyrſt as you did begyn to adde, ſo reckon agayne euery denominatiō by it ſelfe, & when you fynde ſo many ſmall that doo make any other denomination, let them go, and kepe in mynde onely the reaſidue that will make no greater denominatiō, and loke whether there be any ſuche lyke valewe vnder yͤ nether lyne, and if ther be, you haue wel done. and ſo go forth frome one denomination to another, vnto the ende.

But here muſt you note, that in gatheringe of the ſummes ye muſt reckon thoſe figures that are written betwene the lynes wͭ them that are written aboue theym. as for an example. I wyll examine that ſumme that I did laſt adde, which ſtode thus.

li.	ſ.	d.
22	6	8
5	16	6
4	3	
1	1	
32	6	2

Fyrſt I finde 6 and 8 whyche maketh 14, from whyche I take 12, bicauſe it maketh one of the nexte denomination, and there remaineth 1, and vnder that place I ſee a lyke fygure, therefore I knowe that to bee well doone.

Then come I to the d. where I fynde 1, 3, 16, and 6, that maketh 26, I caſt awaye 20, for they make an other denomination, that is to ſay, poundes: and the 6 whiche remaineth is lyke to the 6 that is written vnder them beneth the loweſt lyne, therefore that is well done alſo. And thence I go to poundes, where I fynde 1, 4, 5, 22, that is 32, to whiche ſumme agreeth an other lyke vnder it. Therfore I iudge all well done.

Scho. I perceyue reaſon in this probation. Nowe wyll I attempte the ſame in the ſumme that I dyd adde, whyche whan I had ended addyng, ſtoode thus.

Fyrſt amongeſt the cees I fynde but two, whiche make 1 q, euen, therefore there muſte nothinge bee vnder the lyne for theym: and amongeſt the kewes I fynde 3, of whiche two make 1 q̄, therfore I lette them go, and the one q, that is lefte, hathe an other like vnder his place, therfore that is well done.

Than the farthinges are iuſte 4, whiche make 1 d,

d				
1 1	ob.	q̄.		
2		q̄.	q.	
1 2				
3				
1 0	ob.			
2			q.	c
1 1		q̄.		
1	ob.			c
1		q̄.	q.	
2				
4 s̄ 6.	ob.		q.	

1 d, and therefore I let them go: Amongeſt the halfe pence there is one odde (for 2 muſt I caſt away, bicauſe they made 1 peny) and vnto it anſwereth a lyke ſumme vnder it. The pence are 54, from which I take away 48, that make 4 ß, & the 6 remaynyng agree to a lyke fygure ſet vnder them, and laſte of all, remaineth the 4 ß, whiche the abiected pence dyd make: ſo I perceaue that I haue well doone. Nowe this wyll I not forgette. But wyl thys examination ſerue in all Addition?

Ma. It ſerueth for all Addition of ſundry denominations, if the additiõ be made with two lynes, (as theſe were) elſe it wyll not ſerue, bicauſe that thoſe ſummes which are heere added betwene the lynes in addition by one lyne, are vnderſtand and not written. but I let that wayes paſſe, bycauſe as it is common, ſo is it more deceauable then thys waies, namely if a mans memory be either dulle, other troubled.

Scholar. Yet it were good to knowe that wayes alſo.

M. If you deſyre to know it, this it is in fewe wordes: Do euery thynge as you dyd in this ſorte of Addition, ſaue that where you made here twoo lynes, you ſhall make there but one: and thoſe ſummes that you did here write betwene the lynes, you muſt keepe in youre memorie, & theym (as you dyd here)

An o
ſome
Addi

eche

eche one whan you come to his place.

Sco. Than thei differ not but in this, that this additiō with two lynes leueth nothing to memory, but writeth doune all: and the other way committeth certaine numbres to memory, as you taught me in the fyrſte exāples of Addition of ſmall ſummes of one denomination. But what yf a man vſe it (as you ſaye men do commonly) how ſhall it bee examyned?

Maſt. Seeing you are ſo deſyrous of it, I wyll ſhewe both an example of the addition, and alſo the manner to examyne yt. I propoſe theſe iij. ſummes to be added, and I gather fyrſte the pens, as I did in the other ſorte, & I fynde of them 8, 3, 9, that is 20, of whiche ſumme I bate away 12, which make 1 s. and keepe that 1 in my mynde, and the reſt, that is 8, I write vnder the pens. Than do I adde the ſhyllyngs togyther, & fynde of theym 6, 7, 8, that is 21, whereof I bete 20, that make 1 li. whyche I kepe in mynde, and to the other 1 that remaineth I adde that one that came of the pens, & was in my mynde, whyche make 2, and them I write vnder the ſhyllynges.

li.	s.	d.
12	8	9
6	7	3
3	6	8
		8

li.	s.	d.
12	8	9
6	7	3
3	6	8
22	2	8

Than do I recken the poundes togyther 3,6,12, that is 21, and to them I adde the 1 in my mynd that remayned of ẏ ſhyllinges, whiche make 22, theym doo I write vnder the poundes; and than my ſumme totall appereth to be 22, li. 1, ſ. 8 d.

Nowe to examine this ſumme, and all ſuche lyke, you ſhall doo thus. Fyrſt begyn at the lefte hande with the poundes, & take from them that are aboue the lyne 9, as often as you canne: then that that remayneth ſhall you double and ioyne it with the ſhyllynges, and take awaye 9 from that as often as you can, and what ſo euer remaineth ye ſhall take for it thre tymes ſo muche, and put to the pence. than take from al that ſumme 9, as often as you canne, and what ſo remaineth after you haue withdrawen 9 as often as you can, write that at the ende of a line, as I taught you in the other addition.

An o
forme
proof

And then come to the ſumme vnder the lyne, beginnynge with the poundes, and doo euen as you dydde with the ſummes aboue the line, till you come to your pennies, and if the figure of the ſumme that remayneth after caſting away 9 (as often as you can) do agree with the other that remayned before of the other ſumme, whiche you dyd wryte at the ende of the lyne, than haue you done well, elſe not. and for an exampie.

E.ij. I

6

I wyll examine that laſte ſume whiche was thus ſet.

Fyrſte I ſhall begyn at the left hande with the poundes, putting them togither, whyche make 21, in which ſumme I fynde 9 twyſe, (for twyſe 9 is 18) that I deduct, and there remayneth 3: that 3 muſte I double (as I ſayde) bicauſe it is the remayner of the poundes, and it wyll be 6. Than gather I the ſumme of the ſhyllings whiche is 21, to the whiche I adde the aforeſayde 6, and thã is it 27, wherin I fynde 9 thre tymes, and there remaineth nothing. Thys remayner ſhoulde I take thre tymes: but three tymes nothyng, is nothing: therfore in this place is there nothing left to be added to the pennies. Wherefore I muſte take the ſumme of pennies alone, whiche is 20, from thence yf I take 9 twyſe, there remayneth but 2, whiche I putte at the ende of a lyne, thus.

li.	S.	D.
12	8	9
6	7	3
3	6	8
22	12	8

2 ————

Then I come to the poundes of the vnder numbre or totall ſumme, & there I fynd 22, from whiche I take away 9 twyſe, and there remayneth 4: that 4 I double, and it is 8, then do I adde that 8 to the ſhillinges, and it maketh 10, from whiche I withdrawe 9, and there reſteth one: then do I take that

1 three

1 thre tymes, and it maketh 3, which I adde to the 8 d. and it maketh 11. from whyche if I bate 9, there reſteth 2, whiche is equall to the number noted at the ende of the lyne: ſo thereby I perceaue that I haue done well.

Sc. But I doo not ſee the reaſon of this.

Maſt. No, no more do you of many thinges els, but hereafter wyll I ſhew you the reaſons of all Arithmeticall operations. for this I iudg to be the beſt trade of teachyng, fyrſte by ſomme briefe preceptes to inſtructe a learner ſomewhat in the vſe of the arte, before he learne the reaſons of the arte, and then may you afterwarde more ſoner make hym to perceaue the reaſons: for harde it is for to occupye a yonge learned witte with bothe the arte & ye reaſons of it all at ones: how be it he ſhall neuer be cunning in dede in an art, that knoweth not the reaſon of euery thing touching it. But for this worke, bicauſe ye reaſon is eaſy, I wil ſhowe it you now. You knowe that if one pounde doo remayne, it being tourned into ſhyllings, wold make 20 s. in which number there is 9 contayned twyſe, and 2 s beſide. And therefore for one pounde you ſhall take 2 s, and ſo for euery one pound 2 s.

Scho. I ſee it well. for if there remayned 7 li, after the nynes were caſt awaye, I muſt take 14 s for that 7 li. And ſo haue I caſte awaye 14 tymes 9 s. and yet remayneth of euery pounde 2 s. which maketh. 14 s.

Maſter. Like wayes in ſhyllynges, which contayne 12 d. for euerye ſhyllynge, yf you abate 9 pence, there reſteth 3 pence.

Sc. It is playne ynough. And ſo yf 5 ſhillinges do remaine, I muſt take for it 15 d: that is 3 pence for euerye ſhillynge, and yet in that ſo dooing, I haue caſt away 5 times 9 pence.

M. Other woorkes haue as good reaſon, but I wyll not ſtande about yeldynge reaſons nowe.

Sc. Yet one thinge more I pray you ſhew me, why did you write your numbre that remained (after you had withdrawen all the lynes) at the ende of a lyne? for I ſawe no reaſon why that lyne dyd ſerue.

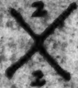
M. Dyd you euer marke a carpenter whā he wrought?

Scholar. Yea many tymes.

M. And haue you not ſeene him whan he hath taken meaſure of a boord that he hath pricked it, & hath with a ſwitche of his hād drawē a line frō the prick that he made?

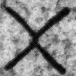
Scho. Yes I haue marked that, and haue ſeene ſome make 3 or 4 lynes by the pricke, & ſome alſo haue I ſeen make a croſſe by it, but that I perceaued was for the eaſye fyndyng of their pricke.

Ma. And euen ſo is this lyne for the eaſy fyndinge of your remayner, and therefore ſome do make a croſſe thus. And ſette the one remainer aboue the croſſe, and the other vnder the nether parte of the croſſe, as if I ſhould ſet my two remayners thus.

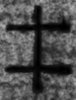


But there is an other ſorte of profe of Addition, to whiche the croſſe ſerueth more meeter: And that is whē the addition is of dyuers denominations, & I would examine euerye denomination by it ſelfe, which waies though it be not much vnlyke to the fyrſte proofe that I broughte of ſuch diuers ſummes, yet I wil declare it, leaſt you ſhuld think that I wold hyde it frō you. You muſt make ſo manye lynes in your croſſe, as you haue ſundrye denominations: as if you haue but two denominations, than you may make it thus, that the ouer parte and the nether part may ſerue for one denomination, and the twoo ſides for the other. And if you haue thre denominations, as poundes, ſhillings and pennies, than muſt you make thre lynes thus.

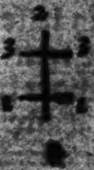
An oth ſorte of proofe.



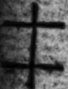
The vpright lyne maye ſerue for the poundes, and the higheſt thwart line for ſhyllings, & the loweſt for pēnes. as for exāple I will take a ſum thus added.



li.	s.	d.
16	12	5
12	8	1
9	2	7
38	3	1



For the proofe of the whyche, bicauſe it contayneth three denominations, I muſte make a croſſe of thre lines, thus. Than I recken fyrſt at the right hand the pennyes: 7, 1, 5, make 13, from whyche I take 12 for the nexte denomination, that is to ſay, a ſhyllyng, and there reſteth 1. whyche I muſte write at one ende of the nether thwart line.



After that I gather the ſum of the ſhyllinges, 2, 8, 12, whych make 22, to theym I put one that I tooke of the pennies, & that maketh 23, from thoſe I take 20, the quantitye of the next greater denomination, that is to ſaye, a pounde, and there reſteth 3. whiche I write at the end of the hygheſt thwart lyne.

Thyrdly, I adde togither the poundes 9, 12, 16 which make 37, to theym I adde the 1 that came of the ſhillinges, and than there is 38, wherin I fynd 4 tymes 9, and 2 ouer, that 2 I wryte on the vpryght lyne.

That doone, I come to the totall ſumme, and examine it, beginnynge at the pennies, where I fynde but 1, and can not take 9 frō hym, therfore I ſet hym at the other end of yͤ nether thwart lyne: than come I to the ſhyllinges, where I fynde only 3, which bicauſe it is leſſe than 9, I ſet it at the other ende of the lyne of ſhyllinges, that is, the ouermoſt thwart lyne.

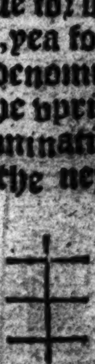
Laſte of all, of the 38 li. I take out 4 tymes 9, whiche is 36, and there remayneth 2, whych I wryte vnder the vpryght lyne.

Then I conſydre euery numbre, comparyng it to the numbre that is againſt it, and bicauſe I fynde them to bee euerye one lyke his matche, I knowe that I haue well doone.

S. This croſſe I perceaue doth ſerue for thoſe thre denominatiōs, pounds, ſhillings, pennies. but what if I had ob. q̃. q. and c?

Ma. You thinke you be at Oxforde ſtyll, you bryng forth ſo faſte your q and c. Theſe three lynes, as I haue ſayde, doth ſerue for three denominations, ſuche as they bee: as here they doo ſerue for poundes, ſhillinges and pennies: but if you haue no pounds in your ſumme, then maye they ſerue for ſhyllinges, pennies and halfe pennies, yea for q̃, q, and c, if you haue no greater denomination, ſo that you remembre that the vpright lyne ſerueth for the greateſt denomination, and the hygheſt thwarte lyne for the nexte, and the loweſt for the leaſte.

And ſo if you haue fower denominatiōs, you muſt make your croſſe with ſo manye lynes. And if that your ſumme bee of more denominations, make ſo many lines in your croſſe. And thus wyll I make an ende of Addition.

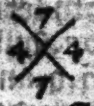


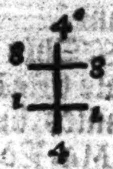
Examples of Addition.

li.	ſ.
262587	6
41635	12
28124	2
471	4
332818	4

li.	ſ.	d.
340	17	10
28	6	8
13	13	4
382	17	10

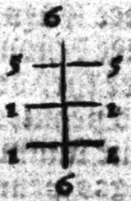
The proofes.





An other example.

li.	ſ.	d.	q.
22		6	2
2	3	4	
	10	2	3
24	14	1	1



SVBTRACTION.

Scholer.

Then haue I learned the two fyrſt kyndes of Arithmetike, now (as I remēbre) dooeth folowe Subtraction, whoſe name me thynketh dooeth ſounde contrarye to Addition.

Maſter. So is it in deede: for as Addition encreaſeth one groſſe ſumme by bryngynge many into one, ſo contrary waies, Subtraction dymyniſheth a groſſe ſumme by wythdrawinge of other from it. ſo that Subtraction or Rebating is nothing els, but an art to withdrawe and abate one ſume from an other, that the Remainer may appeare.

Subtrac

Scholar. What call you the Remainer?

Maſter. You may perceaue by the name.

Scho. So me thinketh: but yet it is good to aſke the trouthe of al ſuch thinges, leaſte in truſting to myne owne coniecture, I bee deceiued.

Maſter. So is it the ſureſt waye. And as I ſee cauſe, I wyll ſtyll declare thynges vnto you ſo playnely, that you ſhall not neede to doubt. How be it, if I do ouer paſſe it ſōe tymes (as the manner of men is to forget ÿ ſmale knowledge of theym to whome they ſpeake) than do you put me in remembrance your ſelfe. and that waye is ſureſt.

And as for this worde that you laſte aſked me, take you this deſcription. The Remayner is a ſumme lefte after dewe workinge, whiche declareth the exceſſe or difference of the two other numbres as yf I woulde deducte 14 oute of 18, ther ſhoulde remaine 4, which is called the Remainer, and is the differēce betwene thoſe two numbres 14 & 18.

Rema

Scholer. I perceaue than what Subtraction is, now reſteth to know ÿ arte to worke by it.

Maſt. That ſhall you do by this meanes: Fyrſte you muſte conſyder, that yf you ſhuld go about to rebate, you muſte haue two ſundry ſummes propoſed, the fyrſte whyche is youre groſſe ſumme or ſumme totall, and it muſt be ſet hygheſt, and then the rebatemēt or ſumme to bee withdrawen, whiche muſte be ſet vnder the fyrſte (whether it be in one parcell or in many) and that ſo, that the firſt fygures be one iuſte ouer an other, and ſo ẏ ſeconde and thirde, and all other folowing, as you dyd in Addition: then ſhall you draw vnder theym a lyne, and ſo are your ſūmes dwely ſet to begyn your working.

Then begyn you at the right hande (as you did in Addition) and withdrawe the nether numbre out of the higher, and yf there remayne any thinge, write that right vnder them benethe the line: and yf there remayne nothynge (by reaſon that ẏ two figures wer equall) then wryte vnder them a cyphre or noughte. And ſo do you with all the other fygures, euermore abatyng the nether out of the higher, and write vnder them the remainer ſtyll, tyll you come to the ende. And ſo wyll there appeare vnder the lyne what remayneth of youre groſſe ſume, after you haue deducted the other ſumme from it, as in this example.

I receyued of your father 4 8 ℔, of whiche

I haue layde out for you 36 li. now woulde I knowe what dothe remaine? and therfore I ſet my numbres thus in ordre: Fyrſt I wryte the greateſt ſumme, and vnder hym the leſſer, ſo that the fygures at the righte ſyde be euen one vnder an other, and ſo the other thus.

li.
48
36

Than do I rebate 6 out of 8, & there reſteth 2, which I write vnder theym ryght beneth the lyne, thus.

Than I go to the ſeconde figures, and do rebate 3 oute of 4, where there remaineth 1, whiche I write vnder them right, & then the holle ſumme and operation appereth thus.

li.
4 8
3 6
2

li.
4 8
3 6
1 2

Whereby it appeareth, that if I withdrawe 36 out of 48, there remayneth 12.

Scholer. Nowe wyll I proue in a greater ſumme: And I wylle Subtracte 1 3 6 7 9 2 4 out of 2 4 6 8 9 4 6. Thoſe ſummes I ſet in ordre, thus.

2 4 6 8 9 4 6
1 3 6 7 9 2 4

Then do I begyn at the ryght ſyde, and deducte 4 out of 6, and there reſteth 2, whiche I write vnder them. Then go I to the ſeconde figures, and withdrawe 2 out of 4, and there remayne 2, whyche I ſet vnder theym alſo: than I take 9 out of 9, and there reſteth 0, whiche I write vnder theym: for you ſaye, that if the fygures bee equall ſo that nothynge remayne, I muſte wryte thys cyphre 0 vnder theym.

M. It was well remembred: now go forth.

S. Then come I to the fourth place, and draw 7 out of 8, and there remaineth 1, whiche I write vnder theym alſo. Then in the fyft place I take 6 from 6, and there reſteth nought, for it I write vnder them a cyphre, 0: Thā in the vi place, 3 rebated frō 4, there remayneth 1, which I write vnder theym: And lykewyſe in the vii and laſt place, 2 taken from 3, there is left one, which I write vnder them: ſo haue I done my whole workinge, and my ſummes appeare thus.

Wherby I ſee, that yf I rebate 2367924 oute of 3468946, there remaineth 1101022.

	3	4	6	8	9	4	6
	2	3	6	7	9	2	4
	1	1	0	1	0	2	2

M. This is well done. And that you maye be ſure to perceaue fully the arte of Subtraction, let me ſee how can you ſubtract 5298478 out of 80003456.

S. Fyrſt I ſet downe the greateſt ſumme, and after that I write vnder it the leſſer nombre, beginning at the ryght ſyde: and then my figures wil ſtand thus.

8	0	0	0	3	4	5	6
	5	2	9	8	4	7	8

Than take I [illegible] from [illegible] and the reſte is [illegible] whiche I write vnder

theym

them. Then do I withdrawe 2 from 5, and there remayneth 3, which I write vnder them: Then take I 7 out of 4, but that I can not, what ſhall I now do?

Maſt. Marke well what I ſhall tell you now, how you ſhall do in this caſe and in all other lyke. If any fygure of the nether ſum be greater then the fygure of ẏ ſum that is ouer him, ſo that it can not be taken oute of the fygure ouer hym, then muſte you put 10 to the ouer fygure, and then conſydre howe muche it is, and oute of that whole ſumme withdrawe the nether fygure, and write the reſte vnder them. Can you remembre this? Note.

S. Yes that I truſt I ſhall. Nowe then in mine example where I ſhould haue taken 7 out of 4 and coulde not, I put 10 to that 4, which maketh 14, from it I take away 7, and there reſteth 7 alſo, whiche I write vnder them.

M. So haue ye done well, but now muſte you marke an other thynge alſo: that when ſo euer you do ſo put 10 to any figure of the ouer numbre, you muſt adde one ſtyll to the fygure or place that foloweth nexte in the nether line, as in this example, there foloweth 4, to which you muſt put 1, and make him 5 and then go on as I haue taught you. Note.

Sc. Then ſhall I ſaye: 4 and 1 (whiche I muſt put to him for the 10 that I added to 4 before) make 5, whiche I ſhoulde take out of 3, but that can not be, therefore muſte I put to it alſo 10, and then it wil be 13. from whiche I take 5, & ther reſteth 8 to be written vnder them: and becauſe of that 10 added to the 3, I muſte adde 1 to 8 that foloweth in ẏ neither line, & that maketh 9, whiche I ſhuld take out of 0 and can not, therefore I put therto 10 & that maketh 10; from 10 I take 9, & there remaineth 1, whiche I write vnder them.

Then do I adde 1 likewiſe to the nexte figure beneth, whiche is 9. and that maketh 10: that 10 ſhuld I take out of the figure aboue, but I can not, for it is 0, therefore I put 10 to it, and ſo take I 10 out of 10, and there reſteth 0 to be writtē vnder them: thē come I to the nexte figure whych is 2, and to hym doo I adde 1, whyche maketh 3, & 3 I can not take out of nought, therefore of that nought I make 10, and thenſe dooe I take 3, ſo remayneth there 7 to bee written vnder them: Lykewiſe doo I put 1 to 5 that foloweth, and then is it 6, that woulde I take out of 5, and can not, therefore I take 10 to that 5, and make it 15, frome whyche I rebate 6, and there remaineth 9, whyche I write vnder them; now haue I ſpente all the nether figures, & what ſhall I do more?

M. You ſhulde haue added 1 to the nexte figure folowyng (yf there hadde ben any) becauſe you added 10 to the laſt figure before of the ouer lyne: But ſeyng there is no fygure folowinge, you muſte adde that one to the place folowing, and then deduct that 1 from the numbre aboue.

Sc. Then ſhall I ſaye, bicauſe I borowed 10 to the ouer 5, I muſte put 1 in the next place beneth, that is vnder 2: then muſte I ſubtracte that 1 from 2, and there reſteth 1, to be writē vnder that 2 in the ninth place; Now I haue no more to ſubtract, for there is neuer a fygure remayning beneth, nother yet any vnitee to be added, bicauſe I borowed not 10 to the figure laſt before, and yet is there 8 remayning in the ouer lyne, whiche (I thinke by reaſon) ſhould be ſet at the ende of the fygures in the loweſt rowe whiche is vnder the lyne, for bycauſe there was nothinge taken from it.

M. That is well conſidered, and reaſon teacheth ſo in deede.

S. But ſyr I beſeche you, ſhal I alwaies when any numbre ſo remaineth alone (as this 8 did) write him vnder the line ſtraight againſt his owne place?

M. Yea what elſe? whether they bee one or manye: and this well remembred, you haue ſufficiently lerned Subtraction. Howe be it, bycauſe of certaine things that might deceaue you, if you dyd not take good heed to your working, I wyll propoſe to you an other example of many numbres to be ſubtracted, as thus.

I receaued of a frende of myne to keepe 2869 crownes, of whiche at one tyme I deliuered him againe 500, at an other tyme 368, and at an other time 440, and an other tyme 80, and an other tyme 64: nowe would I knowe howe many doo reſte behynde? Therefore fyrſte I ſet downe my groſſe ſumme, and vnderneth it I ſette all the parcels, thus: and vnder them a double lyne.

2869
500
368
440
80
64

Then fyrſt I begynne at the firſt place, and gather togyther the ſum of all thoſe lynes (ſaue the ouermoſte) in theyr firſt fygures. and ſo doo I with all the fygures of the ſeconde place, and ſo forthe, as I dydde in Addition, ſaue that I leaue out the higheſt rowe of numbres, and that ſumme ſo gathered betwene the lines, do I ſubtract out of ye higheſt rowe of nūbres, & the remainer do I ſet vnder ye nethermoſt line. as for exāple.

2869
500
368
440
80
64
1452
1417

I ſet the ſummes as before, than doo I gather the firſt figures together, where I fynde but 4 and 8, that make 12, for three cyphres encreaſeth no ſumme in Addition, as you learned lerned before, of ẏ 12, therfore do I wryte ẏ dygite 2, and keepe the article in my mynd tyll I come to the ſecond places, where I fynde, 6, 8, 4, 6, that make 24, to them I put the article in my minde, and it is 25. of whyche I wryte 5 vnder the ſeconde place, and keepe the digette 2 in my mynde for the thyrd place, where I fynd 4, 3, 5, that make 12, to the whiche I adde the 2 in my minde, and that maketh 14, thereof I wryte the 4 vnder the thyrde place: and bicauſe there remayneth no more fygures to bee added, I write the digete 1 in the fourth place, as you ſee in the example.

Than come I to ſubtracting of thys ſumme, and I ſhall ſubtract the ſumme betwene the lines, from the outmoſt ſumme, ſaying: 2 from 9, remaine 7, to bee written vnder them beneth the loweſt lyne: than in the ſeconde place I take 5 from 6, and there reſteth 1, to be written vnder them. Than in the third place, 4 from 8, reſteth 4. Laſte of all in the fourth place, 1 from 2, remaineth 1. And thus I ſe that after thoſe 5 ſummes are ſubtracted from 2869, the Remayner is 1417.

S. This I perceaue: but is there no ſhorter waye and more ſpedier?

Ma. Yes, when you are a whyle exerciſed in it: for you may as faſte as you can gather the numbres togither, withdrawe them out of the higheſt ſumme. if ſo be it, that all the parcelles whiche you doo gather, do not exceede nyne. but and yf they exceede nyne, than muſt you ſubtracte onlye the dygette that is in it, and reſerue the article tyll the nexte place, where you ſhall adde it with the other figures, and ſo ſubtract the whole out of the fygure aboue them: but and if in this place the ſumme of the parcels do excede 9, than (as I ſayde before) ſubtract the digite onely, & reſerue the article to the next place, and ſo ſtill go forth, tyll you haue ended your working.

As for example: in the laſte ſummes propoſed; I gather fyrſte in the fyrſte place 4 and 8, that maketh 12, of whiche I deducte the dygite 2 out of 9, and wryte vnder the remayner, whiche is 7, and the article 1 I kepe in my mynde. Than in the ſecond places I gather the parcels 6, 8, 4, 6, whiche amount to 24, to that I adde the article 1, whyche I haue in my minde, and than is it 25. than do I take 5 (that is the digette in this numbre) from 6, that is in the ſeconde place of the higheſt ſumme, & there remayneth but 1, to bee written vnder theym, and nowe do I keepe the article 2 in my mynd ſtyll. Than in the thyrde place, 4, 3, 5, maketh 12, and the article 2 in my mynde maketh 14, then take I 4 (which is the digete) from 8 that is ouer them, and there reſteth

4, whiche I write vnder them. Then haue I the article 1 yet in my mynde, which I ſhuld adde to the parcels next folowing, but ſeyng there is no nombre folowing, I take that diget alone, and deduct him out of the nexte ſumme aboue, whiche is 2, and than is the remainer 1, which I write in yͤ fourth place vnder 2. Loe, now haue you a ſhorter waye.

Sc. I lyke both wayes well, and I perceaue both well. yet as in the one, the workyng ſeemeth ſomwhat long, ſo in the other it leaneth verye muche (me ſeemeth) to remembraunce, and therfore may cauſe errour quickly, except a man haue a quicke and an exerciſed remembraunce.

M. What, would you then haue ſuche a waye that ſhould not be ſo longe as the one, nor ſo ſhorte as the other?

Sc. Yea, if there were anye ſuche.

M. Than do thus ſtill as you gather your parcels, when they excede a digete, and make 10 or more, take the article, and wryte hym betwene 2 lynes (as in the fyrſt exãple) vnder the nexte place toward the lefte hand, and then deduct yͤ digete from yͤ fygure that is ouer him, & write yͤ remainer. And thẽ whẽ you gather yͤ nexte parcels, you ſhal adde to them the fygure that is vnder thẽ, betwene the two lynes. And if it excede 9, doo as I ſayd before, write the article vnder the nexte place betwene the lynes, and subtracte the diget from the fygure that is ouer those parcels: and if that all the parcels togyther & the numbre betwene the lynes do make but a digete, then deduct it wholly from the fygure aboue: as in this example.

I wold subtract out of 40308964, these thre parcelles, 20003428, 10002342, 10101462.

40308964
20003428
10002342
10101462

Therfore I set them fyrst in ordre due and then I gather the parcels of the fyrst place, which are 8, 2, 2, that is 12, of which I take away the article, and set him under the seconde place betwene the lynes, and the digett 2 that remaineth, I deducte out of 4, and there resteth 2 to be written under the fyrst place beneth the lowest line. Then come I to the second place, and gather the parcelles of it, 6, 4, 2, and the one betwene the lynes, which make 13, of which I take the article, and set him under the third place betwene the lynes, and the digette 3 I take frome 6, and there remayneth 3, which I write under the seconde place beneth the lowest lyne. Than in the thyrde place I fynde 4, 3, 4, which with the 1 betwene the lynes do make 12, therfore I write the article agayn under the fourth place, and the digit 2 I take frome

9, and there remayneth 7, which I wryte vnder them beneth the loweſt lyne.

And than come I to the fowerth place, where I gather, 1, 1, 3, and the 2 betwene the lynes, that maketh 7, whiche bicauſe it is but a diget, I plucke from 8, and the Remainer is 1, and muſt be written vnder them in the fowerth place. After that came I to the fyfte place, wher are only thre cyphars, whiche make nothinge; than ſhould I take that, that is to ſay nothing, from the figure ouer them, whiche is alſo a cyphre, therfore I muſte ſaye thus: if I take nought frome nought, there remaineth nought: ſo muſte I write a cyphre vnder them. Than in the ſyxte place I fynde but 1, which I take out of 3 ouer him, and the Remayner is 2, that muſt be written beneth the loweſt line in the ſyxte place. So go I to the ſeuenth place, wher I fynd only cyphres, and in the greate ſumme ouer them a cyphre alſo, therefore muſt I wryte their remayner (whiche is nothynge) wyth a cyphre alſo. Than in the eyghte and laſte place, I gather 1, 1, 2, that make 4, whyche if I take oute of that 4 that is ouer theym, there wyll nothynge remayne. And that muſte bee noted wyth a cyphre beneth the loweſt lyne, as I haue often ſayde. and ſo haue I ended my worke, and the fygures ſtande thus.

Sco. Syr, I remembre you taught me that cyphres ſhuld not come in the laſt place, for bicauſe they ſerue only to encreaſe the valewe of other fygures whiche folowe them, and ſerue not for thoſe figures that go before them: and nowe in your example you haue ſet two cyphres in the two laſte places.

```
4 0 3 0 8 9 6 4
2 0 0 0 3 4 2 8 }
1 0 0 0 2 3 4 2 }
1 0 1 0 1 4 6 1 }
----------------
        1
----------------
0 0 2 0 1 7 3 3
```

Maſt. I commende you for your remembrance. And truth it is, I would not haue ſet them here, but onlye bicauſe that I woulde make you plainely to perceaue the arte of Subtraction. Therfore ſeyng you doo now perceaue it, whan ſo euer you ſhoulde wryte downe a cyphre, looke whether anye other fygures be yet behinde, And if not, than let go the cyphre alſo, for it nedeth not to wryte him in any latter places, where no other fygure doth folowe: except it be (as I ſayde) to teache the vſe of ſubtraction the plainer. Therfore my fygures muſt ſtande thus, whan I haue ended my worke.

```
4 0 3 0 8 9 6 4
2 0 0 0 3 4 2 8
1 0 0 0 2 3 4 2
1 0 1 0 1 4 6 1
----------------
          1 1 1
----------------
    2 0 1 7 3 3
```

S. So I wold thinke by that you taught me before. And now I beleue I could ſubtracte any lyke ſummes.

Maſt. So maye you, if you haue marked what I haue taught you. But bycauſe this thynge (as all other) muſt be learned ſurely by often practiſe, I wyll propounde here ij. examples to you, wherein if you often exerciſe your ſelfe, you ſhalbe ripe and perfect to ſubtract any other ſumme lightly: for in the is contayned all the obſeruaunces of whole numbre. And bicauſe you ſhal perceaue ſomwhat both howe to doo it, and alſo whether it be well doone when you haue proued to doo it: therefore haue I written vnder them bothe their remayners.

```
 3 0 8 9 6 4                1 2 5 6 1 4
 1 0 3 1 4 5 }                    3 4 2 }
 1 0 2 5 9 7 }                    6 8 3 }
 1 0 1 0 2 4 }              [illegible] }
 -----------                -----------
         1 1                    1 1
 -----------                -----------
     2 1 9 8                1 2 4 3 9 0
```

Sch. Syr I thanke you. But I thynke I myght the better doo it, yf you did ſhewe me the woorkynge of it.

Ma. Yea but you muſte proue your ſelfe to doo ſome thynges that you were neuer taughte, or els you ſhall not bee able to doo any more than you were taughte. And that were rather to learne by rote (as they call it) than by reaſon. And againe there is no thyng in this example or any other of whole numbre, but I haue taught you the rules of them alreadye.

Sco. Than I truſte by practiſe to atteine the vſe of it. And is this all that I ſhall learne of Subtraction?

M. Yea, ſauing that (as you haue ſene in Addition) there are numbres of dyuers denominations, in whiche the working is not muche vnlike, yet without ſome inſtructiõs be gyuen of it, it might ſeeme to a learner more difficult, then in dede it is. Therefore I will briefly ſhewe you the vſe of it onely, by one example or two.

A certaine man owed to me 14 li. 12 ſ. 8 d. of whiche he payed me at one tyme 4 li. 6 ſ. 8 d., at an other tyme 3 li., and at an other tyme 2 li. 3 ſ, 4 d. and laſte of all 6 ſ, 8 d: Nowe wolde I knowe, what remaineth vnpayde yet, therefore I ſet my ſummes thus.

li.	ſ.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8

S. Syr I praye you, why doo you write 2 li? for the cõmon ſpeache vſeth rather to ſay 40 ſ.

M. We muſt here vſe the denomination that is greateſt in any ſum, ſo that we maye not write accordinge as we vſe to ſpeake, ſayng 16 d, 18 d, or like wayes 7 grotes, 8 grotes, 24 ſ, 40 ſ, 48 ſ, and ſuch other, but we muſt write euerye denominatiõ that is in any ſumme by it ſelfe, namely ſhillinges and poundes. So muſte we write for theſe ſummes now named, 1 s. 4 d: 1 li. 6 d: 2 s. 4 d: 2 s. 8 d: 3 li. 4 s: 2 li. 2 li. 8 s: and ſo forth of other lyke.

S. So that we may not write in Arithmetike penies, when the ſumme amounteth to ſhyllinges, nor ſhillinges whan the ſumme maketh poundes. Now if it pleaſe you, ende your example.

li.	s.	d.
14	13	8
4	6	8
2		
3	3	4
	6	8
	1	
4	16	

M. Whan my ſummes are ſo ſet as I ſhewed, than muſte I begynne with the ſmalleſt denominatiõ, ſaying: 8 4 8 are 20, which be ſumme bycauſe it is pence, and 12 pennies do make 1 s. I muſte take from that 20 (whiche commeth of the three parcels) 12, and for them write 1 betwene the lynes under the ſhillinges, than the 8 d. that remayneth, I take out of the hygheſt ſumme (whiche is 8 alſo) and than remayneth nought. Wherefore under the pence I write nothing. Than come I to the ſhillinges, and gather the parcels 6, 3, 6, which with the 1 betwene the lynes, make 16, that muſt I take oute of the ſumme that is ouer it, but ſeyng that ſum is but 13, I can not take 16 out of 13, I muſte borowe 1 of the

14 li. and put to the 12, and that maketh 32, for 1 li. is worth 20 s. then take I 16 out of 32, and there reſteth 16 to be written vnder the ſhyllinges. Than come to the poundes, whoſe parcels are 2, 3, 4, that is in al 9, and 1 more muſte I adde therto, bycauſe of the 1 that I borowed before vnto the 12 s, and than is there 10, whiche I muſt take out of 14, ſo dothe there remaine 4, to bee written vnder the poundes: ſo doothe my remainer appeare to be 4 li, 16 s.

S. This do I perceaue very well. and yf there be none other thinge to bee learned in Subtractiō, thē may I come to Multiplicatiō, for ỹ you reckned to be in ordre nexte.

M. We haue done in dede with ỹ art of Subtraction, as touchīge ỹ working. But yet before we go to Multiplicatiō, I will inſtruct you howe to examine your worke, whether it be well doone or no. and that is by caſting awaye 9 as often as you can fynde it, as you did in Additiō, ſauing that you muſt here examine ỹ higheſt nōbre alone, & note ỹ reſidue of it at ỹ lines ende, as you did in Addition.

profe of Subtraction.

And when you haue done with the higheſt numbre, than examine all the other togyther, caſtyng thence 9 as often as you can: and if the remayner bee lyke the other, then haue you doone well.

But if you haue diuers denominations in your ſumme, yet for them all ſhall you make but one ſeuerall lyne as you dyd in Additiõ, remembryng to begynne the examination at the greateſt denominatiõ, and to double the remayner of poundes, and triple the remayner of ſhillinges, as you did alſo in addition.

As for a profe, I wyl examine this worke, wher in the higheſt line I fynde of poundes 14, frõ thens I bate 9, and there reſteth 5, which I do double, bicauſe they are poundes, and than are thei 10, therto I ad the 12, & it maketh 22, from whiche I take 9 twyſe, and there reſteth 4, whiche bicauſe they are ſhillinges, I triple, and then are they 12, thereto I adde the 8, and than are they 20, thence take I twiſe 9, and yet reſteth 2, whiche I write at the one ende of a lyne, thus. 2———

li.	s.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8
4	16	

Then I examine all the other parcels, and the remayner togither, euery denominacion by it ſelfe. And fyrſte of poundes I fynde 4, 3, 2, 4, that is 13, from which I take 9, & there reſteth 4, that do I double, and it maketh 8, to it do I put the ſhyllinges, 6, 3, 6, 16, that is 31 (for the one betwene the lines muſt not be reckned, nor none in that ſpace) and that maketh in all 39. Where hence I take 9 foure tymes, and there remayneth 3, that do I take thre times, and it is 9, wherfore I caſt it away, thā do I take ẏ pennies 8, 4, 8, that make 20, from whiche I take 9 twiſe, and there reſteth 2. which I write at the other end of the profe lyne, And bicauſe I ſee that thoſe two numbres are equall, I ſaye that I haue well wrought.

And if you wyll, you may make for euery denomination a lyne, as you learned in Addition: but then muſt you begin your examination at the ſmalleſt denomination, as you dyd in Addition, for their profe is altogither lyke, ſauyng that in Addition you examined the nethermoſte ſumme alone, and all the other togither: and in Subtraction you muſte examine the higheſt nūbre alone, and all the other togither. And if you marke it well, it is euen all one, for that ſumme that in Addition is loweſt, in Subtraction is hygheſt, and that ſumme is called ẏ Groſſe or Total ſumme.

Groſſe or Total ſumme.

Therefore if you marke what I ſayde in Addition, you maye eaſilye perceaue what is to be done for the proofe of Subtraction. And to the entent that you may perceaue it the better, I wyll ſhewe you an other proofe of ſubtraction, and that ſhal be by Addition, thus. Draw vnder the loweſt numbre, whyche is your remainer, a lyne: than adde that numbre, and all the other that you did ſubtracte before, togither, and write that that amoun-

other
ofe of ſub-
action.

amounteth vnder the loweſt lyne. and if the ſumme that cõmeth thereof be equall to the higheſt of the ſubtraction, than was the ſubtraction well wrought, elſe not.

As for example: in the laſt ſummes, whiche ſtoode thus.

li.	s.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8
1	1	
4	16	

Fyrſte I adde 8, 4. 8, that maketh 20, whereof I take 12 awaye, bicauſe they make one ſhyllinge, and write for them one vnder the ſhyllinges: and the 8 that is lefte, I write beneth the loweſt lyne. Than adde I the ſhyllinges 6.3.16.1.6. that make 32: from whiche I take 20, and for it I write 1 vnder the poundes, and the 12 that remayneth, I write vnder the ſhyllings. Thã come I to the poundes, addynge theym togither, which are 4.3.2.1.4. that maketh 14: than do I write 14 vnder the li. and ſo haue I ended the addition. And I ſee that the loweſt lyne of numbre and the higheſt bee lyke, wherefore I knowe that I haue wel doone. for my fygures appeare thus.

li.	s.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8
1	1	
4	16	
14	12	8

And thus nowe haue

I taught you the arte of Subtraction, and the meanes to prooue whether it bee well wrouhht or not.

Nowe and you remembre, I omitted in teaching the proofe of Addition one waye, whiche I ſayde was by Subtraction.

S. Truth it is, & thã was it deferred, becauſe yͤ I had not thã learned yͤ feate of Subtractiõ, wherby I ſhold haue proued it, but now I thanke you, I haue well learned the arte of Subtraction, and the proues of it, bothe by 9, and by addition. And nowe I woulde be glad to knowe, how I maye prooue Addition by Subtraction.

The proofe of addition by ſubtraction.

M. Then marke you this. When you haue ended your addition, take the numbres all that you did adde to the higheſt ſumme, and deduct or ſubtract them from the groſſe ſum that doth reſult, and if the remainer be lyke to the higheſt numbre, then haue you doone well, elſe not.

As for example. I take one of the ſumes that I dydde adde before, whiche was thys that foloweth here.

106800
9400
116200

Then doo I come to the myddle nūbre, and ſubtract that from the nether numbre beginning at the right hande, and fyrſt I ſay 0 out of 0, there remayneth 0: that wryte I vnder an other lyne. Then agayne 0 in the ſe-

ſeconde place, from 0 remayneth 0, vnder it I write 0 alſo. Nexte that in the thyrde place 4 out of 2 wyll not be, therfore I adde to that 2, 10, and make it 12, frome that I take 4, and there reſteth 8. Then ſaye I farther: 9 in the fourth place and 1 (whiche I muſte adde for the 10 borowed before) make 10, that muſte I take from 6: and bicauſe I can not, I adde to the 6, 10: and then is it 16: from thence I take 10, and there reſteth 6 to bee written vnder them. Againe in the fyfte place where I fynde nothyng written, I muſt ſette 1 for the 10 laſte borowed, and that 1 do I take from the 1 vnder him, and ſo remayneth noughte; wherefore I write doune a cyphre 0. Nowe haue I doone with the ſubtraction: and yet in the groſſe ſum remayneth 1, whiche I muſte ſet ryght in the ſame place in the remayner, and ſo the remayner appeareth to be lyke vnto the higheſt ſum of the Addition, as here appeareth: wherefore I ſaye that the Addition was well wrought. and thus maye you do in any other ſumme of one denomination or many. Therfore nowe wyll I make an ende of Subtraction, and will inſtructe you in Multiplication.

```
106800
  9400
------
116200
------
106800
```

MVLTIPLICATION.

Multiplication is ſuch an operaciō that by ii ſummes producyth the thyrde, whiche thyrde ſumme ſo manye tymes ſhall cōtaine the fyrſt, as there are vnites in the ſecond. And it ſerueth in the ſteede of many Additions. As for example. When I woulde knowe how many are 30 tymes 48. If I ſhould adde 48 thyrty tymes, it wolde be a longe worke. Therfore was this worke of Multiplication diuiſed, whiche ſhall doo that at ones, that Addition ſhould do at manye tymes.

S. I perceaue the commoditie of it partly, but I ſhall not ſee the full profyte of it, tyll I knowe the whole vſe of it. Therfore ſyr I beſeeche you teache me the workinge of it.

iplic
n of al

M. So I iudge it beſt. but bycauſe that great ſummes can not be multiplied, but by the multiplicatiō of digits, therfore I think beſt to ſhewe you fyrſte the arte of multiplying them: As when I ſaye, 8 tymes 8, or 8 tymes 9. &c. And as for the ſmall digittes vnder 5, it were but follye to teache anye rule, ſeyng they are ſo eaſye that euery child can doo it. but for the multiplication of the greater digites, thus ſhall you doo.

Fyrſt ſet your digites one ouer the other right, then loke how many eche of them lacketh of 10, and write that againſt eche of them, and that is called the Differences: as if I would know how many are 7 tymes 8, I muſt write thoſe digites thus.

8
7

Then do I looke howe muche 8 dooth differ from 10, and I fynde it to be 2, that 2 do I write at ẏ right hande of 8, thus.

8 2
7

After that I take the difference of 7 lykewaies from 10, that is 3, and I write that at the right ſyde of 7, as you ſee in this example.

8 2
7 3

Then do I drawe a line vnder them as in Addition, thus.

8 2
7 3
———

Laſte of all I multiply the two differences ſaying: 2 tymes 3 make 6, that muſte I euer ſet vnder the differences beneth the line: then muſt I take the one of the differences (whiche I wyll, for all is lyke) from the other digit (not from his owne) and ẏ that is lefte, muſt I write vnder the digits. As in this example. If I take 2 from 7, or thre from 8, there remaineth 5, that 5 muſt I write vnder the digits: & then there appeareth the multiplication of 7 tymes 8, to be 56. And ſo like

8 2
7 3
———
5 6

tymes of anye other digittes, if they bee aboue 5, for if they bee vnder 5, then wyll their differences be greater then them ſelfe, ſo that they can not be taken out of them, and agayne ſuche little ſummes euery childe can multiplye, as to ſaye: 2 tymes 3, or 4 tymes 5, and ſuche lyke.

Sc. Truthe it is. And ſeeynge me ſemeth that I vnderſtande the multiplynge of the greater digittes, I wyll proue by an example howe I can doo it. I wolde knowe how many are 9 tymes 6.

M. It is all one in valewe to ſay, 9 times 6, or 6 tymes 9: but yet the ordre is beſte to put the leſſer ſumme fyrſte, ſaying: 6 tymes 9. and ſo of all other ſummes.

Sc. Then woulde I knowe howe manye ar 6 tymes 9. therfore I ſet the digittes thus.

9 1
6 4
5 4

Then doo I ſet their differences at their righte ſyde: the difference of 9 which is 1 agaynſt it, and the difference of 6 whiche is 4, againſt it alſo, as in this example.

9 1
6 4
5 4

And vnder them I drawe a line. Then doo I multiplye the digittes together, ſaying: 1 tyme 4. maketh 4, that 4 do I write vnder the differences thus.

Then take I one of the differences from the other digitte, as one [illegible] from 8, or els 4 from 9, and the whiche there reſteth 5. Whiche I doo write under the digits, and ſo appeareth the multiplication of 6 tymes 9, to bee 54. Thus I ſe the feate of this manner of multiplication of dygits.

M. Nowe myghte you go ſtraight in the multiplication of greater nūbres, ſaue that both for your eaſe and ſuretye [illegible] I wyll draw you here a table [illegible] appeare the multiplication of all [illegible] this is it.

1	2	3	4	5	6	7	[illegible]	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

[illegible] ch fygure when you woulde knowe any m[illegible]cation of digits, take your firſte in the [illegible] ſquares, and [illegible] ſo go [illegible] the [illegible] you [illegible] [illegible] the [illegible] and than the numbre that is in the meeting of their both ſquares, is the multiplication that amounteth of theym. As if you wolde knowe by thys table the multiplication of 7 tymes 9, ſeeke fyrſt 7 in the blacke ſquares, and than go right foorth toward the ryght hande, tyll you come vnder 9 of the hygheſt rowe; and in the meetinge of their ſquares you may ſe 63, which is their multiplicatiõ.

S. [illegible] is verye good and ready. And ſo may [illegible]nde the multiplication of anye dygittes. But nowe howe ſhall I do in greater ſummes?

M. When you wold multiply any ſum by an other, you ſhall marke that it is the meteſt ordre to ſet the greateſt numbre higheſt, whiche is the place of the numbre that muſt bee multiplied: and lykewayes the leſſer numbre vnder it, for that is the place of the Multiplyer or multiplicatour, that is to ſay the numbre by which multiplicatiõ is made: and is in engliſhe alwaies putte before thys worde, Tymes: in ſuche ſpeaking when I ſaye 20 tymes 70. And the numbre that followeth this woorde Tymes, is that whiche muſte be multiplyed.

Therfore when I wolde multiplye one ſumme by an other, I muſt write the greateſt higheſt, and the leſſer vnder it, as in addition. And vnder them muſt I draw a line. As for example. If I woulde multiply 264 by 29 I muſt ſett them thus.

264
29

Than muſte I multiplye euery fygure of the higher rowe, by euerye fygure of the nether rowe: and that that amounteth I muſte ſette vnder the lyne, as thus. Firſt I do multiply 4 by 9, ſaying: 9 times 4 (or 4 tymes 9, which is all one) and that maketh 36, as the table before of digittes dooth declare, of that 36 I muſt write the 6 that is the digitte vnder the 9, and the 3 in the nexte place toward the lefte hande.

264
29
6

Thē come I to the ij. figure of ẏ higher row & ſay: 9 times 6 make 54. of which I write ẏ 4 vnder the 3, & the 5 vnder the nexte place (as ẏ reaſō willeth me) thus.

264
29
536
4

After that come I to the nexte figure, which is 2, and do multiply it by 9, and that maketh 18, whereof I write 8 vnder the thirde place, and the article 1 in the fourth place, thus.

264
29
1536
84

And ſo haue I ended the fyrſte fygure of the multiplier. Then begyn I with the nexte fygure, and multiply it into all the higher fygures, as thus.

264
29
1536
84
8

Fyrſte 2 tymes 4, make 8, that do I write vnder the ſecōd place, for euermore the digite, of fyrſte figure of the multiplication that amounteth of the fyrſte fygure of the hygher numbere, muſt bee ſet vnder the multiplier of it, and the other in their ordre toward the left hand.

Scholar. I vnderſtand you thus: that the digytte of the ſumme amountinge of the multiplycation of the fyrſte fygure of the hygher rowe, by the fyrſte fygure of the lower row, or multiplier, muſt be ſet vnder the fyrſt place: & that that amounteth of the ſame firſte fygure by the ſeconde multiplyer, muſt be ſet vnder the ſeconde place: and ſo of the other, if there be more multipliers.

Ma. So meane I in deede: and if there amounte but a digitte, then muſte yt be ſette vnder the multiplyer. And now to go forth: I multiply by the ſame 2, the ſecond fygure of the hygher rowe, whiche is 6, ſaying: 2 tymes 6, make 12, whereof I write the dygit 2 vnder the thyrd place, and the article 1 I write vnder the fourthe place.

264
29
1536
184
128

Then do I multiply the laſte figure of the higher ſumme, by that ſame 2, ſaying: 2 tymes 2 is 4, whiche I write vnder the fourthe place. And ſo haue I ended the whole multiplication: And the ſummes ſtand thus.

264
29
1536
184
428

Than muſte I drawe a lyne vnder all thoſe ſummes that amounte of the multiplication: and muſte adde all them into one ſumme, as in this exaumple you may ſee.

```
 264
  29
1536
184
428
7656
```

where in the fyrſte place I fynd but 6, and therfore write I it vnder the lyne: Then in the ſecond place 8, 4, & 3, make 15, whereof I write 5, & kepe 1 in my minde. and ſo forth as you lerned in Addicion. And ſo appeareth the whole ſumme to be 7656, which amounteth of the multiplication of 246 by 29.

Sco. If there be no more to bee obſerued in it, then can I do it I ſuppoſe, as by this exāple I ſhall proue: I wolde multiply 1365 by 236, wherfore I ſet theym thus.

```
1365
 236
```

Than do I multiplye 5 by 6, ſaying: 6 tymes 5 make 30, of whiche I writ the cyphre in the firſt place, and the article 3 in the ſeconde place.

```
1365
 236
  30
```

Thand doo I by the ſame 6, multiply the ſecond fygure of the higher ſum which is 6, ſayinge 6 tymes 6, make 36, of whiche I write the 6 vnder the ſeconde place, and the 3 vnder the thirde place.

```
1365
 236
 330
  6
```

Than doo I multiplye the thyrde fygure whyche is 3, by

B.v. the

the ſame 6, and that maketh 18, of that I ſette the 8 vnder the thyrd place, & the 1 in the fourth place.

1365
236
1330
86

Than come I to the laſte fygure of the higher ſumme, and multiply it by 6 ſaying, 6 times 1 make 6, that doo I write vnder ẏ fourthe place.

1365
236
1330
6

And ſo haue I ended the fyrſte multiplier.

1365
236
1330
686

Than begyn I with the ſecond multiplier, and ſaye fyrſt: 3 tymes 5, that maketh 15, of which I ſet the 5 vnder the ſecond place, bicauſe that the multiplier is there, & the 1 I ſet vnder the thyrde place.

1365
236
1330
686
15

Than come I to the ſeconde fygure that is 6, and multiplye it by 3, which maketh 18, of whiche I ſette the 8 vnder the thirde place, and the article 1 in the fourth place.

1365
236
1330
686
115
8

Thã come I to the third figure whiche is 3, and multiplye it by 3 ſaying: 3 times 3 make 9, which bicauſe it is but one dygite, I ſet vnder the fourth place.

1365
236
1330
686
115
98

And then commyng to the laſte fygure 1, I multiplye it by 3, and it maketh 3, whyche I ſette in the fifte place, & than haue I ended twoo of the multipliers, and the ſummes ſtand thus.

1365
236
1330
686
115
398

Then come I to the thyrde multiplier, and multiply it into euerye figure of the higher ſumme, and fyrſte I ſay: twoo tymes 5, make 10, of whyche I ſette the cyphre vnder the multiplier in the thirde place, and the article 1 in the fourthe place.

1365
236
1330
686
115
398
10

And ſo multiplying the ſeconde fygure 6 by that ſame 2, there amounteth 12: whereof I wryte the digite 2 vnder the fourth place, and the article 1, vnder in the fyft place.

1365
236
1330
686
115
398
110
2

Nowe doo I multyplye by the ſame 2, the thyrde fygure of the hygher ſumme, whyche is 3, and that maketh 6: whyche I ſett vnder the fyfte place.

1365
226
1330
686
115
398
110
62

Than come I to the laſt place, and multiplye that 1 by 2, and there amounteth 2, whiche I ſette in the fyfte place, and then dooth the ſummes ſtande thus.

$$\begin{array}{cccccc} & & 1 & 3 & 6 & 5 \\ & & & 2 & 3 & 6 \\ \hline & & 1 & 3 & 3 & 0 \\ & & 6 & 8 & 6 & \\ & & 1 & 1 & 5 & \\ & 3 & 9 & 8 & & \\ & 1 & 1 & 9 & & \\ 2 & 6 & 2 & & & \\ \hline 3 & 2 & 2 & 1 & 4 & 0 \end{array}$$

And ſo haue I ended the whole multiplication.

But nowe (as you taught me) to know what thys whole ſumme is, I muſt adde all thoſe parcels togyther, and than vnder the lyne wyll appeare as you ſee, the groſſe or totall ſumme, that is 322140.

Maſter. That is well doone.

S. Than me thinketh I wold call it well doone, when I knewe whether I had well done or no.

M. It may be tried by 9 as addition was, but the ſureſt proofe is by Diuiſion, & therfore I wyll reſerue that tyll you haue learned the arte of Diuiſion.

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anner of
ltiplica-
on.

And before we paſſe from Multiplicatiõ, I wyll yet ſhewe you an other manner of multiplication, whiche are counted of ſome men bothe more ready, and more certain: of whiche the one differeth nothing from thys that I haue taught you, ſaue that it doothe vnderſtande alwayes the articles, and ioyne them to the nexte ſumme, and therfore

I

I wyll declare it only by an example.

If I woulde multiplye 1542, by 365, I muſte ſette theym as I ſayde beefore, and then doo I multiplye 2 by 5, and it ma[illegible]keth 10, of whyche I write the [illegible] the fyrſt place, and keepe th[illegible]gite 1 in my mynde.

[illegible]
365
0

Than ſaye I forthe: 5 times 4 doo make 20, and the 1 in my minde, are 21. thereof I wryte the 1 vnder the ſeconde place, and kepe the 2 in my mind.

1542
365
10

Than come I to the third figure 5, ſaying: 5 tymes 5 make 25, and the 2 in my mynde make 27, whereof I write the 7 vnder the third place, and kepe the article 2 in my minde.

1542
365
710

Thā coming to the laſt figure I ſay: 5 times 1 make 5, and 2 in my mynde make 7: that doo I wryte vnder the fourth place.

1542
365
7710

And than haue I ended my fyrſte multiplier.

Than do I lykewaſes with ỹ ſecond multiplier, ſaying: 6 ty[illegible] 2 make 12, therof I write ỹ digitte 2 vnder the ſecond place, & keepe the article 1 in my mynde.

1542
365
7710
2

Than ſaye I forthe: 6 times 4 maketh 24, and the 1 in my mynde make 25, ſo I ſette that 5 vnder the thirde place, and keepe the 2 in my mynde.

```
 1542
  365
 7710
   52
```

Than multiplye I foorth, ſaying: 6 times 5, maketh 30, and 2 in mind make 32, wherof I write the 2 vnder the iiij. place, and keepe the 3 in my mynde.

```
 1542
  365
 7710
 252
```

Than do I multiplye the laſte fygure 1 by 6, and it maketh 6, to that I adde the 3 in my mynde, and it maketh 9, which I write in the fyſte place.

```
 1542
  365
 7710
9252
```

And ſo haue I ended 2 fygures of the multiplyer.

Than with the third and laſte multiplyer doo I lykewaies, and ſaye fyrſte 1 3 tymes 2, make 6, whiche I write in the thirde place vnder the multiplier.

```
  1542
   365
  7710
 9252
  6
```

Than by that 3 do I multiplye likewaies the ſeconde fygure 4, & it maketh 12, wherof I write the digitte 2 vnder the fourth place, and the article 1 I keepe in mynde.

```
  1542
   365
  7710
 9252
 26
```

Than come I to the third fygure 5, ſaying: 3 tymes 5, maketh 15, and the 1 in my mynde, make 16, thereof I write the 6 vnder the fyfte place, and keepe the article 1 in my mynde.

```
  1 5 4 2
    3 6 5
  -------
  7 7 1 0
9 2 5 2
6 2 6
```

Than come I to the laſte fygure whyche is 1, and multiplye it by 3, and it maketh 3, therto I adde the one in my mynde, and it maketh 4, whiche I write in the 6 place. And than haue I ended the multiplication, and the fygures ſtande in ordre thus.

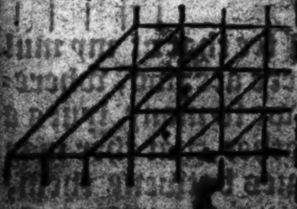
```
    1 5 4 2
      3 6 5
    -------
    7 7 1 0
  9 2 5 2
4 6 2 6
```

Whyche parcels yf I adde into one ſum, it wyll be 562830, which is the groſſe or totall ſum of all that multiplication.

Scho. Well, this manner of Multiplication I perceaue: but what other ſortes haue you?

An other waye of multiplication.

Maſt. There is one way that is wrought by a checker table, made thus.

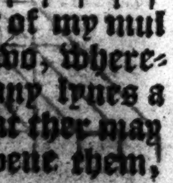


Looke howe manye places your ſum hath that you wold multiply, ſo many ſquares muſte you make in your table, from the right ſyde to the lefte, and ſo manye from the hygher part to the lower, as there be places in your multiplier. Than ſette downe your greateſt ſumme fyrſte on the toppe of the table, euery figure in due ordre, in a ſquare alone, I meane in thoſe ſquares that be open and vncroſſed. And lykewiſe in thoſe lyke ſquares at the right hande, ſette downe your multiplicatour or multiplier, the laſt figure in the higheſt place, & ſo downwarde, that the firſte figure may be in the loweſt place.

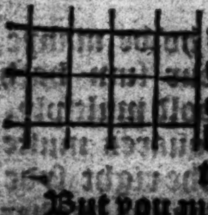
Sc. Syr if it pleaſe you, me thinketh than I vnderſtande you beſte, whan you doo not ſtand longe in telling the rule before examples. But propoſe ſome example, and then in declaryng it, brynge in the rules withall.

M. In deede that way is eaſieſt for a yong learner, therfore wyll I euen ſo doo. Take this example nowe: I wolde multiply 2036 by 23.

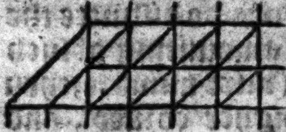
Fyrſte I conſidre that my greateſt numbre hath four fygures or places, and therfore I make ſo manye romes betwene lynes, thus.



Than I ſee that of my multipliers there are two, wherefore I drawe ſo many lynes a croſſe the other, that ther may be two romes betwene them, thus.

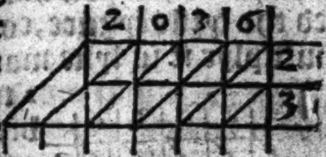


But you muſte not forget to let the endes of the lynes runne oute, as it appeareth in thys this patron, for in thoſe open ſquares muſt your twoo firſt numbres, and all the totall ſumme be ſet.

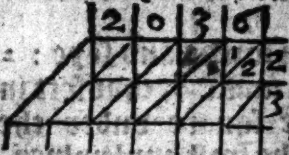


Than drawe a croſſe bar through euery cloſe ſquare, ſo that it maye reach doun to the loweſt ouerthwart lyne, as in this forme. And than is your checker forme prepared.

Than ſett downe your firſt or greateſt ſumme on the toppe & your multiplier on the right ſyde in the open ſquares, thus.

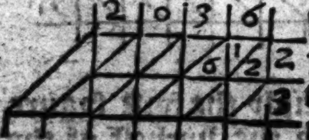


Than begyn to multiply the fyrſt fygure of the higher ſumme by the higheſt of the multiplier, ſaying: 2 tymes 6 make 12. that 12 muſte you write in the ſquare that is againſt the 2 and the 6, but of ſuche mãner, that the digit be ſet in the nether corner of yͤ ſquare, & the article in the hygher corner. as you maye ſee in this exãple.

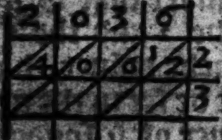


And ſo of euerye other multiplication, what euer amounteth you muſt write in the common ſquare, which is againſt both thoſe fygures, by whiche you doo multiplye. And if that ſumme do make but one digite, than muſt it be ſet in yͤ lower corner of the ſquare but if it make an article, than write the article in the higher corner, and let the cyphre go (if you wyll) euermore: for here it ſerueth for nothing, ſeeyng the lynes do diſtincte the places: but if the ſumme amounting of ſuch multiplication do make a mixt numbre, then write the article in the higher corner, and the dygite in the lower corner, as I dydde by that 12.

Then whan you haue multiplied and ended the fyrſte fygure, come to the nexte, and multiplye it in lyke manner, as in ſayinge: 2 tymes 3 is 6, that 6, bicauſe it is but a digit, you ſhall ſet in the nether corner of ẏ ſquare, next vnder 3, thus.

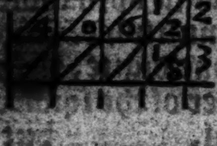


Then go forth ſaying: 2 tymes 0 is 0, ſet that vnder the barre (if you lyſt) in the third ſquare.



Than forthe and ſaye: 2 tymes 2 make 4, that ſett in the laſte ſquare vnder the bar, ſo haue you ended the fyrſt multiplyer.

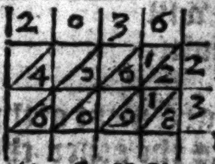
Come now to the ſecond multiplier, and ſaye: 3 tymes 6 make 18, of whiche ſumme the article 1 muſt be ſette aboue the bar, in that ſquare that is next to the 1, (as you ſee heere) and the 8 vnder the barre.



Then ſaye: 3 tymes 3 maketh 9, ſett it in the nexte ſquare beneth the barre, Then 3 tymes 0 is 0, write it in the nexte ſquare, or let it go, for all is one.

S. I perceaue it well: for heere the lynes diſtincte the places, wherefore cyphres doo only ſerue, and therfore heere they need not to bee.

M. Then ſaye farther: 3 tymes 2 make 6, write that in the laſt ſquare, then wyll the whole fygure ſtand thus.



S. Now coulde I (me ſemeth) do lyke agayne: But howe ſhall I doo nowe to gather the ſumme?

Ma. Marke fyrſte the ordre of the places in this fygure, and ſo ſhall you perceaue the reaſon of gatherynge them into a ſumme.

The ſloape barres doo parte the places. ſo that the fyrſte place is the loweſt corner, (in all ſuch figures) of the nethermoſt ſquare next the right hand: and all the half ſquares betweene that barre and the nexte, ſtandeth for the ſeconde place, and ſo the roume betweene that and the nexte barre is the third place: and ſo foorthe. Now yf you perceaue this, then muſte you adde all the fygures of one place togither, as if you had an Addition of dyuers ſummes.

S. If I vnderſtand you right, then mu [illegible] take here in this example 8 to be in the [illegible] place. 9, 1, and 2 in the ſeconde; 0, 6, 1, in the thirde: 6, 0, in the fourth: 4 in the fyfte: and the ſyxte place hath no fygure.

M. You ſaye well, and the reaſon is bycauſe the multiplication ſeruinge to that ſquare, made but a digitte.

Scho. Then it is all one, as if they ſtood thus.

12
061
46098

Ma. Euen ſo it is: and nowe adde this ſumme, and there will appeare the totall of the multiplication to be 46828.

roofe
houe
ies,

And if you will ſee the agreement of this manner of multiplycation, & the other that you learned before, then multiplye thoſe twoo ſummes (that is 2036, by 23) after the fyrſt maner without ſquares.

2036
23

S. You mean to ſet them thus in ordre.

2036
23
18
609

And then multiplye 3 into 6, make 18. 3 tymes 3 make 9. 3 tymes 0 is 0. then 3 times 2 make 6: whiche muſt be ſet thus.

Then doo I lyke wayes with the ſeconde multiplyer, ſayinge: 2 tymes 6 make 12, 2 tymes 3 make 6, 2 tymes 0 [illegible], and 2 tymes 2 make 4, whych when I adde to the other, then wyll the whole multiplication ſtand thus.

Ma. So that you maye ſee in euery place the ſame fygures, as they were in ỹ multiplicatiõ by ſquares, though thei differ i [illegible] & lownaſſe of places, but being added togither, they make one ſume.

2036
23
18
609
12
406

And thus now ye haue learned thre ſortes of multiplication, whiche you lyketh beſte, that may you vſe.

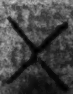
Yet are there other formes, but ſyth they nothing differ from theſe thre in effecte, but onely in ſetting of the nũbres, I wyll ouerpaſſe them tyll a more meater place & tyme. And nowe will I inſtructe you in Diuiſion, ſo that you thynke your ſelfe ſufficientlye to perceaue what I haue taught you.

Scholer. Yes ſyr I thanke you: but I doo not perceaue howe to examine my worke, to try whether I haue well doone or no.

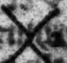
M. That is commonly vſed by the proofe of 9, as you learned before in Addition and Subtractiõ; ſaue that it hath this waies dyuers from them.

Proo
multip
cion.

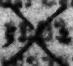
Firſt you muſt make a croſſe after this manner.



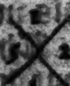
Then muſt you examine your ſum that ſhoulde bee multiplied, and looke what remayneth after caſting awaye of 9, that ſet you at the one ſyde of the croſſe; then examin the multiplier, and what ſo euer remayneth in it after caſtyng awaye 9 as often as you can, write that at the other ſide of the croſſe, then muſt you multiply thoſe ij numbres together, and looke what amounteth therof, if it be vnder 9, write it at the higher parte of the croſſe: but if it bee aboue 9, then take thence 9 as often as you can, and write the reſte at the head of the croſſe. As in the laſte example of multiplication, the numbre to be multiplied is 2036, wherin is ones 9, and 2 remaineth, whiche I write at one ſyde of a croſſe, thus.



Than doo I examine the multiplyer, whiche is 23, wherin there is no 9, but 5 in all, that 5 therfore I ſette at the other ſyde of the croſſe, thus.



Then do I multiplye 5 by 2, and it maketh 10, from whiche I withdrawe 9, and there reſteth 1, that 1 do I ſet at the head of the croſſe. than do I examine the groſſe ſum amountyng of the multiplication, which is 46828, wherein I fynde 9 three tymes and 1 remaynyng, that 1 I ſet at the fote of the croſſe, and then I ſee it to agree with the other 1 at the topp of the croſſe, and ſo know I that I haue doone well: for if they twoo dyd differ, than were my worke vaine,



and the multiplication falſe.

This is the cōmon proofe, but the moſte certaine proofe is by Diuiſion, of whyche I wyll anon inſtructe you.

S. Syr what is the chiefe vſe of Multiplication?

M. The vſe of it is greater then you can yet vnderſtande, howe be it, theſe plain commodities it hathe, that if you would reſolue any great and whole value into many ſmal and leſs portions: as if you would chaunge poundes into ſhyllinges, pennies or anye other greater or ſmaller parcelles, by multiplication yee ſhall doo it ſpeedilye and eaſilye. Alſo if you ſhoulde neede to adde one ſumme to it ſelfe, or to anye other oftentymes, you ſhall doo it by Multiplication muche more ſpeedily, readily, eaſyly & ſurely than by often and ſundrye Additions. Take you theſe commodities groſſely ſhewed for an anſwere at this tyme, and hereafter I ſhall more abundantly make you to perceaue the vſe of it.

DIVISION.

Scholar.

Vel ſyr, then in Diuiſion I pray you to inſtructe me. But me thinketh by the name of it, that it ſhould be all one with Multiplication: for I call that Diuiſion, when anye thinge is parted into dyuers and many partes.

M. You take it as it is take comonly, how be it, if you marke well, you ſhall perceaue that it is quite contrary to Multiplication, and dooth not parte one thinge or fewe thinges into manye, but contrarye waies it bringeth many parcels into few, but yet ſo, that there fewe taken togyther, are equall in valure to the other many. for by diuiſion pennies are tourned into ſhyllinges, and ſhyllinges into poundes. as for example of 120 ſhyllinges, it maketh 6 pounde, ſo are 120 turned into 6, whiche is a ſmaller numbre: but then yf you conſidre the denominatours, you ſhall ſe that they are ſuche, that one of the latter is equall to 20 of the fyrſte, and ſo in valewe the ſummes are one, though in numbre thei do farre differ, and the latter ſumme is the leſſer, and ſo is it alwayes in diuiſion, howe be by it, yet in ÿ workinge the summe is parted by an other, & therof doth it take the name.

Sc. I thinke I shall better vnderstand the reason of the name, when I knowe the vse of the worke, therfore nowe wold I gladly learne that.

Ma. Diuision is a partition of a greater summe by a lesser. Therfore (as you maye perceaue) vnto Diuision are required twoo numbres: the fyrste, whiche shoulde be diuided, and that muste be the greater: and the seconde, by whiche the other muste bee diuided, and that is the lesser, and is caled the Diuisor.

The fyrste muste be fyrst written, and the seconde so set vnder it, that the laste fygure of the lower numbre be right vnder the last of the higher, contrary wayes to the worke of the other kindes of Arithmetike: for in the the twoo fyrste fygures were set euer meete one vnder the other, but in Diuision the last fygures muste be set meete, excepte it chaunce so that the last fygure of the Diuisor be greater then the laste of the hygher numbre, for then you shall set the laste of the Diuisour vnder the laste saue one of the hygher numbre, as for example.

If you shoulde dyuide 365 (which are the the sum of the daies of a yere) by 28 (which ar the dayes of a common moneth) then should you set them thus.

But if you would diuide thoſe 365 daies, by 52, whiche is the numbre of weekes in one yeare, then ſhuld you ſet them thus.

365
52

Lykewaies if I wolde diuide the ſame 365 by 4, which is the ſum of the quarters of a yeare, then muſt I ſette them thus.

365
4

Scholar. Sy, thys doo I vnderſtand, but howe nowe ſhoulde I doo to diuide the one by the other?

M. You muſt begyn with the laſte figure next the lefte hand, and ſee how meny times the laſte fygure of the diuiſor, maye bee taken out of the laſte fygure of the ouer numbre, and that ſhall you note within a crooked line toward your ryghte hande. As for example.

365
28

I would diuide 365 by 28, then ſet I thoſe ii. ſummes thus.

And I looke howe many tymes I maye fynde 2 (which is the laſte fygure of the diuiſour) in 3 (which is the laſte of the numbre to be diuided) and conſidering that I cā take 2 out of 3 but ones, I make a crooked lyne at the right hande of the nūbres, & within it I ſette 1, and that is called the Quotient numbre. Then bycauſe that when 2 is taken out of 3 there remayneth 1, I muſte write that 1 ouer 3, and deface or cancell the 3 and the 1, then wil the figures ſtand thus.

1 6 5
8

Then muſt I go to the next fygure of the diuiſor, and take it lykewayes ſo manye tymes oute of the fygures that be ouer it, and loke what doth remain, that I muſte write ouer theym, and cancell them, as in this example.

I take ones 8 out of 16, and there remaineth 8, whiche I muſte ſet ouer the 6, and cancell or croſſe out the 16, and the 8 of the diuiſour: And then will the figures ſtand thus. And ſo haue I ones wrought.

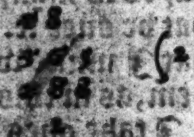
8
1 6 5
8

Sch. So I perceaue that you take the nether fygure not onlye oute of the other that is right ouer him, but out of that with the other alſo that remaineth beſide it, and are written toward the left hande.

M. So muſt you doo, for you muſt ſo take the diuiſor out of the ouer numbre, that there remaine not ouer it ſo greate a numbre as it ſelfe is: for then were your worke in vaine.

But yet againe here muſt you marke, that when you ſeeke howe manye times the laſte fygure of the diuiſour may bee founde in the numbre ouer him, that you looke alſo whether you maye as often fynde all the figures folowing in thoſe that are aboue thē, if not, take your Quotient leſſe by one, & thā proue againe: and ſo ſtylle, tyll you fynde a meete Quotient.

Whan you haue thus wroughte ones, than muſt you begyn againe, and write your diuiſour a newe, nerer towarde the righte hande by one place, as in this example, you ſhall ſette 2 vnder 8, and 8 vnder 5, thus.



Then (as before) ſeeke howe manye tymes you maye take the laſte diuiſour out of the numbre ouer him.

S. That maye I do here 4 times.

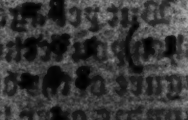
M. Truth it is that you may fynd 2 foure tymes in 8: but than marke whether you cā fynde the fygure folowinge ſo manye tymes in the other that is ouer him. Can you finde 8 foure times in 6?

S. No, nother yet ones.

M. Therfore take 2 out of 8, ones leſſe.

S. That is 3 tymes.

M. Wel, the 3 times 2 make 6: if I take 6 out of 8, ther remaineth 2: which 2 w the 5 folowing make 25, in which ſū I may find 8 iii. times alſo: and therfore I take 3 as a true quotient, & write it in the croked line of the quotient before the 2, thus.



Than ſaye I: 3 tymes 2 make 6, than 6 oute of 8 reſteth 2, therefore I cancell the 2 and the 8, and wryte ouer it the 2 that doth remaine: thus.

```
  2
1 8
3 6 5   (13
2 8 8
  2
```

Then do I take 8 as many tymes out of 25, ſaying: 3 tymes 8 make 24, and if I take 24 oute of 25, there remaineth 1, ſo then I cancell 25 and 8, and ouer ye 5 I ſet 1 thus.

```
  2
1 8 1
3 6 5   (13
2 8 8
  2
```

And nowe haue I doone whith diuidyng, for I can fynde my diuiſour 28 no more in the ouer ſumme.

S. No, except you wold part the 1 that remaineth into 28 partes.

M. That is well ſayd. and ſo muſt we doo in ſuche caſes, when there remayneth anye thynge: but I will lett that paſſe nowe, and will make you perfect in whole diuiſion, and will hereafter teache you peculiarly of broken numbre, called Fractions.

Nowe if you do perceaue the ordre of diuiſion, than doo you diuide this ſumme, 136280 by 452.

Sch. Fyrſte I ſet downe the numbre that ſhuld be dyuided, than doo I ſet the diuiſour vnder it, ſo that the laſt figure of it, be right vnder the laſte fygure of the ouer numbre.

Than wyll it be thus.

136280(
452

M. Can you take the laſte of your diuiſor (which is 4) out of 1, whiche is the laſte of the ouer numbre?

Sc. I had forgotten, bicauſe the laſt of the diuiſor can not be taken out of the laſt of the ouer numbre, in as much as it is the greter, therefore muſte I ſett the diuiſor one place more forwarde towarde the ryght hande thus.

136280(
 452

And then muſte I looke how often I may fynd the laſt fygure of the diuiſor (that is 4) in 13, which thing I may doo 3 tymes, therfore doo I ſay: 3 times 4 is 12, which I take out of 13, and there remaineth 1. Then doo I make at the ryghte hande of my ſummes a crooked lyne, and write before it my quotiente 3, and I cancell 13 and 4, and ouer the 3 I ſette the 1 that remayneth, and thā the fygures ſtande thus.

 1
~~13~~6280(3
 ~~4~~52

Than doo I multiplye the ſame quotient into euerye fygure of the diuiſor, and withdrawe the ſumme that amounteth out of the numbres ouer theim, as fyrſt I ſaye: 3 times 5, make 15, which I take from 16, and ther reſteth 1, I cancel therfore 16 and 5, & wryte ouer the 6 the 1 that remayneth, thus.

 11
~~136~~280(3
 ~~45~~2

Than do I ſay lykewiſe

Than

tymes 2 make 6, whiche I take out of 12, and ther reſteth 6, therfore I cancell the 12 and the 2, and ouer the 2 I wryte 6 that remaineth, thus.

2 2 6
2 3 5 2 8 0 (5
4 5 2

Then ſhould I ſette forwarde the diuiſor into the next place toward the right hand thus.

2 2 6
2 5 5 2 8 0 (5
4 5 2
4

M. But you may ſe, that ouer the 4 is no fygure, therfore muſt you ſette the diuiſor yet forwarder by an other place.

And marke whan ſo euer it chaunceth ſo, that you ſhould ſet forwarde the diuiſor, and that it can not ſtande there, bicauſe there is no number ouer the laſte place, or if there be any, it is leſſer then the laſt fygure of the diuiſor, than muſt you remoue the diuiſor yet ones againe: and bycauſe that his fyrſt place of remouing fayled him, therefore ſhall you write in the quotient a cyphre 0. And if you ſhuld by chaunce need to do ſo often times, for euery tyme write a cyphre in the quotient. The reaſon of this, will I ſhewe you hereafter. Nota

2 2 6
2 5 6 2 8 0
4 5 2 5 2 (50
4

Sc. Then muſte I ſette my ſummes thus.

And bicauſe I remoued the diuiſor, ſo that I ouerſkipped one place, I muſte write a cyphre in phre in the quotiente: and than muſt I ſeeke a newe quotient, as in this example I muſte ſaye: Howe many tymes 4 is there in 6? and ſyth it canne be but ones, therfore do I wryte 1 in the quotient, and than ſaye I: 1 tymes 4 taken out of 6, remaineth 2, I cancell the 6 and the 4, and write 2 ouer them thus.

    2
2 2 6
1 5 6 2 8 0 (301
4 5 2 5 2
    4

Then ſaye I againe: ones 5 out of 28, remaineth 23, I let the 2 ſtād as it dyd, and ouer the 8 I ſet 3, cancelling yͤ 8 and the 5 vnder it, thus.

    2
2 2 6 3
1 5 6 2 8 0 (301
4 5 2 5 2
    4

M. You might as well haue ſayde, ones 5 out of 8, and ſo remayneth 3. but nowe go forth.

S. Than ones 2 out of 0, can not be, what ſhall I nowe doo?

M. Borow of the next numbre that is behynd (for there is 230) and doo as you learned in Subtraction in a lyke caſe.

Scholer. Than muſte I borowe 1 of the 3 commyng behinde next, and make that 0 to bee 10, and then take I 2 oute of 10, and there reſteth 8. And bycauſe I borowed one of the 3, I muſte cancell the 3, and wrytte 2 ouer it, than doth the fygure

ſta

ſtande thus.

$$\begin{array}{cccccc}
 & & & 2 & 2 & \\
 & \cancel{} & \cancel{} & 6 & 5 & 8 \\
\cancel{1} & \cancel{3} & \cancel{6} & \cancel{2} & 8 & \cancel{0} \\
 & 4 & 5 & 2 & 5 & 2 \\
 & & & 4 & &
\end{array}$$

(301

M. Nowe haue you doone, and yet remayneth 228, & your quotient dooth ſhewe you, yͭ if you diuide 136280 by 452, you ſhall fynde your diuiſor in youre greater numbre 301, that is CCC tymes & ones, and 228 remayning.

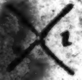
And in the other example where I diuyded 365 by 28, the quotiente was 13 and 1 remayned: whereby I knowe that in a year (whiche contayneth 365 dayes) ther are 13 monethes, reckening 28 dayes (or 4 weekes) iuſte to a moneth, and 1 daye more.

Sc. Why than doo we call a yeare but 12 monethes?

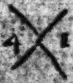
M. Of that at a more conuenient time wil I fully inſtructe you: but now it is not conuenient to entangle your mynde with other thynges, than do dyrectlye pertaine to your matter. Therfore yf you cā remembre what you haue hearde, you haue learned a ſhorte manner of diuiſion, whiche I woulde haue you often to practiſe, ſo that you maye bee perfecte in it, and hereafter I wyll ſhew you certaine other proper pointes touchynge it.

S. Then I praye you, yet tell me howe I ſhall examine and trye my wooꝛke, whether I haue done well or no, that though no man be by me to tell me, yet I maye perceaue it my ſelfe.

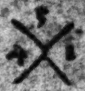
M. Some men (yea and cōmonly) do trye that by the rule of 9, as in all the other kindes, ſaue that their ordre is this. Fyrſt they caſt awaye 9, as often as they can out of the diuiſor, and that that remaineth, they ſet at one ſyde of a croſſe: As in our fyrſt example, the diuiſor was 28, from whiche you maye take 9 thre tymes, and 1 remaineth, which they ſette by a croſſe, thus.



Than doo they lykewiſe examine the quotient (whiche in our exaumple is 13) and frō thence they caſt away 9 as often as they can, and the remainer they ſet at the other ſyde of the croſſe, and than multiplye they togither thoſe 2 remainers: and to it that amounteth, thei adde the remainer of the diuiſion if there were any, from that whole ſumme they withdrawe 9, as often as they canne, and the reſte they ſet at the head of the croſſe, as in our exāple the quotiente is 13, whiche maketh only 4, and therefore muſte you ſette 4 at the other ſyde of the croſſe, thus.



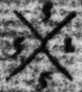
Then multiplye 4 by 1, and it yeldeth but 4. thereto adde the remainer of the diuiſion (whiche was 1) and it will bee 5, whiche ſumme doothe not amounte to 9, and therefore muſte bee ſette wholly at the headde of the



the croſſe, as you ſee here.

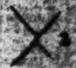
And this numbre on the heade of the croſſe is the fyrſte proofe, to whyche if you fynd an other lyke in the numbre that was diuided, then haue you doone well.

Therfore now ſhall you likewiſe examine the whole ſumme that was diuided, & take awaye 9, as often as you can, and that that remayneth, ſet at the foote of the croſſe: and if it bee equall to that in the heade of the croſſe, than haue you well doone, elſe not. As in our example the whole ſumme was 3,6,5, which maketh 14, frō that take 9, and there reſteth 5, whiche ſet at the foote of the croſſe, thus. And you ſhall ſee that they agree: therefore haue you well doone.

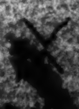


5. Nowe will I lykewiſe examine our ſeconde example, wher the diuiſor was 4,5,2 whiche maketh 11: from thence I take 9, & the 2 that remayneth, I ſet at ye right ſide of a croſſe, thus.

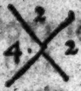
A ſecōde proofe.



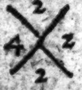
Than examine I the quotient, whiche was 3,0,1, wher I fynd but only 4, that doo I ſet at the other ſyde of the croſſe, thus.



Than doo I multiply 4 by 2, and it maketh 8, to that doo I adde the remayner of the diuiſion, (whiche was 2,1,8, and maketh 12) and they twoo make 10, wherein I fynd twiſe 9, and 2 remayninge, that 2 muſt I ſet at the head of the croſſe thus.



Than doo I examine the whole numbre to bee dyuided, which was 136280, wher I fynde twyſe 9 and 2 remainynge, whiche I ſette at the foote of the croſſe, thus.



And bicauſe that it doothe agre with the fygure at the head of the croſſe, I know that the diuiſion was well wrought.

M. This is the common pro[illegible]ow be it, the more certayne woorkinge is [illegible] the contrary kinde, as to proue diuiſion by multiplication, thus.

ofe by ltiplica- n.

Multiplye the quotient by the Diuiſour, and if the ſumme that amounteth bee equal to the ſumme that ſhould be diuyded, then haue you well diuided, elſe not.

Howe be it, this muſte you marke, that if there remained any thing after the diuiſion, that muſt you adde to the ſumm that amounteth of the multiplication: as in our fyrſt example the quotient was 13, and the diuiſour was 28. Now multiply the one by the other and the ſumme will bee 364: ſo that if you adde the one that remained after the diuiſiō, then wyll it bee 365, which was the ſumme that ſhould be diuided. and therfore I know that I haue well doone.

S. Nowe will I proue the ſame in the ſe-

cond

conde example, whoſe diuiſor was 452, and the quotient 301: theſe do I multiply togither, and ther amounteth 136052, to which if I adde the 228 that remained, than will it be 136280, whiche was the whole ſumme to bee diuided, and therfore I perceaue that I haue well doone.

M. This is the ſureſt way to examine Diuiſion by Multiplication: & contrary wiſe the ſureſt profe of Multiplicatiō, is Diuiſiō.

And therfore nowe will I ſhewe how you maye proue Multiplication by Diuiſion.

A proo
of mul
cation
Diuiſ

When you haue ended Multiplication, and woulde knowe whether you haue well done or not, ſet the groſſe ſum that amounteth of the Multiplication ouermoſt, and diuide it by the multiplier: and if the quotient be the ſame number that ſhould be multiplied, then haue you well wrought, elſe not: as in that example, where we multiplyed 264, by 29, the groſſe ſumme was 7656.

Nowe if you will knowe whether that Multiplication be true, you ſhall diuide that 7656, by the Multiplier 29: and you ſhall perceaue that the quotiente wyll bee 264, and that is a token that you haue well wrought.

S. By your patience, I wyll proue that: and fyrſt I ſet downe the groſſe ſumme and the Multiplier, not after the rule of Multiplication, but after the rule of Diuiſion, for now that numbre is become the diuiſor, that was before the multiplier, I ſhal ſet theym therfore thus.

7656
29

Then ſhall I ſeeke how many times 2 in 7, that maye bee 3 tymes, and 1 remaineth: but then may not 9 be found ſo often in 16, therfore muſt I take a leſſer quotient, that is to ſay, 2. then ſay I: twiſe 2 maketh 4, whiche I take out of 7, & there remayneth 3. then do I cancell 7 and 2, and ouer 7 I wryte 3, and in the quotient I ſette 2, ſo the fygures ſtande thus.

3
7656 (2
29

Then ſaye I forthe, 2 times 9 make 18, whiche I bate oute of 36, and there reſteth 18, then cancell I 3, and ouer hym ſet 1, and lykewiſe I cancell 6 and 9, and ouer them I ſet 8, ſo that thus ſtande the fygures.

1
38
7656 (2
29

Then doo I ſette forwarde the diuiſor by one place, and ſeeke a newe quotient, that is to ſaye, howe many tymes 2 are in 18, whiche I fynde to be 9 times, but then canne I not fynde 9 ſo many tymes in 5, therfore I take a leſſer quotient, as to ſay, 8: but yet is that to greate, for if I take 8 times 2, oute of 18, there remayneth but 2, and I canne not fynde 8 tymes 9 in 25; therefore yet I take a leſſer quotient, yͭ is 7, which is alſo to great, for if I take 7 tymes 2 out of 18, ther reſteth 4, but nowe I can not take 7 tymes 9 out of 45, therfore yet I ſeeke a leſſer quotient, as to ſay, 6: the ſay I, 6 times 2, make 12, that I take out of 18, and there remayneth 6, ſo I cancell that 18 & the 2, & write 6 ouer 8, thus.

26
58
7656 (26
299
2

The ſay I forth: 6 times 9, maketh 54, that take I out of 65, and there remaineth 11, and the figures ſtand thus

26
581
7656 (26
299
2

Then muſt I ſet forth the diuiſor againe and ſeeke a newe quotient, which will bee 4, for though I may find 2 in 11 fiue times, & 1 remaine, yet I can not fynd 9 ſo often in 16, therfore I ſette the figures thus.

1
26
581
7656 (264
2999
22

And the 4 in the quotient I multiply into the fygures of the diuiſour, ſaying; 4 tymes 2, maketh 8, whiche I take oute of 11, & there reſteth 3, therfore I cancell the 11 and the 2, and ſet 3 ouer the fyrſt place of 11, thus.

2
263
582
7656 (264
2999
22

And then do I ſay forth 4 tymes 9 maketh 36, whiche I take from

36, and there remaineth nothynge, ſo that the quotient of this diuiſion, where 7656 is diuided by 29, is 264. Whiche dooth declare, that if 264 be multiplied by 29, the ſumme wil be 7656. And thus I perceaue now how both Multiplication is prooued by Diuiſion and Diuiſion alſo by Multiplication.

M. Nowe haue I ended the fyue moſt cõmon kynds of Arithmetike: for as touching Mediation, Duplation, Triplacion & ſuche other, they are no ſeuerall kyndes of Arithmetike, but are contayned vnder the other: for Mediation is contained vnder Diuiſion, and is nothing elſe but diuiding by 2: and ſo ar Duplacion and Triplacion contayned vnder Multiplication: for Duplacion is nothinge elſe but multiplynge by 2, and Triplacion is multiplying by 3, of which I wil onlye propoſe examples, for the rules you haue heard already.

exãple Media-

If you woulde Mediate or diuide into 2 this ſumme 4531010, you ſhall ſette 2 for the diuiſor, and worke as you learned before, as thus.

```
4 5 3 1 0 1 0 (
2
```

Then I fynde 2 in 4 two tymes, therfore my quotient muſte bee 2, ſo I cancell 4 and 2 and remoue the diuiſor forward, thus.

```
4 5 3 1 0 1 0 (2
2 2
```

Then againe I fynde 2 in 5 twyſe, and 1 remairemayning, ſo I write 2 againe for my ſecond nūbre of the quotiēt, and cancell 5 and 2, & ouer 5 I ſette 1, thus.

```
 1
4531010 (22
22
```

Then remoue I the diuiſor forward and ſeeke a newe quotient, whiche is 6: then ſay I, 6 times 2 make 12, take that oute of 13, & there reſteth 1, ſo I cancell 2 and 13: and ouer 3 I ſet 1, thus.

```
 1111 3
4531010 (226
2222222
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Then remoue I the diuiſor forward, and ſeeke a newe quotient whiche is 5, then take I twyſe 5 oute of 11, and ther reſteth 11, ſo I cancell the 2 and the laſte fygure of 1, & let the firſt ſtande thus.

```
  11
4531010 (2265
2222
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Then remoue I the diuiſor forward and ſeeke a new quotiēt, which is 5: thē take I 2 fiue times out of 10, and there reſteth nothinge.

```
  11
4531010 (22655
22222
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Then remoue I againe the diuiſor forwarde, thus.

But bicauſe I can not find the diuiſor in the nūbre ouer it, I muſt ſet a cyphre in the quotient, and re

```
  11
4531010 (226550
2222222
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moue the diuiſor to the next place as appeareth in the fygure before.

Then ſeeke I a newe quotient whiche I fynd to be 5, for ſo many tymes may I haue 2 in 10. Then haue I fullye ended this Mediation or Diuiſion by 2, and the quotente is this, 2265505, which is the half of 4531010 as you maye trye by Duplacion: for double that quotient, or multiplye it by 2, and the ſame numbre will amount.

placiō

I will no longer tarry about theſe, ſeyng they are but membres of the other kindes. But here nowe will I teache you certaine eaſye formes both of Multiplication and of Diuiſion, and fyrſte of Multiplication.

y for
s of mul
cation.

If you would therfore multiply any ſum by 10, you ſhall need to do no more but adde a cyphre before his fyrſte place: as for example: 36 multiplied by 10, make 360. Likewaies if you will multiply any ſumme by 100, put two cyphres at his beginnyng.

So if you would multiplye anye ſumme by a thouſand, adde three cyphres to the beginnyng of it.

S. This do I wel perceue, & alſo yͤ reaſō of it

M. I will omitte all reaſons tyll our next meting, whē I ſhall tell you the reaſon of all other partes of Arithmetike alſo: and as to our matter nowe, loke (as I haue told you) that you do both remembre it, and alſo often practiſe it.

But if you woulde multiply any numbre by 5, marke fyrſt whether the numbre be euen or odde, and if it be euen, take the halfe of it, and wryte a cyphre at the beginning of it, as for example: I would multiplye 2564 by 5, I take the halfe of it, which is 1282 (as you maye knowe by Mediation) and before it I ſette a cyphre thus, 12820, and this is 5 tymes 2564.

And thus maye you doo with anye other euen ſumme that you woulde multiply by 5.

But if the ſumme be odde, as for example 2563, then muſt you take the leſſer halfe of it, or (if you wyll) take awaye 1 frome the fyrſte figure (as here take 1 from 3) and then take the halfe of the reſte, and ſette before it 5: as of 2563, the leſſer halfe is 1281, for heere I take but 1 for the halfe of 3, and if I putte 5 before that leſſer halfe, then haue I multiplied it 5 tymes, as thus, 12815.

S. What meane you by the leſſer halfe?

Maſt. There is no iuſte halfe of any odde numbre, therefore if we diuide an odde numbre into two partes as nygh equal as canne be, yet wyll the one halfe excede the other halfe by one, as for example: The two moſt neareſt halfes of 9, are 5 and 4, and likewiſe of 15, are 7 and 8, wher you ſee that the one parte ſtyll is greater then the other by one. Nowe it is eaſy to know which is the greater halfe, and which is the leſſer halfe.

S. Then I perceaue you, and can do likewaies (I doubt not) with any ſumme. For if it bee not verye eaſye to part into halfe, then will I doo it by Mediation eaſilye inoughe.

M. That is a ſure waye. And nowe haue you learned howe to multiplye eaſilye by 5, 10, 100, 1000: and of lyke manner may you do with any other of that ſorte.

But now if you will multiplye by 20, 30, 40, and ſo forthe: or by 200, 300, and ſuche lyke, where there is one cyphre in the fyrſte place, or manye ordrely in the fyrſte places, you ſhall take awaye thoſe cyphres, and multiplye the ſumme onely by the other fygure, or fygures (if they bee many) and then at the beginning of the ſum that amounteth, ſhall you ſet ſo many cyphres as you toke awaye.

Example of 2873, whiche I wolde multiply by 300. Fyrſte I caſt away the twoo cyphres from the multiplier, and I multiplye the ſumme by onely 3 that is lefte, and it amounteth to 8619: before whiche I putte the twoo cyphres that I tooke away before, and then is it 861900. And that is the ſumme that amounteth, when 2873 is multiplyed by 300.

S. And if there were two or more figures beſyde the cyphres, I muſt onely take away the cyphres, and multiplye by the other fygures, as I learned before. as if I woulde multiplye 93648 by 25000, I ſhoulde take

awaye

away the thre cyphres, & multiplye the ſame by 25, and then at the beginnynge of that totall ſumme ſhould I adde the thre ciphers againe.

M. Euen ſo. but and if it chaunce the nūbre that ſhould be multiplyed, or bothe the ſummes, as well the numbre that ſhould be multiplied as yͤ multiplier, to haue cyphres in their fyrſte places, euermore caſte awaye the cyphres, and worke by the reſte: But remembre to reſtore as many cyphres to the amountyng ſumme, as you bated before. as in this example: 30200, ſhalbe multiplied by 206: I ſhal only take away the two cyphres from the greater numbre, and then multiply 302 by 206, and afterward adde the two cyphres againe. But if I wolde multiply the ſame 30200 by 2060, I ſhall not only take away the two ciphres from the numbre that ſhoulde be multiplied, but alſo I maye take awaye the one cyphre from the multiplier, & then muſte I adde 3 cyphres to the ſum that amounteth: but take heede that you take awaye no cyphre that commeth after anye ſignifying fygure, as in this laſt example, you may not take awaye that in the fourth place of the higher nūbre, nother that in the third place of the multiplier: howe bee it, yet this you may doo: If one cyphre or more come in the middeſt of your ſummes, you may multiply by the other figures, and ouer ſkippe

theym

them, but ſo that you giue euery figure his due place, as thus: I will multiplye 3026 by 2004, therfore I ſette theym thus.

3026
2004
24
1208

And thus I multiplye them: fyrſte 4 tymes 6, make 24. I ſet the 4 vnder the fyrſte place, and keepe the 2 in my mind, or write it downe for eaſye remembraunce: than ſaye I agayne: 4 tymes 2, maketh 8, and 4 tymes 0 maketh 0, then 4 tymes 3 maketh 12.

But now whan I come to the next cyphre bicauſe that it multiplyeth nothing, I lett it go. And lyke wayes the ſeconde cyphre, but then whan I doo come to the 2, and multiplye it into the 6 of the ouer numbre, you muſte take hede (accordynge as I taughte you in Multiplication) that the fyrſt numbre amountinge of the Multiplication, be ſette vnder the multiplyer right, and the other ordrely towarde the left hand, according as you may ſee in this example.

3026
2004
24
1208

Where if you had expreſſed the cyphres after the cōmon rate, than ſhould the figures ſtande as foloweth.

12
604

But in effecte all is one, ſaue that the firſt waye by ouerſkipping of the cyphres is the ſhorter and eaſyer waye: for that in effecte they be bothe one, the addition of the partels wyll declare, whiche in bothe wyll appeare thus.

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   3016
   2004
--------
     24
   1208
  0000
 0000
   12
 604
--------
 6064|04
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And nowe wyll I make an ende of this matter.

Sc. Syr I thanke you: for I ſe great eaſe in this waies of Multiplicatiõ, and if you canne ſhewe me ſuche lyke in Diuiſion, you ſhall greatlye further me.

Maſt. Yes I wyll teache you ſome eaſye waies in Diuiſion alſo, and firſt this: If you would diuide any ſumme by 10, you ſhal only with your penne make a ſquare lyne betweene the fyrſte fygure of your ſumme, and the ſeconde. and than haue you doone: for the whole numbre that foloweth the lyne, ſtandeth for the quotiente, and the fygure that is before the lyne, is the remainer. as for example 3648 diuided by 10, wyll ſtand thus. Where 364 is the quotient, and betokeneth that ſo many times are 10 in 3648: & the 8 after þ line is the remainer, which can not be diuided into 10. but by brekyng it into Fractions, wherewith I wyll not medle yet.

Eaſy for
mes of
uiſion.

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364|8
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And ſo lyke waies if you woulde dyuide anye ſumme by 100 with your penne, you ſhall cut away the twoo fyrſt figures: and yf you woulde diuide by 1000, you muſte cutte

awaye

away the 3 fyrſt fygures: and ſo of any other diuiſor, whoſe laſt fygure is 1, and the other be cyphres, locke how many cyphres the diuiſoure hathe, and ſo manye fygures at the beginning ſhal you cut away with the ſquire lyne, & they ſtande alwais for the remayner, bycauſe they are leſſe then the diuiſour, and canne not be dyuided by it. and the other fygures that be behynde the lyne, ſtand for the Quotient.

But nowe if your diuiſour haue anye other fygure in his laſte place than 1, and in all his other places haue cyphres, looke howe many cyphres there bee, cutte awaye ſo many of the fyrſte fygures of the numbre that ſhoulde bee diuided, and diuide the reſte that foloweth the lyne, by that fygure that is in the laſte place, as if it were the whole diuiſour.

Example of 64284, whiche I would dyuide by 300, here muſt I cut away the twoo fyrſte fygures, (for ſo many cyphres my diuiſour hath) and muſte diuide the reſt by 3, which is the figure in the laſte place of the diuiſour. Fyrſt therefore I parte awaye the twoo fyrſte fygures, and the ſum ſtandeth thus.

642|84.

Than doo I dyuide 642 by 3, and the quotient is 214, for in 6, I fynde twiſe 3, and in 4 ones, and 1 remayning, whiche 1 with the 2 were before, doothe make 12, wherein

I

I fynde 2 foure tymes. and this is a readye waye to tourne ſhyllinges into poundes: for ſyth 1 pounde dooth containe 20 ſhillinges, I muſte diuide the whole numbre of ſhyllinges by 20, therefore eaſilye to do it, I ſee that my diuiſour hath one cyphre, and therfore I cutte awaye one fygure from the beginninge of the whole ſumme of ſhillinges, and than I do mediate or diuide by 2, the other fygures or ſumme that foloweth.

S. I wyll put an example.

If you woulde diuide 64287 ſhillings by 20, that is to ſay: If I would tourn ſo many ſhillynges into pounds, I muſt cut away the fyrſt fygure, that is 7, & diuyde the reſt, that is 6428 by 2, ſo ſhall the quotiente bee 3214, whereby I knowe that 64287 ſhyllynges make 3214 pounde, and 7 ſhillinges remaining.

M. Nowe proue by Multiplication whether you haue well doone or no.

S. The quotient is 3214, which I do multiplye by the diuiſour 2, and it doth amount to 6428.

Ma. Hereby maye you perceaue not onlye that you haue well doone, but alſo howe by diuiſion you maye tourne ſhyllinges eaſilye into poundes: And contrary waies, by Multiplication you maye tourne poundes into ſhyllinges.

But heere ſhall you ſee amongeſt diuers

men, diuers formes of ſuche diuiſion, but if you marke what I haue tolde you, you ſhall perceaue eaſily all their wayes: for ſum men doo not cut awaye ſo manye of the firſte fygures of the ſumme that they would deuide, as ther ar cyphres in the firſt places of theyr diuiſor, but they ſet all their cyphres orderly vnder the fyrſte places of the numbre that they woulde dyuide, and than with the other fygure (or fygures if they be many) thei diuyde the reſte of their ſumme.

Example. If they woulde dyuide 725931 by 3400, thy ſette theyr ſummes thus.

725931
34 00

And then do they dyuide orderly tyll they come to the cyphres: for there they ſtay and ende their woorke, as in this example: They ſeeke how often 3 may be found in 7, whiche is 2 tymes, and 1 remaynyng, therfore they ſette 2 in the quotient, and cancell 3 and 7, and ouer 7 they ſette that 1 that remeyned thus.

1
725931 (2
34 00

Than do they go foorth ſayinge: twoo tymes 4, maketh 8, which thei take out of 12, and there remaineth 4, thus.

1 4
725931 (2
34 00

Than renewe they the diuiſor forwarde, and ſeeke howe often 3 maye be found in 4,

whyche

Whiche is but ones, and 1 remayneth, than ſet they 1 in the quotient, and cancel 3 & 4, and ouer theym they ſet that 1, thus.

$$\begin{array}{cccccccl}
 & 1 & & & & & & \\
\cancel{1} & \cancel{4} & & & & & & \\
\cancel{7} & \cancel{2} & 5 & 9 & 3 & 1 & (& 21 \\
\cancel{3} & \cancel{4} & 4 & & 0 & 0 & & \\
 & 3 & & & & & &
\end{array}$$

Than take they ones 4 oute of 15, and ther reſteth 11, or els more eaſily: Take ones 4 oute of 5, and there reſteth 1, ſo they cancell the 4 and 5, & ſett 1 ouer theym thus.

$$\begin{array}{cccccccl}
 & 1 & & & & & & \\
\cancel{1} & \cancel{4} & 1 & & & & & \\
\cancel{7} & \cancel{2} & \cancel{5} & 9 & 3 & 1 & (& 21 \\
\cancel{3} & \cancel{4} & \cancel{4} & & 0 & 0 & & \\
 & 3 & & & & & &
\end{array}$$

Then ſette they forthe the diuiſor againe, and ſecke howe many times 3 are in 11, whiche they finde 3 times, and 2 remayning: ſo thei ſet 3 in the quotient, & cancell 11 and 3, and ouer them ſetteth 2, thus.

$$\begin{array}{cccccccl}
 & & \cancel{1} & 2 & & & & \\
 & \cancel{1} & \cancel{4} & \cancel{1} & & & & \\
\cancel{7} & \cancel{2} & \cancel{5} & 9 & 3 & 1 & (& 213 \\
\cancel{3} & \cancel{4} & \cancel{4} & 4 & 0 & 0 & & \\
 & \cancel{3} & \cancel{3} & & & & &
\end{array}$$

Then doo they multiply 4 by 3, which maketh 12, that withdraw they out of 29, & there reſteth 17, of which the 7 muſte bee ſet ouer the 9, and the 1 ouer 2, thus.

$$\begin{array}{cccccccl}
 & & 1 & & & & & \\
 & \cancel{1} & \cancel{2} & & & & & \\
\cancel{1} & \cancel{4} & \cancel{1} & 7 & & & & \\
\cancel{7} & \cancel{2} & \cancel{5} & \cancel{9} & 3 & 1 & (& 213 \\
\cancel{3} & \cancel{4} & \cancel{4} & \cancel{4} & 0 & 0 & & \\
 & \cancel{3} & \cancel{3} & & & & &
\end{array}$$

And nowe are the 2 cyphres next enſuing ſo that the diuiſor cã no more be ſet forward and therfore is the diuiſion ended, and the remayner is 1731

Now the quotient, whiche is 213, doothe declare, that if you diuide 725931 by 3400, you ſhall fynde it therein 213 tymes, & there remayneth 1731, ſo ſhall you fynd it, yf you woorke as I taught you by cuttinge awaye the two fyrſte fygures, bycauſe of the two ciphres. But this muſt you mark (as you may perceaue by this laſte example) that yf there be lefte anye other remayner in the ſumme that was behynde the ſquire lyne, that the remainer muſte bee ſette to the latter end of the fyrſte remayner, which was cutte awai with the ſquyre lyne: as if you wold diuyde 725931 by 3400, after the forme ẏ I taughte you, than woulde your ſummes appear thus.

1
2 2
2 4 1 7
7 2 5 9|31 (213
3 4 4 4
3 3

So that 17, whiche remayneth after the line, muſte be ſette to the 31 (that was cutte away with the line) in higher places, as you ſee here: where that 17 with the 31 do make 1731.

ther in
en of
Dis

And here woulde I make an ende of Diuiſion, ſauinge that there commeth to my mynde one late inuention of eaſye Diuiſion, which I will brieflye ſet forth to you, ſo that if you fynd eaſe in it, you maye vſe it. Bycauſe that the hardeſt point in diuiſion is the ready and eaſy findyng of the quotient numbre:

bre: and againe, if that bee truely knowen, all the reſte is but lyghte to bee doone: therfore this waies ſhall you quickly and truely fynd the quotient.

Fyrſt write the nine fygures of numbre: I meane 123456789, not along as I haue ſette them nowe, but vp and doune as in this forme. And at the lefte ſyde of them draw a longe lyne as you ſee heere: then conſidre the diuiſor by whiche you intend to woorke, and ſet it on the lefte ſyde of the longe lyne, righte againſt 1. and for a diſtinctiō draw a line vnder it: then multiply your diuiſor orderly by eche of thoſe fygures, beginninge with 2, and ſo go downwarde tyll you haue ended all: And looke what doothe amounte of the multiplication of eche figure into the diuiſor, than do write againſt the figure whereby you did multiply.

| 1
| 2
| 3
| 4
| 5
| 6
| 7
| 8
| 9

Sc. By example I may perceaue it better.

M. Take thys example 263345 to bee diuided by 64, then muſte I ſette the 9 fygures as I ſaide before, and the diuiſor muſte I ſette againſt the 1, thus.

64 | 1
| 2
| 3
| 4
| 5
| 6
| 7
| 8
| 9

Than muſt I multiply that diuiſor by eche fygure orderly: firſte by 2, and it maketh 128, whiche I muſt ſet againſt 2 at the left hand.

Than

Then multiplye 64 by 3, and it maketh 192, whiche is ſette againſt 3. Then 4 tymes 64, make 256, that ſette I by 4. Then ſay I, 5 tymes 64 make 320, that ſette I agaynſte 5. Than 6 tymes 64, make 384, that ſette I againſt 6. Then 7 tymes 64, make 448, whiche I ſette againſt 7. Further I ſay: 8 tymes 64, make 512, whiche I ſette by 8. And laſte of all I ſaye: 9 tymes 64, make 576, whiche I ſet againſt 9. And thā they wyll ſtande thus.

64	1
128	2
192	3
256	4
320	5
384	6
448	7
512	8
576	9

And ſo is the table ended, by whyche you maye eaſilye fynde the Quotient, as you ſhall ſee by example nowe.

Doo you ſet downe the numbers (as you learned before) accordyng to the ordre of Diuiſion.

263845
64

Scholar. That is thus.

M. Nowe looke what nũbre ſtandeth ouer the diuiſor, reckenyng therto all them that be behinde it towarde the lefte hende.

S. Then are there ouer the diuiſor 263.

M. That is iuſte: nowe ſeeke in the table on the lefte ſyde, whether you can fynd 263.

Scholar. It is not there.

M. Then take that numbre, that is nexte to it, beneth it: I meane a leſſer numbre then

263, but of all the leſſer numbres that the table hath, take you that, that goeth nigheſt to 263.

Scholar. That is 256.

Ma. So is it: and marke this euermore, whan you can not fynde iuſtelye in the table that ſumme that is ouer your diuiſour, then note that that is nexte beneth it of any ſum that is in the table, and loke at the right hand of the lyne what fygure or dygitte that is againſt that ſumme, and take that digite for your quotient, & then worke on, as you learned before: for now haue I told you yͤ whole vſe of this table.

Howe beit, yet that you may be ſure to vnderſtande it, I wyll ſet you ende this enſample of Diuiſion by it. Now therfore begin agayne.

Sc. Fyrſt I ſet downe the ſummes after the common manner, thus.

263845
64

Then doo I looke ouer the diuiſor, and fynde there 263. Nowe to know how many tymes 64 maye be taken out of 263, I reſort to the table aforeſayde, and ſeeke for the numbre 263, but it is not there, therefore as you bad me, I take a leſſer numbre, the nexte to it that I can fynde in the table, and that is 256, whiche numbre hathe againſte it on the right hande this digite 4, whiche I muſte take for the fyrſte fygure of my quotient.

Then doo I (as I learned before) multiply that quotient into euery figure of the diuiſor orderlye, withdrawing the ſumme therof, amounting out of the ouer ſum. as here I ſaye fyrſte: 4 times 6 make 24: ſo I take that out of 26, ſaying: 4 oute of 6 remayneth 2, whiche I write ouer the 6: then 2 out of 2 remayneth nothing. then cancell I 2 and 6, and alſo 6 in the diuiſor, and the ſum ſtandeth thus.

$$\begin{array}{ccccccl} & 2 & & & & & \\ \cancel{2} & \cancel{6} & 3 & 8 & 4 & 5 & (4 \\ & \cancel{6} & 4 & & & & \end{array}$$

Then doo I lyke wyſe ſaye foorth: 4 tymes 4 make 16, which I take out of 23, and there reſteth 7 to bee ſet ouer 3, and that 3 with the 2 behind it, and yͤ 4 vnder it, muſt be canceled, as you ſee heere.

$$\begin{array}{ccccccl} & \cancel{2} & 7 & & & & \\ \cancel{2} & \cancel{6} & \cancel{3} & 8 & 4 & 5 & (4 \\ & \cancel{6} & \cancel{4} & & & & \end{array}$$

Then haue I done with the firſt fygure of the quotiente.

Ma. Nowe ſet forwarde your diuiſour, and ſeeke a newe quotiente, as you ſought this.

$$\begin{array}{ccccccl} & \cancel{2} & 7 & & & & \\ \cancel{2} & \cancel{6} & \cancel{3} & 8 & 4 & 5 & (4 \\ & \cancel{6} & \cancel{4} & 4 & & & \\ & & 6 & & & & \end{array}$$

Sc. Than thus ſtandeth the fygures, ſo that ouer the diuiſor I ſe 78, which I ſeeke in the table, and can not fynde it: therfore I take the next leſſer, and that is 64, the dyuiſour it ſelfe.

M. So muſt you do whē there is none other.

```
 27
265845(41
 644
  6
```

S. Than againſt it I finde this dygit 1, which I muſt ſet in the quotient before 4, thus.
Then multiply I 6 by 1, and it is but 6 ſtill.

Ma. You neede not go about to multiplye whan the quotiente is 1, for 1 dooth nother multiplye nor diuyde. but in ſuche caſe onlye ſubtract the diuiſor out of the numbre that is ouer it. Note.

S. Then I take 4 out of 8, and there reſteth 4, and 6 out of 7, there remayneth 1, ſo I cancell thoſe numbres, & wryte the remayners ouer their places thus

```
  1
 274
265845 (41
 644
  6
```

Then ſette I forward the diuiſour agayne, thus:

```
  1
 274
265845 (41
 6444
  66
```

Where I ſee ouer the diuiſour 144, whiche I ſeeke in the table, & fynd it not: therfore I take the numbre in the table that is nexte therto beneth it, whiche I fynde to be 128, agaynſt whiche in the right ſyde I fynde 2, whiche I take for my quotient, and that doo I multiplye fyrſte 2. into 6, and therof cometh

```
    2 2
  1 2 7 4
2 6 3 8 4 5 (412
  6 4 4 4
    6 6
```

2 tymes 4 are 8, whyche I take oute of 24, and ther remaineth 16, of whiche I write the 6 ouer 4, & the 1 ouer 2, and caucell 2, 4 and 4, thus.

```
      1
    2 2
  2 7 4 6
2 6 3 8 4 5 (412
  6 4 4 4 4
    6 6 6
```

next leſſer, whiche is 128, againſt whiche I fynde 2, ẏ do I ſet into the quotient, and by it I multiply firſt 6, and thereof commeth 12, which I take out of 16 & there reſteth 4, then cācel I 1, 6, & 6, & ouer 6 I ſet 4, thus.

12, whiche I take out of 14, and then remayneth 2, that 2 I ſette ouer 4, and cancell the other fygures, 1, 4 and 6, thus:

Than ſaye I forth:

```
     1
    2 2
  2 7 4 6
2 6 3 8 4 5 (412
  6 4 4 4
    6 6
```

Now agayne I ſet forwarde the dyuiſour thus.

And ſeyng ouer it 16, I ſeeke that in the table, but fynde it not, therefore I take the

```
      2
    2 2 4
  2 7 4 6
2 6 3 8 4 5 (4122
  6 4 4 4 4
    6 6 6
```

Then doo I multiplye 4 by 2, and it maketh 8, whiche I take out of 45, and there remaineth 37, as in this example.

			4	3			
		4	5	4			
	9	7	4	6	1		
2	6	5	8	4	6	(	4422
	6	4	4	4	4		
		6	6	6			

And nowe haue I doone.

M. Well, nowe I ſee that you can worke by this kinde of diuiſion, as farre forth as I taughte you.

S. Yea ſyr, I thanke you, and I fynde in it muche eaſe and certainneſſe.

M. Yet one thynge I doubt whether you perceaue: What if you did fynd in the table the numbre that ſtandeth ouer the diuiſour, what woulde you next doo?

S. I thynke I ſhuld take the digit againſt it on the lefte hande for the quotient.

M. So is it: and as often as you ſeeke in the table and fynd your numbre iuſt, the dygite againſt it is your true and iuſt quotient. I call that a true quotient, if it bee the right quotient that you ſhould take, though your diuiſour multiplied by the ſame, do not cleerly ſubtract the numbre ouer it, but there doth ſomewhat remayne, as it chanced in all the examples that you did worke by. But if it ſhould chaunce (as it dooth often) that your diuiſour multiplied by your quotiente, doo ſubtracte cleane the numbre ouer it, then cal

I call that quotient not onlye a true quotiente, but alſo a iuſte quotient, bycauſe it doth iuſtlye conſume the numbre ouer the diuiſor: and that chaunceth euermore whan the numbre ouer the diuiſor is iuſtlye found in the table.

S, This I ſhall remembre.

M. But yet one eaſye pointe more I will tell you in this ſorte of Diuiſion, therefore marke it well.

Whan you haue found in the table, other the ſame ſumme that is ouer the diuiſor, other the nexte beneth (for lacke of the other) than looke what digite ſtandeth againſte it, take that for your quotient. And bicauſe it is ſome payne to multiplye the diuiſor by the quotient, you ſhall not need to do it, but only take the numbre that you found in the table, and ſubtract that from the ouer numbre: for if you doo multiply the diuiſor by the quotiente, that wyll bee the numbre that ſhall amounte, therefore is this waye more eaſyer.

Schol. So is it. and alſo more certayner for ſuche as I am, that myght quickly erre in multiplying, eſpecially being ſmally practiſed therein.

Maſter. Then proue in ſome brief example whether you can doo it, and ſo wyll wee make an ende.

S. I woulde diuide 38468 by 24, therfore fyrſt I ſette the table as here foloweth.

24	1
48	2
72	3
96	4
120	5
144	6
168	7
192	8
216	9

Then ſet I the two ſummes of Dyuiſion, thus.

58468
24

And ouer the diuiſoure I fynde 38, whiche I ſeeke in the table and finde it not, therfore take I the nexte beneth it, whiche the table hathe, and that is 24, the diuiſour it ſelfe. againſt whiche is ſet 1, which I take for the quotient, whiche I ſet in his place: And nowe I nede not to multiplye the diuiſor by it, but onely to withdrawe the diuiſor oute of the 38, that is ouer it, and ſo remayneth 14, as thus.

14
58468 (1
24

Then ſet I forward the diuiſor, and fynd ouer it 144 as appeareth: then ſeeke I that numbre in the table and fynde it, & againſt it is 6, therefore I ſet 6 before 1, for my quotient, and I take that 144 for the iuſte multiplication of the dyuiſour by that quotient, and therfore without any new multiplication I doo ſubtract that 144, from the other 144, and there reſteth nothyng, as you may ſee.

14
58468 (1
244
2

14
58468 (16
244
2

Therefore I ſet forwarde the diuiſor. but ſeeyng

ſeeyng it will not be in the nexte place, (for then ouer 2 woulde be nothing) I ſet it forward twiſe, as you ſe heere.

2 4
3 8 4 6 8 (160
2 4 4
2 2

And for bicauſe that I could not ſette it in the nexte place folowing, therfore I ſette a cyphre in the quociente, as you ſee.

Than looke I ouer the diuiſour, and find 68, whiche I can not fynd in the table, therfore take I the nexte beneth it, which I find in the table, and that is 48, and againſte it ſtandeth 2, whiche I take for the quotiente. And thē without any multiplying of the quotient in to the diuiſor, I doo ſubtracte that 48 from 68, and there reſteth 20, as heere appeareth.

2 4 20
3 8 4 6 8 (1602
2 4 4 2 4
2 2

And ſo haue I ended the whole diuiſion.

M. Now can you ſufficiently ſkyll in theſe kyndes of Arithmetike. And now for the vſe of theſe two laſte, that is Multiplication & Diuiſion, I will briefly ſhewe you the feate of Reduction.

REDVCTION.

Eduction is, by which all ſummes of groſſe denomination, may be tourned into ſummes of more ſubtill denomination: And contrarye wayes: all ſummes of ſubtyle denomination maye bee broughte to ſummes of groſſer denomination.

Sc. What call you Groſſe denomination, and Subtyle denominatino?

M. That I call a Groſſe denomination, whiche doth contayne vnder it many other ſubtiler, or ſmaller: As a pounde in reſpecte to ſhyllinges, is a Groſſe denomination, for it is greater then ſhyllinges, and contayneth manye of them: And ſhyllinges in compariſon to poundes, are a Subtyle denomination, for bicauſe they are leſſer then poundes, and many of them are contayned in one of the other: as ſo lykewaies of other thinges whatſo euer thynge is compared to other, if it be greater and containeth many of thē, it is a Groſſer denomination: but if it bee leſſer, ſo that many of them are in the other, then are they called Subtyle denominations: whereby you maye perceaue, that one de-

Groſſe de
nominati

Subtyl de
nominati
on.

denomanation may be called a Groſſe denomination, and alſo a Subtyl (that is to ſey, a great and ſmall) in dyuers compariſons. For ſhyllinges compared to poundes, are a Subtyll or ſmall denomination: but compared to pennies, they are a Groſſe or great denomination.

Sc. Nowe I vnderſtand the name, I pray you teache me the vſe.

M. The vſe is eaſily learned, if you remembre what you haue learned before. For if you will reduce any ſum of a Groſſe denominatiō into a ſumme of a Smaller or ſubtyler denomination, you muſt cunſydre how manye of that Subtyler denomination, do make one of the Groſſer denomination, and by that numbre or numerator doo you multiplye the other ſumme: as if you wolde reduce 20 poundes into ſhyllinges, you muſte conſidre that in a pound are included 20 ſhillinges, therfore multiplye the one 20 by the other 20, and there will amount 400, wherby you maye knowe, that in 20 pound ar cōtayned 400 ſhillings. Likewaies if you wold reduce 30 ſhyllinges into pennies; conſideryng that in 1 ſhyllinge are 12 pennies, you muſt multiplye 30 by 12, and it will be 360. whereby you fynde, that in 30 ſhillinges are contayned 360 pennies. And thus may you reduce any groſſe denomination into a more ſubtiller, by multiplication, if you know how

many

manye of the leſſer doo make the greater: of whiche thyng I wyll anone gyue you a brief table for the moſt accuſtomed kyndes of money, weightes, meaſures, and tyme, and ſuch lyke, whereby you maye knowe howe often eche ſubtile denominatiõ is contayned in the groſſer, whan you ſhall neede it for the foreſayde kynd of Reduction. And alſo the ſame ſhall ſerue you, if you wolde reduce any ſum of a ſubtiler denomination, into a ſumme of a groſſer denomination: For in ſuche reduction you muſt conſidre (as in the other form) how many of the ſmaller do make the greater, and by that numbre muſt you diuide the other ſumme, and the quotient wyll declare how many of the greater denomination are comprehended in that ſumme; as for example, If you woulde knowe how many ſhillynges are contayned in 3240 pens: conſidre that 12 pennies do make 1 ſhilling, you muſt diuide that 3240 by 12, & your quotient wyll be 270, wherby you know that ſo many ſhillynges are in 3240 pennies. But & you wold knowe farther, howe manye poundes are in thoſe 270 ſhillinges, ſeyng that euery pound conteyneth 20 ſhyllinges, dyuide that 270 by 20, and it will be 13, and 10 remaynyng, wherby you maye knowe, that in 3240 pennies, or 270 ſhyllinges, are 13 poundes and 10 ſhyllinges. For euermore the remayner muſt bee named by the name of denominatiõ of the ſumme that was diuided, whiche in this place were ſhyllynges. And thus maye you doo with anye other kyndes of denominations.

Wherefore to the intente that you maye haue a light knowledg in yͤ common coines, weightes, meſures an ſuche other, I haue prepared heere a briefe table, which ſhal ſuffyſe to you at this time, tyll herafter at more conuenient opportunitye I may inſtruct you more exactly in the ſame.

A table for Engliſh coynes.

ſhe
es.

A Soueraygne.
Halfe a ſoueraygne.
A Royall.
Halfe a Ryall.
A quarter Royall.
An olde Noble.
Halfe an olde Noble.
An Angell.
Halfe an Angell.
A George noble.
Half a George noble.

A quarter Noble.
A Croune.
Halfe a Croune.
A Croune.
A grote.
A harpe grote,
A penny of 2 pence
A dandy pratte.
A Penny.
An halfe penny.
A farthyng.

The valewe of Engliſhe coynes.

valew
nglyſh
es.

A Soueraygn is the greateſt engliſh coyn, and contayneth 2 Royalles, or 3 Angelles, eyther 9 halfe crounes, or 4 crounes and an halfe, that is to ſaye, 12 ſ. 6 d.

Halfe a Soueraygn is equall with a Royall.

A Royall contayneth an Angell and a halfe, that is to ſay. 11 ſ. 3 d.

Halfe a Royall containeth 5 ſ. 7 d. ob.

A quarter of a Royall contayneth 2 ſhyllinges, 9 d. ob. q.

An olde Noble, called an Henrye, is worth 2 crounes, or a noble and halfe, that is 10 ſ.

Halfe an olde noble is woorth 5 ſ.

An Angell contayneth a croun and halfe, or 3 halfe crounes, that is 7 ſ. 6 d.

Halfe an angell is worth 3 ſ. 9 d.

A Noble, called a George, is worth 6 ſ. 8 d.

Halfe a Noble is worthe 3 ſ. 4 d.

A quarter of a Noble (which in the olde ſtatutes is called a farthynge) contayneth 20 d.

A Croune contayneth 5 ſ. and the halfe crowne 2 ſ. 6 d. Howe be it ther is an other crowne of 4 ſ. 6 d. whiche is knowen by the roſe ſyde: for the roſe hathe no croune ouer it, as in the other croune, but it is enuironned on the 4 quarters with 4 floure deluces, whereby you maye beſt knowe it. But I wil retourne to ſpeak of the valu of the coynes, for I intend not now to deſcribe the formes of them. Nowe of golde are there no more common coynes.

In ſyluer, the greateſt is a Grote, which containeth 4 pennies. Than is ther another Grote called an Harp, which goeth for 3 d.

Then nexte is the Pennye of 2 d. And then a Dandy pratte, worth 3 halfe pence. Nexte it a Penny, then a Halfe penny, and laſt and leaſte of all a Farthynge, whoſe coyne is on the one ſide a croſſe, and on the other a purculles. This I tell you, bicauſe I ſee many that can not knowe a farthinge from a ſmall halfe pennye.

Nowe haue I tolde you all the englyſhe coynes bothe of golde and ſyluer, but yet of the two moſte common valewes of money ſpake I nothing: that is to ſay, of poundes and ſhillynges, which though they haue no coynes, yet is there no name more in vſe then they: of which the ſhyllings containeth 12 pennies or 3 grotes, and the pound 2 olde Nobles, 3 George nobles, or 4 Crownes, that is to ſaye 20 s.

Here wolde I nowe expreſſe the valewes of ſundry other coynes of dyuers centreyes but for three cauſes I nowe refrayne. The firſt and chiefeſt is, bicauſe they are not currant by the ſtatutes of this realm. An other cauſe is, by reaſon they are ſo vncertayne, that they bee neuer longe at one rate. And agayne they are ſo different in ſo menye places, that it were matter inoughe for a great boke to ſpeake ſufficiently of them all. How be it, yet bicauſe you ſhall not be all togither ignorant of them, I wyll ſhewe you the valewes of ſome that are moſt in vſe: and firſt of Fraunce.

Frenc
coyne

The moſte common money are Deniers, Soulx and Frankes. 12 Denyers make 1 S. 20 ſoulx make 1 Franc. ſo that as you ſee theſe 3 kindes are lyke in the rate to pennies, ſhyllinges and poundes with vs, but that this is the difference, that their Denier is but the 9 parte of our pennye, and ſo their ſoulx (commonly called ſowſes) go 9 to our ſhilling, and 9 of their Francs to an engliſh pounde of mony: ſo that 3 of their Francs make a noble. And by thoſe 3 may you practiſe howe to reduce Frenche mony into Engliſhe mony. And as for the reſt of their coynes I wyll omyt tyll an other time, when I entende to ſhew you the rate of ſundry other kyndes of mony.

Flaund
coyne

But now as for the coynes of Flaundres be ſo changeable, that you muſt know them from tyme to tyme, elſe you can not reduce them into our monny certainly. But yet bycauſe that you ſhal haue an example of their monny to exerciſe you withall, you ſhal take thoſe that bee moſte common, as Styuers, bothe ſyngle and double, Grotes flemmiſh, Carolus and Gyldens. A flemmyſh Grote is a lyttle aboue 3 farthinges engliſhe. A ſyngle Styuer is 1 d. ob. q. The double ſtyuer is 3 d. q. The ſyluer Carolus ſyngle 2 d. q. c. The double ſyluer Carolus, is 4 d. ob d̃.q. Then is there alſo the Carolus gylden, whiche is worth 20 ſtyuers. And the Flemmyſh noble is worth 3 Carolus gyldens, and xii. ſtyuers.

But I will let them paſſe now, exhorting you to practiſe to reduce thoſe kyndes into engliſh money, accordynge as I haue ſette forthe here folowinge: 2160 deniers, make 840 d̃, or 20 s̃. 3240 deniers make, 360 d̃, or 30 s̃: 8352 denieres, make 928 d̃, or 3 li, 17 s̃ 4 d̃: 2160 ſoulz, make 240 ſhillinges; and ſo of other in lyke rate.

But if you will reduce Flemmyſh monye iuſtly, you muſt reduce it fyrſt into the ſmalleſt part of engliſh mony that is in that coyn as for example. If I wold reduce 368 double ſtyuers into Engliſh money, conſidering that a double Styuer contayneth 3 d̃, q̃, you ſhall fyrſt looke howe many q̃, be in the double Styuer, and you ſhall fynd them 13, therfore multiplye the ſumme of the ſtyuers by 13, and than haue you their valewe in farthinges, which is 4784. Nowe yf you diuide that by 4, then wyll there appeare the numbre of pence: but better it were to dyuide it by 48 (for ſo manye farthinges are in 1 ſhillynge) and then will the quotient declare the ſumme of the ſhyllynges.

Lyke waies yf you woulde reduce anye ſumme of ſyngle ſtyuer into Engliſh mony, you muſt multiplye the ſumme fyrſte by 13, and than haue you the summe of q, whiche summe if you diuide by 8, then will amount the summe of pennies: or if you dyuide it by 96, the summe of shyllings wyll appeare.

But thys marke in all diuision whan yee doo reduce to bryng 1 denomination into an other, if there be any remainer after the diuision, that must be named by the denomination of the grosse summe that was diuided. as for example: I would bryng 254 q into pence, therefore I do dyuide this 254 by 4, for so manye farthinges make 1 penye, and the quotiente is 63, whiche is the summe of the pence, and then remayneth yet 2, whyche are farthinges styll, as you maye proue by dyuiding. And this must be marked in all diuision, namely when it is doone for reduction.

Thus muche haue I sayd of mony. Now will I shewe you in like sorte the distinction of waightes, after the statutes of Englande, where the leaste portion of waighte is commonly a Graine, meaninge a grayne of corne or wheat drye, and gathered out of the myddle of the eare. Of these graynes in time passed, 32 wayed iust 1 penny of Troye. And than was but 20 penies in an Ounce. But now are there 46 pennies in an Ounce, so that there are not fully 14 graynes in one pennye.

1 gray

1 peny
Troye.

An Ou

But nowe of Ounces after Troye rate

(whiche is the ſtandarde of England) 12. ounces make 1. pounde.

rdes
weigh

But commonly ther is uſed an other waight called Haberdepoiſe, in whiche 16. ounces make a pounde. Therfore whan you woulde reduce ounces into poundes, you muſte conſider whether your waightes be Troye waightes or Haberdepoiſe: and if it bee Troye waight, you muſte diuide your ounces by 12 to bringe them to poundes, but if it be Haberdepoyſe, you muſte dyuide them by 16.

undred
ght.

Nowe agayne, there be greater waightes whiche are called an Hundred, halfe a hundred, and a quartern, and alſo halfe a quarterne. &c.

S. Why? ſo there maye bee reckened 20 pound, 40 pound, 200 pounde, and ſuche innumerable.

M. All theſe are numbres of waight, but they haue not common waightes made to their rate, as the other haue. And againe, theſe that I dyd name are not iuſte in numbre, as they ſeme by their name, for an hundred is not iuſte 100, but is 112 pounde. And ſo the halfe hundred is 56, the quarter 28, and the halfe quarter 14. And this is the common waightes uſed in moſte thinges that are ſold by waighte.

e
ghts.
ce.

Howe bee it, there are in ſome thynges other names: as in Wolle 28 pounde is not called a quarterne, but a Todde: and the 14 pounde is not named halfe quaterne, but a Stone. and the 7 pound Half a ſtone. Other names bycauſe they differ in many places, & agree in fewe, I let theym paſſe.

Stone

But a Sacke of Woll by the ſtatutes is lymitted to bee 26 Stone.

Sacke

Now in cheeſe, though it bee ſolde by the hundred, and by the ſtone in ſome places, yet the verye weightes of it are Cloues, and Weyes, ſo that a Cloue ſhoulde containe 7 pounde, and a Weye 32 Cloues, that is 224 poundes. How be it ſome ſtatute bokes ſay, that a Cloue ſhoulde be 6 pounde: and ſome ſaye alſo that a Weye doothe contayne 36 Cloues, and that is commonly uſed; for the cõmon Wey is of 256 pound, ỹ is 36 cloues, reckeninge 7 pound to the Cloue, and there is 4 pound ouer waight. Let this ſuffice you at this time touching waightes.

Cheeſe w
ghtes.
a Cloue
Weye.

Nowe of waightes are made other meaſures, bothe for grayne and lyquour. For a Pounde in waight maketh a Pynte in meaſure, ſo that 8 Pounde (or 8 Pyntes) doo make a Gallon. Halfe a Gallon is named a Pottell: and Halfe a Pottell is called a Quarte, whyche contayneth twoo Pyntes. Nowe aboue a Gallon the nexte meaſure is a Fyrkin, than a Tercian, a Kylderkyn or halfe Barrell, and a Barrell. And by theſe meaſures are ſold commonly Ale, beer, wine and oyle, butter, and ſope, ſalmon, Heryngs

Meaſur
for liquo
A Pynt
Gallon
Pottell
Quarte
Fyrkyn
Tercian
Kylder
Barrel

and Eeles.

But as theſe bee unlyke thinges, ſo the meaſure of their veſſels doo differ: for the meaſures of Ale are as foloweth.

Ale,	the Fyrken ý kilderken the Barrell	containeth	8 16 32	gallons
beer	the Fyrken ý Kylderkē the Barrel	containeth	9 18 36	gallōs

Sope meaſures, both Fyrkyn, Kylderkin and Barrell, ſhuld be all equall to Ale meaſures. Moreouer the ſtatutes dooth lymitte the weight of euery of thoſe thre veſſels being emptye.

A Barrell Half Barrell A Fyrkyn	to weighe emptye	26 13 6½	poundes.

Hearynges alſo be ſold by the ſame meaſures that Ale and Sope be ſolde by.

Hearinges alſo are ſolde by the tale, 120 to the hundred, tenne thouſan to the laſt.

Salmon and eles haue a greater meaſure

Salmō & Eeles	the butte ý barrell half bar. the firke	doth hold	84 42 21 10½	gallōs.

Nowe here, ſome ſtatutes dyd limit Eele veſſels equall with Hearing veſſels.

Now as for wyne veſſels ſeldom ar ſmaller then Hogges headeſ, whiche are of 63 gallons: euery Hogges hed is two Barrels. Yet there are many other wyne veſſels, but of them all ſe this table, and marke the meaſures one to an other.

Of wine and oyle	the Rondelet	holdeth	18	gallons
	the Barrell		31½	
	ỹ hogges hed		63	
	the Tertian		84	
	the Pype		126	
	the Tonne		252	

But you ſhall marke that there be other kindes of Tercias, for ther be Tercias (that is to ſaye) Thyrdes of Pypes, of Hogges heades, and of Barrels, as well of other thinges as of wyne.

Alſo of Malueſeyes and Secke, &c. the halfe tonne is not called a Pype, but rather a Butte.

And thus muche haue I thought meet to tell you at this tyme.

Scholar. And is that alwaies true?

Ma. I haue tolde you howe it ſhoulde bee, but howe it is I maye not ſaye: howe they doo differ daily from their iuſte meaſure the Gagiers can tell you better then I. But I wyll leaue this paſſe nowe, and ſpeake briefly of the other meaſures.

And as of weightes there dyd ſpryng the liquide meaſures (whereof I ſpake laſte) ſo of the ſame ſpryngeth drye meaſures, as Peckes, Buſhels, Quarters, and ſuch lyke, whereby are meaſured corne ⁊ lyke graines: alſo ſalt, lyme, coales and other lyke. And this is the ordre and quantitie of them.

ecke. A Pecke is the meaſure of 2 Gallons.

ſhell. A Buſhell contayneth 4 Peckes.

uarter. A Quarter holdeth 8 buſhels.

ye. A Waye containeth 6 quarters.

Theſe ar the common names and meaſures but in diuers places there be diuers ſortes.

The buſhell in many places is 2 buſhels: but then is the buſhell ther called a Stryke. And in ſome places halfe a quarter is called a Cornoke. But theſe diuerſities are to many to tell you briefly them all. And againe ſyth they are againſt the lawe and ſtatutes, I counte them vnmeete to be vſed.

ryke.
noke.

But nowe remaineth yet an other kinde of Meaſure, whereby men meate lengthe and breadth, and thoſe are an Ynche, a Foote, and ſuche other: whoſe names and lengthe this table ſheweth.

eaſures
meare
[illegible]
ynche.

yuche. 3 graynes of barley, make an Ynche.

oore. 12 Ynches, make a Foote.

rde. 3 Foote make a Yarde.

e. 3 Foote and 9 Ynches make an Elle.

5 Yardes and a halfe make a Perche.

rche. 1 Perche in bredth ⁊ 40 in length, do make a Rodde of land, which ſome cal a Rood, ſome a Yarde lande, and ſome a Farthendele.

odde.

lfhdele. 2 Farthendels, make half an acre of groũd.

4 Farthendeles make an Acre. Acre.

Here mought I tel you many thinges els touching meaſure, and alſo howe to reduce ſtraunge meaſures to our meaſures, but becauſe it can not well bee doone withoute the knowledge of Fractions, whiche as yet you haue not learned, I will let theym paſſe tyll an other tyme, when I ſhall inſtructe you in Geometry, wherin I ſhould be enforced els to repeate the ſame often againe.

The pa
of tym

Sc. But yet ſyr of the partes of tyme, I praye you tell me ſomewhat.

A Da
an Ho
Weeke
Mone
Yeare.

M. You knowe that a naturall daye hath 24 houres, and euery houre hathe 60 minutes. It needeth not to tell you that 7 daies make a Weeke, and 4 Weekes make a cōmon moneth, & 13 monethes make a yeare, lackynge 1 daye and certayne houres and minutes. But of that I ſhall enſtructe you hereafter.

Heere will I make an ende of Reductiō for this tyme, whiche though it be counted no kynde ſeuerall of Arithmetike, yet you ſe it is no leſſe needfull to be knowen, nor eaſier to bee doone, than of any of the other.

S. Mary ſyr it ſemeth vnto me much harder than any other ſort, for it requireth the knowledge of ſo many thinges. but now ſyr whē you ſee time, I am ready to learn forth, for as much of reductiō as you haue taught me, I remembre, but and if I do at any time forgette, I ſhall haue recourſe to the tables, whiche you haue ſet forthe for me.

M. So do you, for it will not bee remembred without exerciſe. Nowe Progreſſion I will beginne.

PROGRESSION.

Progreſſion is a brief and quicke addition of diuers ſummes, proceedynge by ſome certaine and reaſonable order: I mean whã the diſtaunce of euerye 2 numbers is equall to the diſtance of the ſecond nũber from the fyrſt.

S. I vnderſtande you not well.

M. By an example it will be playner, as here: 1,2,3,4,5,6. heere you ſee the ſeconde to differ from the fyrſt but by 1, and ſo dooth all the other one exceede an other by 1 ſtyll to the ende.

S. This I perceaue.

M. And lykewaies heere: 4,7,10,13,16,19,22,25.

S. Yea, they proceed by the difference of 3.

M. And ſo I meane, that Progreſſion is an arte howe to adde all ſuche numbers and other lyke togither, much quicklyer then by common Addition, and that by this meanes: Tell howe manye numbers there are, and if

they bee odde, write their ſumme downe by
it ſelfe. as in this example, 2,4,6,8,10,12,
14. where the numbres are 7 as you maye
ſee, therfore ſette downe 7 in a place alone,
as I haue doone in the margent heere: then 7
adde togither the firſt numbre and the laſte,
as in this example: adde 2 to 14, and that
maketh 16. take halfe of it and multiplye by
the 7, which you noted for the numbre of the
places, & the ſumme that amounteth is the
ſumme of all thoſe figures added togyther.
as in this example: 8 multiplyed by 7 make
56, & that is the ſumme of all the figures.

Sc. That will I proue by an other example. I woulde knowe howe muche this ſum is, 5,8,11,14,17,20,23,26,29. I tell the places, and they are 9, that I note. Then I put the firſt numbre 5, and the laſte 29 togither, and they make 34, I take the half of it, that is 17, & multiply by 9, and it maketh 153. That you ſaye is the ſumme of all the numbres.

M. So ſhall you finde it, if you trye it.

S. Howe ſhall I trye it?

Ma. By Addition: for if you adde all the parcels togither, you ſhall ſee the ſame ſum amounte, if you did worke well. And that manner of Addition trieth all kindes of ſummyng any Progreſsion.

S. Then can I ſumme a Progreſsion, if the numbres of the parcels be odde. But what if they be euen, as in this example, 1, 2, 3, 4, 5, 6, 7, 8?

Ma. When the numbre of the parcels is euen, then note that alſo as you dyd before, and lykewaies adde the fyrſte ſumme to the laſte, and by the halfe of the numbre of the places do you multiplye it. as in your example, the parcels are 8, that note I: then adding the fyrſt ſum to the laſte, ther amounteth 9, that doo I multiplye by the halfe of parcels, that is by 4, and it maketh 36, whiche is the ſumme of the 8 parcelles.

But if you wyll take one rule for theſe both, do thus. Multiply the halfe of the one by the other whole, and the ſumme will amounte all one. For ſometyme it chaunceth that the numbre of the parcels bee odde, ſo that their half can not bee taken: and ſometyme it chaunceth the addition of the fyrſte numbre and the laſte to bringe forth an odde numbre, ſo that the halfe of it can not bee taken: but they will neuer be bothe odde.

Sc. Then I perceaue this, if there bee no more longyng to it.

M. This is inough for Progreſsion Arithmeticall. howe be it there is an other maner of Progreſsion called Geometricall, when the numbres increaſe by a lyke proportion, that is, if the ſeconde numbre contain the fyrſt, 2, 3 or 4 tymes, and ſo forthe, then the thirde contayneth the ſeconde ſo manye tymes alſo: and ſo the fourthe the thyrde, and the fift the fourth: wherfore I ſette thoſe three examples.

3,6,12,24,48,
1,3,9,27,81,
2,10,50,250.

Here in the fyrſt example you ſe, that euery numbre contayneth the other (that goeth nexte before hym) 2 tymes: and in the ſecond example 3 tymes: in the third example 5 tymes. Nowe if you wyll knowe howe to fynd eaſilye the ſumme of any ſuche numbres, do thus: conſidre by what numbre they be multiplyed, whether by 2, 3, 4, 5, or anye other, and by the ſame numbre do you multiply the laſte ſumme in the Progreſſion.

S. I praye you worke it by this example, 2,8,32,128,512,2048, whiche ſum I haue encreaſed by the multiplication into 4.

M. Then muſt I multiply the laſt ſum (which is 2048) by 4 alſo, and it wil be 8192. Nowe muſt I bate frō this ſum yͤ firſt numbre of yͤ progreſſiō, which here is 2, thē reſteth 8190 which ſum I muſt dyuide by 1 leſſe thē was the numbre yͭ I multiplyed by. Seyng than I multiplied by 4, I muſt dyuide by 3, ſo dyuidyng 8190 by 3, the quotient wil be 2730, which is the ſumme of all the Progreſſiō. And now to proue whether you can do the ſame, I giue you theſe numbres to adde, 3,15,75,375,1875,9375,46875.

Sco. I can not well tell by what numbre

this Progreſsion dooth encreaſe.

M. In any ſuche doubte do thus: Diuyde the ſecond numbre by the fyrſt, and the quotient will ſhewe you the numbre that engendreth the Progreſsion.

Sc. Then is that numbre in this exemple 5. for ſo many tymes is 3 in 15.

Ma. So is it. Nowe worke as I taught you.

S. The laſte numbre is 46875, whiche I multiplye by 5, and it yeldeth 234375, from whiche I bate the fyrſt numbre of the Progreſsion, that is 3, and ther reſteth 234372, whiche I dyuide by 4, for that is 1 leſſe then 5, and the quotient is 58593, whiche is the whole ſumme of the Progreſsion.

M. Yf you remembre well this, you haue learned the art of Progreſsion, both Arithmeticall and alſo Geometricall: whiche you may proue other by ſubtraction of eche numbre alone from the ſumme, and ſo will there nothing remaine, other by addinge together of all the parcels, for ſo wyll the ſame ſumme amounte.

And nowe for the vſe of this rule I wyll put forthe to you certaine queſtions, whiche ſome do referre to Addition, but not ſo iuſtly as I doo vnto this rule of Progreſsion: & ſome, as vncircumſpectlye, referre the ſame to Duplacion.

The

The fyrſte queſtion is this.

If I ſolde vnto you a horſe, hauinge 4 ſhoes, and in euery ſhoe 6 nayles, with this condition, that you ſhall paye for the fyrſte naile, 1 ob. for the ſeconde 2 ob: for the thyrd 4, and ſo foorth, doublyng vnto the ende of all the nayles. Nowe I aſke you howe muche woulde the whole pryce of the horſe come vnto.

S. Fyrſt to knowe the numbre of the nayles I muſte multiplye 6 by 4, and that maketh 24, then I wyll do thus. I wyll wryte the numbre of the nayles, euerye one in ordre frome 1 to 24, and againſt eche numbre of nayles the ſumme of halfepennies duely, as the ordre of Duplacion teacheth, and in this next fygure folowing, appeareth.

Than doo I reſorte to the rule of ſumming this Progreſsiō, wher I conſidre, that the encreaſe of thys ſumme proceedeth by multiplication of 2, and therfore I do multiplye the laſte ſumme by 2 alſo, and it yeldeth 16777216, frome whiche I abate the fyrſte numbre whiche is 1, and then reſteth 16777215, whiche I ſhould diuide by 1 leſſe than I dyd multiplye: but ſeeyng that it is 1, I neede not to diuide it, for 1 (as I haue before ſayd) dooth nother multiply nor diuide: therefore I take that ſum 16777215 for the whole ſumme of the halfe pennies,

1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	13
8192	14
16384	15
32768	16
65536	17
131072	18
262144	19
524288	20
1048576	21
2097152	22
4194304	23
8388608	24

which by reduction I fynd to be 699050 ſ, and 7 d, ob. that is, 34952 li, 10 ſ, 7 d. ob.

Maſt. That is well done. but I thynk you wyll bye no horſe of the price.

Sc. No ſyr if I bee wyſe.

Ma. Well, than anſwere me to this queſtion.

A Lorde deliuered to a bricklayer a certaine numbre of lodes of brycke, whereof he wylled hym to make 12 walles, of ſuche ſort, that the fyrſt wal ſhould receaue 2 thirdeles of the whole nũbre, and the ſecond 2 thyrdeles of that that was lefte: and ſo euerye other 2 thirdeles of that, that remayned, and ſo dyd the brycklayer: And when the 12 walles were made, there remayneth one lode of brycke.

Nowe I aſke you howe many lode wente to euery wall, and howe mannye lode was in the whole?

S. Why ſyr, it is impoſſible for me to tell.

M. Naye it is verye eaſy, if you marke it well. Marke well that I ſayde, that euery wall ſhoulde receaue 2 thirdeles of the ſum that was lefte. Nowe take away 2 thirdeles from anye ſum, and you muſte needs graunt that that whiche remayneth is 1 thirdele of the ſumme laſte before: exaumple of 9, from which if you take 2 thirdeles, there wyll remayne 3, whiche is one thirdele of 9. Likewaies frome 3 bate 2 thirdeles, and there will remayne 1.

Sch. This is true. and nowe I perceaue, that the leaſte wall hadde but twoo loade of bricke.

Mayſter. And by the ſame reaſon maye you know: how manye loade euerye walle hadde, accordynge as this figure folowyng doth ſhewe. and lyke wayes what the whole ſumme of bryckes was. for if you make 12 ſummes, multiplyinge by 3 ſtyll frome the laſte remayner, as you ſee heere on the lefte ſyde of the table, there wyll appeare all the remayners after euerye walle: and yf you multiplye the laſte of thoſe 12 ſummes by 3 alſo, than wyll that be the ſumme of the loades whiche were delyuered to the brycklayer.

1	12	2
3	11	6
9	10	18
27	9	54
81	8	162
243	7	486
729	6	1458
2187	5	4374
6561	4	13122
19683	3	39366
59049	2	118098
177147	1	354294

531441

Againe, if you do double euery remainer, as you ſee at the righte ſyde of this table, thoſe numbres will ſhewe the ſum of lodes that wente to eche wall: whereby alſo you maye perceaue that eche wall was 3 tymes ſo greate as the nexte leſſer.

S. Lo, nowe it appeareth eaſye ynoughe. Now ſurely I ſe that Arithmetik is a right excellent arte.

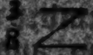
Ma. You wyll ſaye ſo whan you knowe more of the vſe of it: For this is nothing in comparison to other pointes that maye bee wroughte by it.

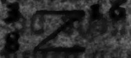
Sch. Than I beſeeche you ſyr, ceaſſe not to inſtructe me farther in thys wonderfull cunnynge.

Maſter.

M. BY the order of the ſcyences (as men haue taught it) ther ſhuld follow next the Extraction of Rootes of nombre, whiche bycauſe it is ſomewhat harde for you yet, I will leue it paſſe for a whyle, and will teache you the feate of the rule of Proportions, whiche for his excellency is called the Golden rule. Whoſe vſe is, by thre nūbres knowen, to fynde out an other vnknowen, whyche you deſyre to knowe, as thus. If you paye for your borde for thre monethes, 16 ſhyllinges, how muche ſhall you paye for 8 monethes?

To knowe this and all ſuche lyke queſtions, you ſhall conſydre which two of youre 3 numbres be of one denomination, and ſet thoſe two the one ouer the other, ſo that the vndermoſte be it, that the queſtion is aſked of. As in my queſtion 3 and 8, be both of one denomination, for they bothe be monethes, and bycauſe 8 is the numbre that the queſtion is aſked of, I ſet them one ouer the other, and 8 undermoſt thus, with ſuch a crooked draught of lynes.





Then do I ſet the other nūbre which is 16, againſt 3 at the right ſyde of the lyne, thus.

And nowe to knowe my queſtion, this muſt I doo: I muſte multiplye the lowermoſt on the left ſide, by that on the right ſide and the ſum that amounteth I muſt diuide

by

by the hygheſt on the lefte ſyde. Or in playner woordes thus: I ſhall multiplye the nūbre of whiche the queſtion is aſked (whiche is called the Thyrd numbre) by the numbre of an other denomination (whiche is called the Second) and that ſumme that amounteth muſte I diuide by the ſumme of lyke denomination, whiche is called the Fyrſte. Than for the knowledge of this queſtion, I multyplye 8 into 16, and there amounteth 128, which I dyuide by 3, and it yeldeth 42 ſhillinges, and 2 ß remayneth, whiche I turne into pennies, and they be 24 d, of which the thyrd part is 8 d: ſo the thyrd part of 128 ß is 42 ß, 8 d. which ſum I write at the right hand of the figure againſt 8, thus.

3 16 ß
8 42 ß, 8 d.

Hereby I knowe that if three monethes bordyng do come to 16 ß, that 8 monethes bordyng wyll come to 42 ß, 8 d. & lykewyſe of any other lyke queſtion.

But heere muſte you marke, that the firſt numbre and the thyrde be of one denomination, and alſo the Seconde and the Fourth, for whiche you ſeeke: or elſe be of ſuche denominations, that you in workyng may bryng them into one. As if a man ſhoulde aſke me this queſtion.

Twelue weekes iourneying coſt me 14 nobles, howe many poundes is that in one yeare? Heere you ſe no two numbres of one denomination, but yet in workyng you maye tourne theym into lyke denominations, as thus. Tourne the one yeare into 52 wekes, and the fourthe ſumme wyll bee nobles, by the ordre of the woorkinge. Then to knowe this queſtion, multiplye the thyrde ſumme 52 by the ſeconde 14, and the ſumme wyll be 728, that dyuide by 12, and it will be 60, and 8 remaining, whiche if you tourne into ſhyllinges, they will be 53 ſ̃, 4 d. whiche if it be dyuided by 12, will yelde 4 ſ̃, 5 d. and and the thirde parte of a penny: putte this 60 nobles (which maketh 20 li) with the 4 ſ̃ 5 d and q̃, and lyttle more: for the ſum that anſwereth to the queſtion, and it is the expence of a yeare, and ÿ ſums will be this.

12 Z 14 nobles.
52 Z 60, 4 ſ̃, 5 d̃, q̃, 2 m

And take this for a generall rule, that euermore the Thyrd numbre be it that ÿ queſtion is ioyned with, and the fyrſte, the numbre that is of the ſame denomination, then muſte the ſeconde neede be that other. A gen rule.

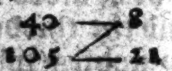
Remembre alſo, that the place of the fyrſte numbre is the higheſt on the left ſide, and the place of the ſeconde right agaynſte it on the righte ſyde: the place of the Third numbre is under the fyrſte, as by thoſe exāples you haue ſeene.

Scholar. This I truſte I can doo.

Ma. But and the queſtion be aſked thus:

general

In 8 weekes I ſpende 40 ſ, howe longe will 105 ſhyllinges ſerue me? Thoughe the ordre ſeeme vnlyke, yet take you 105 for the Thyrde numbre, and 40 beeyng of the ſame denomination, for the Fyrſt, and then 8 for the Seconde. Then multiplye 105 by 8, and it will be 840, whiche if you diuide by 40, it will yelde 21, which is the Fourth numbre, and ſheweth howe manye weekes 105 ſ will ſerue, if you ſpende 40 ſ in 8 weekes.



The fygure of this queſtion is this: as if you ſhuld ſaye: Yf 40 ſ ſerue for 8 weekes, 105 ſerue for 21 weekes.

Other diuerſities there bee of woorkyng by this rule, but I had leuer that you ſhold learne this one well, then at the beginning to trouble your mynde wyth manye formes of woorking, ſyth this way can do as much as all the other, and herafter you ſhal learn the other more conueniently.

But yet before wee make an ende of thys rule, this ſhal you note, that there is an other ordre quite contrarye to this that you haue learned. For in this rule hitherto, euermore loke how muche the Thyrd numbre is greater then the Fyrſt, ſo muche the fourth numbre is greater then the Seconde. And contrary wayes, looke how muche the Fyrſte ſumme is greater then the Thyrd, (if it doo

ore.

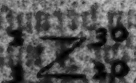
chaunce ſo) ſo muche is the Second ſumme greater then the Fourth. But there is a contrarye ordre as thys: That the greater the thyrde ſumme is aboue the fyrſte, the leſſer the fourthe ſumme is beneth the ſecond, and this rule you maye call the Backer rule, as in example

The Ba
ker rule.
Queſtiõ
bying clo

If I haue bought 30 yardes of clothe, of 2 yardes bredth, and wold bye canuas of 3 yardes broad to lyne it withall, how many yardes ſhoulde I neede?

S. Why, there is none ſo broade.

M. I doo not care for that. I doo put this example only for your eaſye vnderſtandyng: For if I ſhoulde put the example in other meaſures, it would be harder to vnderſtand. But nowe to the matter: If you wold know thys queſtion, ſet your numbres as you dio before: but you ſhall multiplye now the firſt numbre by the ſecond, and that ariſeth therof, you ſhall diuide by the third: which thing if you doo here, I meane if you multiplye 30 by 2, it will be 60: whiche ſumme if you dyuide by 3, there wyll appeare 20. wherby I knowe that if 30 yardes of clothe of two yardes broade, ſhould be lyned with canuas of 3 yards broad, 20 yardes of canuas wold ſuffice, as this figure ſheweth.



And nowe bycauſe yee founde fault at my example exaumple, howe ſaye you, perceaue you thys?

Scholar. Yea ſyr: I ſuppoſe.

An other like queſtiō

Maſt. Than anſwere me to this queſtion. Howe manye elles of canuas of elle breadth wyll ſerue to lyne 20 yardes of Saye, of 3 quarters of a yarde broad?

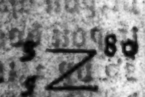
S. In good fayth ſyr, I can not tell, for I knowe not how to bryng the ſummes to like denominations.

M. Than I will tell you: ſith there is mention here of quarters, and againe euery one of the meaſures bothe elles and yardes may be parted into quarters, do you part theym ſo bothe in the breadth and length, and than put forthe the queſtion by quarters.

S. Than I ſhall ſaye thus. Howe manye quarters of canuas of 5 quarters brode, will line 80 quarters of 3 quarters broad?

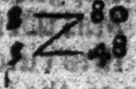
M. Now anſwere to the queſtion.

S. Fyrſt I will ſet them downe in their fourm thus, for 5 is ioyned with the queſtiō, and is therfore the thirde numbre: then is 3 the numbre of the ſame denomination, I mean becauſe thei be both referred to bredth Nowe I multiplye 80 by 3, and it is 240, which I dyuide by 5, and it yeldeth 48. The ſay I, that 48 quarters of 5 quarters brode, wil ſuffice to line 80 quarters of 3 quarters broade.



M. Turne the quarters againe into elles and yardes.

S. Then I ſaye, that 9 elles and 3 quarters of a yarde of elle broad, wyll ſerue to lyne 20 yardes of 3 quarters broad, as this figure ſheweth.



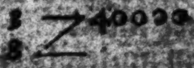
Maſter. This rule is ſo profytable for all eſtates of menne, that for this rule onlye (if there were no more but it) all menne were bound highlye to eſteeme Arithmetike.

By this rule maye a Capitayne in warre worke many thinges, as I wyll hereafter inſtructe you abundantlye, onely now I wil ſhewe you this one example.

Queſtiõs of prouiſion, touching an armye.

If it ſhoulde chaunce a Capitaine which hath 40000 ſouldiers, to be ſo encloſed with his enemie, that hee coulde haue no freſhe purueyance of vittailes, and that the vytayles whyche hee hathe, woulde ſerue that armye but only 3 monethes, how many men ſhould he dimiſſe, to make the vitayle to ſuffice the reſidue 8 monethes?

Sc. As you taught me, I ſette the numbres thus, ſayinge: If 3 monethes ſuffice 40000, to how many will 8 monethes ſuffice?



To know this, I multiply the firſt numbre 3 into the ſeconde 40000, and it yeldeth 120000, which ſumm I dyuide by 8, and ther

wyll

will be in the quotient 15000, whiche if I doo ſubtract from 40000, ỹ remainer wyll declare that hee muſte dimiſſe 25000: as this fygure ſheweth.

3 — 40000
8 — 15000

Ma. Well, ſyth you perceaue nowe the vſe of this rule, I wyll ſhewe other whiche enſue of the ſame. And fyrſte the Double rule, which is ſo called, bicauſe there is in it double working, by which thing only it differeth from this.

he Dou-
le rule.

S. Then by an example I ſhall vnderſtãd it well inoughe.

M. So ſhal you. and let this be the example: If the cariage of 100 pounde weight 30 myles, doo coſte 12 d, howe much wyll the cariege of 500 weight coſt, beyng caryed 100 myles?

f cariage

S. I pray you ſhew me the working of it.

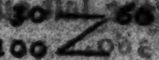
M. You muſt make 2 workinges of it: the fyrſte thus. If 100 pound weight coſt 12 d, howe muche wyll 500 pounde coſte? Set your fygure thus.

100 — 12
500

And multiply 500 by 12, and thereof amounteth 6000, which if you diuide by 100, the quotient will be 60, that is the price of 500 for 30 myles.

Then begin the ſecond worke, ſaying: If 30 myles coſte 60 pence, howe muche will 100 myles coſt?

Set your fygure thus.



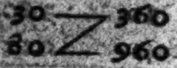
Than multiplye 100 by 60 whereof amounteth 6000, whiche beyng dyuided by 30, will yeld 200. Than you maye ſay, that ſo many pennyes ſhall coſte the cariage of 300 pound waighte 100 myles, after the rate of 12 pence for the 100, caryed 30 myles.

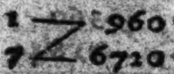
Scholar. Nowe I perceaue it alſo.

M. Then anſwere me to this queſtion. 30 buſhels of wheate ſowed, yeldeth in one yeare 360, howe manye will 80 buſhels yeld in 7 yeare? I meane ſowing euerye yeare of thoſe 7, ſtyll 80 buſhels.

Queſtio of ſowing.

S. Fyrſte I ſay, that if 30 buſhels wyll yelde 360 in 1 yeare, then 80 buſhels wil yeld 960 in one yeare. Then for the ſeconde worke I ſay: Yf one yeare yelde 960, then 7 yeare wyll yelde 6720, as theſe twoo fygures doo ſhewe





But nowe ſyr if I ſet forth 30 buſhels of corne to an other man for 7 yeare, agreeyng ſo, that he ſhall ſowe euerye yeare the whole encreaſe of the corne, and I at the ende of thoſe ſeuen yeares to haue the halfe of the whole encreaſe: I would knowe howe manye buſhels will there amount to my parte, ſuppoſynge the encreaſe to bee after the rate of the

[illegible] of corne.

the laſt queſtion, for 30 buſſhels in one year 360.

Ma. In ſuche a queſtion you muſte haue ſo manye ſeuerall workinges, as there bee yeares: as for exaumple. In the fyrſte yeare 30 buſſhels yelde 360: than to knowe the yeldyng of the ſecōd year, I muſt ſay: If 30 yeld 360, how many yeldeth 360? Work by your rule, and you ſhall fynd 4320. Then ſaye for the thirde year: If 30 yeld 360, how many will 4320 yeld? you ſhall haue 51840. and ſo euery yeare, multiplyinge the whole encreaſe by 360, and dyuidyng it by 30, the encreaſe of the nexte yeare will amount, as theſe 7 fygures doo ordrely declare.

a

30 — 360

b

30 360
360 4320

c

30 360
4320 51840

d

30 360
51840 622080

e

30 360
622080 7464960

f

30 360
7464960 89579520

g

30 360
89579520 1074954240.

Where I haue ſet 7 letters for the 7 yeares, of whiche the fyrſt is ſet without acre: bycauſe that is the encreaſe whiche you doo preſuppoſe, and the laſte numbre of eche other doothe ſhewe the encreaſe of the yeare that it ſtandeth for, whiche the letters dooth declare: ſo that the encreaſe of the 7 yeare, is 1074954240 buſhelles: how many quarters that is, and alſo how many waies, you maye by Reduction ſoone fynde.

Nowe with one queſtion more I wyll proue you. If 6 mowyers do mowe 45 acres in 5 dayes, howe manye mowyers wyll mowe 300 acres in 6 dayes?

One
mow

S. If 45 acres do require 6 mowyers, the 300 acres requyreth 40. Nowe agayne: yf 5 dayes require 40 mowyers, then 6 dayes nedeth but 33 mowyers.

M. Why do you not make mention of the 2 that remayneth in the laſt diuiſion? for the laſte part of the queſtion is wrought by the Backer rule, where the fyrſte numbre 5, is multiplied into the ſecond, that is 40, wherof amounteth 200, whiche if you diuide by the thyrd numbre 6, the quotient wyll be 33 as you ſayde, but then will there remayn 2, which can not well be diuided into 6 partes: howe be it, you may vnderſtand by the firſte parte of 2, the thirde parte of one mannes worke, whiche you muſte put to the 33, or els you maye ſaye that 33 workemen wyll ſuffice.

ende all the 300 acres in 6 dayes, ſaue twoo mennes worke for one day, or 2 daies worke for one man. But ſuche broken numbre called Fractions, you ſhall hereafter more better perceaue, when I ſhall whollye inſtructe you of them.

The rule of Fellowſhyppe.

BUt nowe will I ſhewe you of the rule of Fellowſhippe or Company, whiche hathe ſundry operations, accordynge to the dyuers numbre of the companye. This rule is ſometyme withoute difference of tyme, and ſometymes there is in it dyfference of tyme. Fyrſt I wyll ſpeake of that withoute difference of tyme, of whiche lette thys bee an example.

Four marchantes of one company made a banke of monye diuerſly, for the fyrſt layd in 30 li, the ſecond 50 li, the third 60 li, and the fourth 100 li, which ſtock they occupyed ſo longe tyll it was encreaſed to 3000 li. Nowe I demaund of you, what ſhulde eche man receaue at the partynge of this mony?

Sco. I perceaue that this rule is lyke the other, but yet there is a difference, which I perceaue not.

M. Then will I ſhewe it to you. Fyrſt by Addition you ſhall brynge all the particular ſummes of the marchants into one ſumme, whyche whiche ſhalbe the fyrſte ſumme in your working by the Golden rule, and the whole ſum of the gaynes with the ſtocke ſhalbe the ſecond ſumme. Now for the thyrd ſumme, you ſhall ſet the portion of eche man one after an other, and then worke by the Golden rule, and the fourth ſum will ſhew you eche mans gaynes: as in example.

The parcels of thoſe foure marchauntes make in one ſumme 240 li: ſette that in the fyrſt place, the gaynes in the ſecond, and the fyrſte mans portion of ſtocke in the thyrd place, thus.

240 3000
30

Nowe multiply the ſecond by the thyrde, and it wyll bee 90000, whiche you ſhall diuyde by 240, and there wyll appeare 375 li. thus.

240 3000
30 375

And that is the gaynes for the fyrſt man.

Nowe for the ſeconde man ſet the 50 li. that he brought in yͤ third place, and worke as before: and his part will be 625 li. as this fygure ſheweth.

240 3000
50 625

Likewaies for the thyrd man ſet his money, whiche was 60 li, and hys parte of gaynes wyll be 750 li, as here appeareth.

240 3000
60 750

And ſo for the fourthe man, if you ſet his ſumme whyche is 100 li, his gaynes will bee 1250 pounde, as the proofe wyll declare.

240 3000
100 1250

S. This I perceaue, but is there any way to examine whether I haue wel done or no?

M. That muſte you do by one common proof whiche ſerueth to the Golden rule, and all other enſuing of the ſame: and that is this. Chaunge the ſtandinges of the numbres, and ſet the third in the fyrſt place, the fourth in the ſecond place, and the fyrſt in the third place, and than woorke by the Golden rule. and if you haue done well, the fourth numbre nowe will bee the ſame that was the ſeconde before. As for example. I will take the laſte woorke whiche was thys.

240 3000
100 1250

Whiche to examine, I alter as I ſayde thus.

100 1250
240

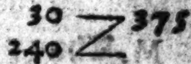
Now if I multyply the ſecond numbre by the thyrd, and diuide that that amounteth by the fyrſte, then wyll the fourthe numbre bee 3000, which was the ſecond before, as you ſee heere.

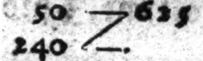
100 1250
240 3000

Whiche is a token, that I haue well doone.

But as in a ſyngle rule one profe thus is ſufficient, ſo in a rule where many operations be, you muſt tourne euerye of them as I haue doone with this one.

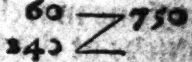
S. Then for the proof of the fyrſte woorke of thys rule I ſhoulde tourne the numbres thus.





And for the ſeconde, thus.

And for the thirde thus.



And in eche of them if the woorkinge were true, the fourthe numbre wyll be ſtyll 3000.

M. Well, nowe an other example will I put to you, not of gaines, but of loſſe: for one reaſon ſerueth for bothe.

On loſſe

If three marchauntes in one ſhyp and of one fellowſhip, hade bought marchandiſe, ſo that the fyrſt had layd out 200 li: the ſecond 300 li, and the third 500 li, and it chaunced by tempeſt that they dydde caſt ouer boorde into the ſea marchādiſe of the valew of 100 pounde, howe muche ſhoulde eche man bear in this loſſe?

S. If I ſhall do in this, as you dyd in the other queſtion. then muſte I ioyne their thre portions togyther, 200, 300, and 500. which maketh 1000: than ſay I; if 1000 leeſe 100 then ſhall 200 leeſe 20, & 300 ſhall leeſe 30, & 500 ſhall leeſe 50, as by theſe thre fygures it doth appeare plaine.

1000 100 1000 100
200 20 300 30

1000 100
500 50

M. Thus you perceaue the vſe of the rule without time. And that you may as wel perceaue the ſame with diuerſitie of time, I propoſe this Example.

le of ſhyp yme.

ns of cke.

Four marchauntes made a cōmon ſtock, whiche at the yeares ende was encreaſed to 35145 li. Nowe to knowe what ſhalbe eche mannes portion of gaines, you muſt knowe eche mannes ſtocke & tyme of continuance.

The fyrſte manne of theſe foure layde in 669 li, whiche he dyd take from the ſtocke againe at the ende of 10 monethes. The ſeconde man layde in 810 li, for 8 monethes. The thirde layde in 900 li, for 7 monethes. And the fourthe layde in 1040 li. for 12 monethes.

This queſtion ſhall you examine as you dyd the other before, ſauinge that where as in the thyrd place of the fygure you ſet eche mannes ſumme alone, here you ſhall ſet the ſame beyng multiplyed by the nūbre of their tyme: & lykewaies in the fyrſt place of the fygure, you ſhall ſette their whole ſummes ſo multiplyed by their time, and added into one ſumme, as thus.

The firſt mannes ſumme is 669 li, which I multiplye by 10 (that was the numbre of hys his time) and it maketh 6690. The ſeconde mannes ſum 810 li, multiplied by 8 (which was his tyme) maketh 6480. The thyrde mannes ſumme 900 li, multiplied by 7 (for ỷ was his time) yeldeth 6300. The fourthe manne ſumme was 1040 li, and his tyme 12. multiply the one by the other, and it will be 12480.

These foure ſummes thus multiplyed by theyr tyme, muſte be ſet ordrely in the third place of the fygure; and in the fyrſte place muſte be ſette the whole ſumme of all four, whiche is, 31950. Nowe to ende the queſtiō I ſaye fyrſte: If 31950 dydde gette 35145, what dyd 6690 get? Anſwere 7359 li, as by this fygure appeareth.

a

31950 — 35145

6690 — 7359

Likewaies the ſecōd manne hadde to his parte 7128 li. the thyrd muſte haue 6930 li. And the fourth manne ſhall haue for hys parte 13728 li. as theſe figures do partelly declare.

b

31950 — 35145

6480 — 7128

c

31950 — 35145

6300 — 6930

d

31950 — 35145

12480 — 13728

S. This I lyke very well, but what profe is there of this worke?

Ma. The ſame that I taughte you for the

other. How be it, there is vſed bothe for this worke & the other alſo this mãner of profe, to adde all the portions togither: and if they agree to the whole ſumme, then ſeemeth it well doone, but this is no ſure rule.

S. Yet will I proue it in this example. The foure parcels are theſe, which if I adde togither, there will amounte 35145, and that was the whole ſumme. ſo is this rule true here.

7359
7128
6930
13728

M. And ſo will it be ſtill, when the worke is truely doone.

But if you lyſte to ſee it proued falſe, take 10000 li. from the fourth man, and putte it to anye of thother 3, and than be you ſure that you haue not doone well, and yet wyll that proofe alowe it, for the Addition wyll ſtyll bee all one.

S. It muſte needes be ſo: but what haue I nowe to learne?

M. There are manye other excellent partes behynde, of whiche I will not as nowe make mention, bicauſe that withoute the knowledge of Fractions, they can not bee duely be taught, and much leſſe vnderſtand. Therfore will I propoſe to you two or thre queſtions more, whereby you maye practiſe the better the feat of the rule of Felowſhip, and ſo make an ende for this tyme.

But this may not be forgotten: that in all ſuche queſtions, if the money bee of dyuers kindes, you muſt by Reduction brynge into one kynde, that is to ſaye, to the leaſte value that is named in the queſtion. And like waies ſhall you do, if the tyme be of diuers kyndes, as ſome yeares, ſome monethes, weekes and dayes, you ſhall make all monethes weekes or daies, accordynge as the leaſte name of tyme in the queſtion is: As for example.

Note.

Fyrſte in diuerſitie of monye. Thre companions bought 2000 ſheepe, and payde for them 241 pound, 13 ſ, 4 d. of whiche ſum one payde 101 li, 10 ſ. The ſeconde payd 82 li, 17 ſ, 10 d. And ỹ thyrd paid 57 li, 5 ſ, 6 d. Howe many ſhepe muſt eche of them haue? Anſwere: The fyrſte ſhall haue 840. The ſeconde 686. And the thirde 474. And that muſte you worke thus.

Questio
of shee

Fyrſt conſideringe that your money is of dyuers denominations, you ſhall (by Reduction) brynge it all into the ſmalleſt denomination whiche is in it, that is to ſaye, pence. and ſo will the totall ſum bee 58000 pence. Nowe if you tourne eche mannes money into pennies alſo, the fyrſte mannes ſum wyll be 24360 pence: The ſecond mannes ſumme 19894 d. And the third mans money will be 13746 pence.

Nowe to knowe how many ſheepe euerye man ſhall haue, ſet the whole ſum of moeny that is 58000 d, in the fyrſte place, and in the ſeconde place ſette the numbre of ſheepe, and then orderly in the thyrde place ſet eche mannes money, and then multiplyinge the thyrde and the ſecond ſummes togither, and dyuidyng that that amounteth by the fyrſt, there wil appeare the numbre of ſhepe that eche man oughte to haue, as theſe three figures do ſhewe.

a
58000 2000
24360 840

b
58000 2000
19894 686

c
58000 2000
13746 474

Scolar. Why do you ſette the mony in the fyrſte place, ſeynge in the queſtion you ſay 2000 ſheepe coſte 58000 d? and not thus, 58000 d coſt 2000 ſheepe?

M. You remembre, I taughte you at the beginnynge of this Golden rule, that the fyrſt and third numbre muſt be of one name and of lyke thinges: & euermore the numbre that the queſtion is aſked of, muſt be ſette in the thyrde place. Now is the queſtion plainly this: If foure men boughte 2000 ſheepe for 58000 pence, howe manye ſheepe ſhall eche manne haue?

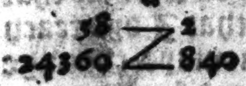
But ſeeyng in this queſtion there oughte more reſpecte to bee hadde to the ſumme of moneye, than to the ſumme of the perſons, (for in the ſummes of money is there proportion toward the ſheepe, and not in the numbre of perſons) therfore muſt we turne the queſtion thus.

If 58000 pence bought 2000 ſheep, how many dydde 24360 d̃ bye? agayne how many did 19894 d̃ bye? and how many bought 13746 pence?

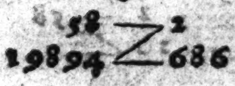
S. I perceaue it reaſonable, and ſo ſhall I do in all lyke queſtions.

M. Euen ſo. But for eaſineſſe of the worke marke this: When ſo euer the fyrſte and ſecond nũbre hath cyphres in their firſt places you may bothe in the multiplication and in the diuiſion leaue out thoſe cyphres, ſo that you leaue out like many out of both ſũmes, as in this queſtion the fyrſte numbre 58000 hath three cyphres, and ſo hath the ſeconde, that is 2000: therfore caſte awaye their cyphers, and ſo will the fyrſte numbre bee 58, and the ſeconde 2. ſet theym in their places, and worke accordyng to the rule, and you ſhall perceaue tha it will be all one, ſauynge that this is the ſhorter and eaſier waye, as theſe thre fygures do ſhewe. Note.

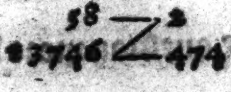
a



b



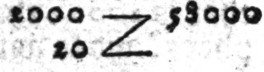
c



And this you ſee is bothe the eaſier and alſo the more certain way to knowe the anſwere to this queſtion.

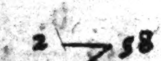
S. Truth it is as you ſay: but ſyr me ſeemeth I might aſke a farther queſtion here, not onlye how many ſheep eche man ſhoulde haue, but alſo what euery ſheepe coſt?

M. That queſtion doth not only belonge to thys rule, but maye alſo bee diſcuſſed by Diuiſion, eſpecially if the queſtions numbre bee one onlye: as thus. Dyuide the totall ſumme 58000 pence, by 2000 (other 58 by 2, omittynge the cyphres) and the quotient wyll bee 29 pence. that is, 2 ſ, 5 d. howe bee it, by this rule you maye doo it, and beſte whan the numbre of the queſtion doothe exceede 1: as if I ſhoulde aſke this queſtiõ. 2000 ſheep coſt 58000 d, howe muche dydde 20 coſte? Then ſhall I ſette my figure thus.



And doing after the rule, ther wyll amount 580 pence. (that is 2 li, 6 ſ, 4 d) the price of one ſcore.

But if you wyll vſe that eaſy way that I dyd teach you, you maye chaunge the fyrſte and ſecond numbre thus.



Yet nowe one queſtion more wyl I moue

(that you maye perceaue the vſe of all other like) and ſo make an ende.

There is in a Cathedrall churge 20 Cannons, and 30 Vicars, thoſe maye ſpende by yeare 2600 li. but euerye cannon muſt haue to hys parte 5 tymes ſo muche as euery Vicar hath: how muche is euery mannes portion ſaye you?

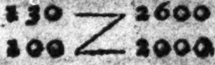
Queſtio
Cannon

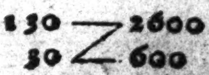
S. I pray you make the anſwere your ſelf, ſo ſhall I perceaue beſte the meanes to anſwere to ſuche other lyke.

Ma. In this queſtion you muſte doo as in thoſe that haue diuerſity of tyme, for here is diuerſity of portions: Therfore ſhal you multiply the numbre of the perſons by their difference of portion: (as you dyd in the other by tyme.) Then muſte you multiplye the 20 (which is the numbre of Caunons) by 5, (for that is the numbre of their portion) ſo wyll it bee 100: Then 30, (that is the numbre of Vicars) by 1, (that is the numbre of theyr portion) and it will be 30: put thoſe two ſummes togither, and they make 130. then ſaye thus: If 130 ſpend 2600 li, what may 100 ſpend? The rule ſheweth 2000 li.

Againe for Vicars: If 130 ſpende 2600 pound, what may 30 ſpend? Anſwer, 600 li. as theſe fygures ſhewe.

Vicare





But if euery Cannon ſhuld haue ſo often tymes 4 li, as the Vicar ſhuld haue 3 li, then ſhoulde I multiply 20 by 4. (that were 80) and 30 by 3 (that were 90) and then bothe were 170. Then ſhould the fygures be ſette thus.

li. ſ. d. li. ſ. d.
170 Z 2600 170 Z 2600
80 Z 1223, 10, 7 90 Z 1376, 9, 5.

But this ſorte is to harde for you by reaſon of the Fractions, therfore I wyll let it reſt to that place. And by this rule you ſee what the 20 Cannons maye ſpende: whiche ſumme if you diuide by 20, you ſhall ſee eche Cannons portion: and ſo of the Vicares, yf you diuide theyr ſumme by 30, the quotiente wyll declare euerye Vicars portion.

The ſecond Dialogue.

The accomptyng by counters.

Maſter.

Nowe that you haue learned the common kyndes of Arithmetike with the penne, you ſhall ſee the ſame arte in counters: whiche feate dooeth not onely ſerue for theym that can not write & read, but alſo for thē that can do both, but haue not at ſom times their penne or tables ready with them.

This ſorte is in two formes cōmonly: The one by lynes, and the other without lynes. In that that hath lines, the lines do ſtād for the ordre of places: and in that that hath no lines, there muſt be ſet in their ſteed ſo many counters as ſhall neede, for eche lyne one, and they ſhall ſupply the ſteede of the lines.

S. By examples I ſhould better perceaue your meanyng.

M. For example of the lynes, Lo here you ſe vi. lines, whiche ſtand for ſyx places, ſo that ẏ nethermoſt ſtandeth for the

—100000————————
—10000————————
✕1000————————
—100————————
—10————————
—1————————

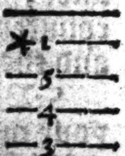
fyrſte place, and the nexte aboue it for the ſeconde: and ſo vpwarde tyll you come to the higheſt, which is the ſyxte line, and ſtandeth for the ſyxte place.

Valewe. Nowe what is the value of euery place or line, you may perceaue by the figures which I haue ſet on them, which is accordynge as you lerned before ỹ in Numeratiō of figures by the penne: for the fyrſte place is the place of vnites or ones, and euery counter ſet in that lyne betokeneth but one: and the ſeconde lyne is the place of 10, for euery coūter ther ſtandeth for 10. The thyrd lyne the place of hundredes, the fourth of thouſandes: and ſo forth.

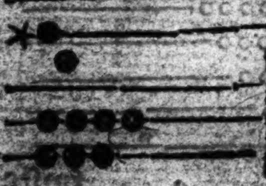
S. Syr I do perceaue that the ſame ordre is here of lynes, as was in the other figures by places, ſo that you ſhall not need longar to ſtande about Numeration, except ther be any other difference.

M. If you do vnderſtand it, then how will you ſette 1543?

S. Thus as I ſuppoſe.



M. You haue ſet the places truly but your fygures bee not meete for this vſe: for the meeteſt fygure in this behalfe, is the figure of a counter, round, as you ſee here, where I haue expreſſed that ſame ſumme.

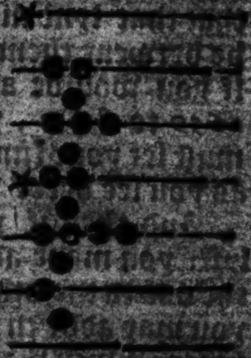


Scholar.

S. So that you haue not one figure for 2, nor 3, nor 4, and so forth, but as manye digettes as you haue, so manye counters you ſette in the loweſt lyne: and for euery 10 you ſet one in the ſeconde lyne: and ſo of other. But I knowe not by what reaſon you ſette that one coūter for 500 betwene two lines.

M. You ſhall remembre this, that when ſo euer you nede to ſet downe 5, 50, or 500, or 5000, or ſo forth anye numbre, whoſe numerator is 5, you ſhall ſette one counter for it in the next ſpace aboue the lyne that it hath his denomination of: as in this example of that 500, bicauſe the numeratour is 5, it muſte be ſette in a void ſpace: and bycauſe the denominator is hundred, I knowe that his place is the voide ſpace nexte aboue hundredes, that is to ſaye, aboue the thirde lyne.

And farther you ſhall marke, that in all working by this ſorte, if you ſhall ſet downe any ſumme between 4 and 10, for the firſt part of that numbre, you ſhall ſet downe 5, and then ſo many counters more, as there reſt nūbres aboue 5. And this is true bothe of digettes and articles. And for example, I will ſet downe this ſum 287965, whiche ſumme yf you marke well, you nede



none other examples for to learne the numeration of this forme.

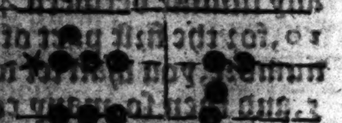
But this ſhall you marke, that as you dyd in the other kynd of Arithmetik ſet a pricke in the places of thouſandes, in this worke you ſhall ſet a ſtarre, as you ſee before.

S. Then I perceaue Numeration: but I praye you, how ſhall I do in this art to adde twoo ſummes or more together?

ADDITION.

Maſter.

The eaſieſt way in this arte, is to adde but two ſummes at ones togyther: how be it, you maye adde more, as I wil tel you anone. Therefore whenne you wylle adde two ſummes, you ſhall fyrſte ſet downe one of them, it forceth not whiche, and then by it drawe a lyne croſſe the other lynes. And afterwarde ſette downe the other ſumme, ſo that that lyne maye be betwene them: as if you woulde adde 2659 to 8342, you muſt ſet your ſummes as you ſee here.

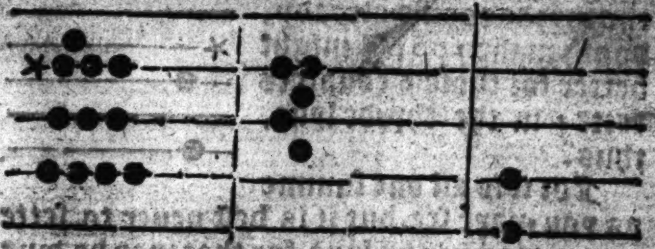


And then if you lyſt, you maye adde the one to the other in the ſame place, or els you may adde them both togyther in a newe place: which way, bycauſe it is moſt playneſt

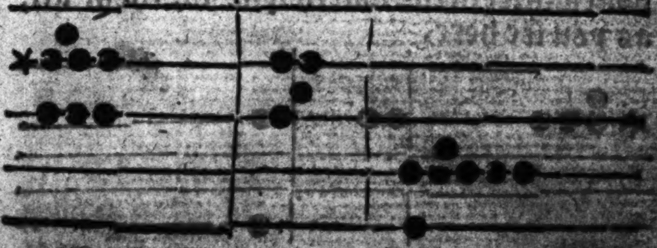
...ition twoo ſummes.

I will ſhow you fyrſt.

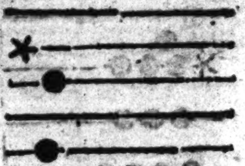
Therfore will I beginne at the vniters, whiche in the fyrſte ſumme is but 2, and in the ſeconde ſumme 9, that maketh 11. Thoſe do I take vp, and for theym I ſet 11 in the newe roome, thus.



Then do I take vp all the articles vnder a hundred, which in the fyrſt ſumme are 40, and in the ſeconde ſumme 50, that maketh 90: or you maye ſaye better, that in the firſt ſumme there are 4 articles of 10, and in the ſecond ſumme 5, whiche maketh 9. but then take heede that you ſette them in their right lynes, as you ſee heere.



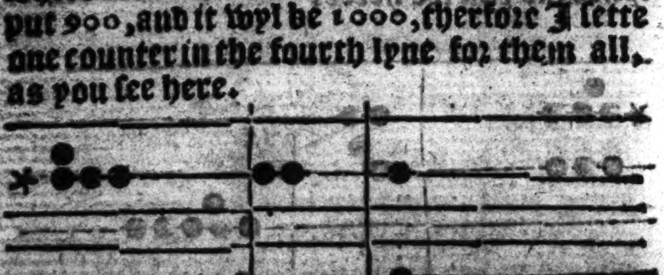
Where I haue taken awaye 40 frome the fyrſte ſumme, and 50 from the ſecond, and in their ſteede I haue ſet 90 in the third roum, whiche I haue ſet plainly, that you myghte well perceaue it: howe be it, ſeyng that 90 with the 10 that was in the thyrde roome already, dooth make 100, I might better for thoſe 6 counters ſette 1 in the thyrde lyne, thus.



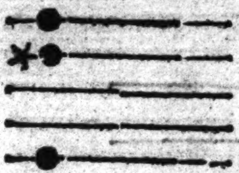
For it is all one ſumme as you maye ſee, but it is beſt neuer to ſette 5 counters in any lyne, for that may be done with 1 counter in a higher place.

Sch. I iudge that good reaſon, for manye are vnneedfull where one wyll ſerue.

M. Well, then will I adde foorth of hundredes: I fynde 5 in the fyrſte ſumme, and 6 in the ſeconde, whiche maketh 9000, then hon I take vp and ſette in the thyrde roume where is one hundred already, to whiche I put 900, and it wyl be 1000, therfore I ſette one counter in the fourth lyne for them all, as you ſee here.



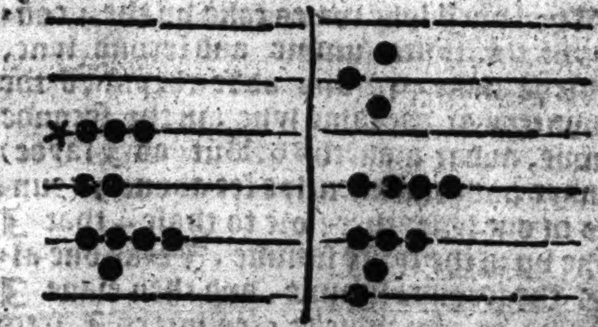
Then adde I the thouſandes togyther, whiche in the fyrſte ſumme are 8000, and in the ſecond 2000, that maketh 10000: them doo I take vp from thoſe twoo places, and for theym I ſet one counter in the fyfte line, and thẽ appeareth as you ſee to bee 11001. for ſo manye doothe amounte of the Addition of 8342 to 2659.



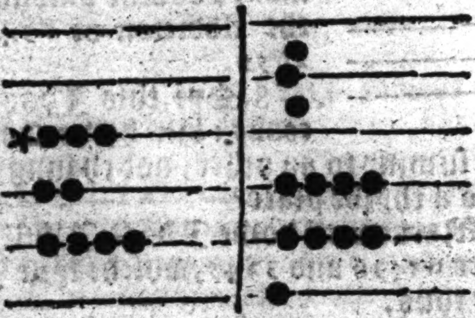
To adde two ſum togythe

S. Syr, this I doo perceaue: but howe ſhall I ſet one ſumme to an other, not chaungynge them to a thyrde place?

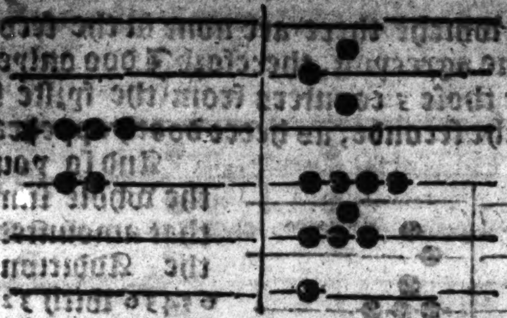
M. Marke well howe I do it: I will adde togither 65436 and 3245, whiche fyrſt I ſet downe thus.



Then doo I begynne with the ſmalleſt, whiche in the firſte ſumme is 5, that doo I take vp, and wolde put to the other 5 in the ſeconde ſumme, ſauynge that two counters can not be ſette in a voyde place of 5, but for them bothe I muſte ſet 1 in the ſecond line, whiche is the place of 10: therfore I take vp the 5 of the fyrſt ſumme, and the 5 of the ſecond, and for theym I ſette 1 in the ſeconde lyne, as you ſee here.

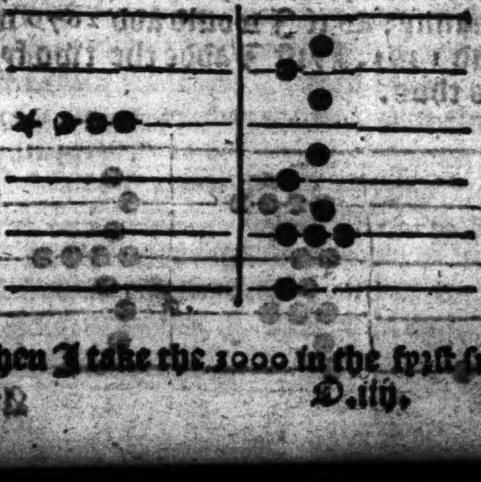


Then do I lyke wayes take vp the 4 counters of the fyrſte ſumme and ſecond lyne, (whiche make 40) and adde theym to the 4 counters of the ſame lyne, in the ſeconde ſumme, and it maketh 80. But (as I ſayde) I maye not conuenientlye ſet aboue 4 counters in one lyne, therefore to thoſe 4 that I tooke vp in the fyrſte ſumme, I take one alſo of the ſeconde ſumme, and then haue I taken vp 50, for whiche 5 counters I ſette downe one in the ſpace ouer the ſecond line, as here doth appeare.

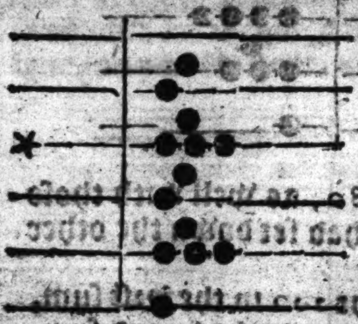


And than is there 80, as well with thoſe 4 counters, as if you had ſet downe the other 4 alſo.

Nowe do I take the 200 in the fyrſt ſum, and adde them to the 400 in the ſecond ſum, and it maketh 600, therefore I take up the 2 counters in the fyrſt ſum, and 3 of them in the ſeconde ſumme, and for them 5, I ſette 1 in the ſpace aboue, thus.



Then I take the 1000 in the fyrſt ſumme, vnto whiche there are none in the ſeconde ſumme agreeyng, therfore I doo onlye remoue thoſe 3 counters from the fyrſte ſum into the ſeconde, as heere dooth appeare.

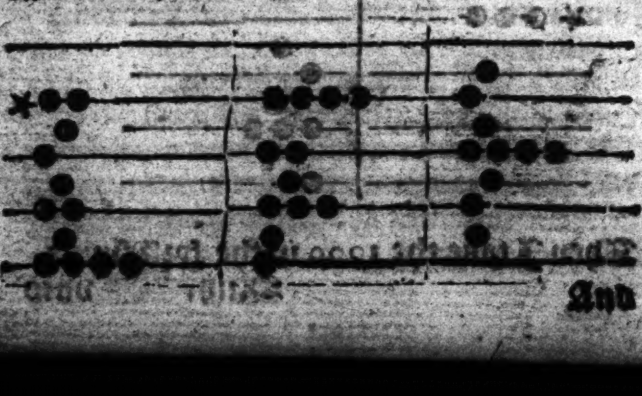


And ſo you ſee the whole ſumme that amounteth of the Addition of 65436 with 3245, to be 68681.

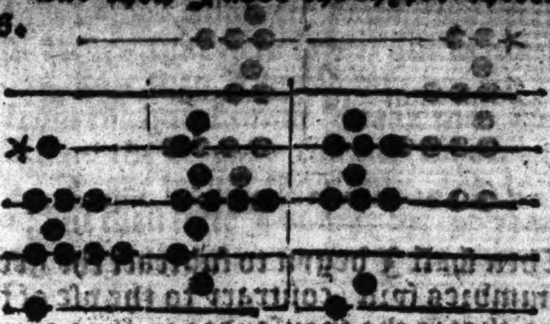
And if you haue marked theſe two exãples well, you need no farther inſtruction in Addition of 2 only ſummes: but if you haue more then twoo ſummes to adde, you maye adde them thus.

on
ee
es.

Fyrſte adde two of them, and then adde the thyrde and the fourthe or more if there bee ſo manny: as if I would add 2679 with 4286 and 1321. Fyrſt I adde the two fyrſte ſummes thus.



And then I adde the thyrde thereto, thus.



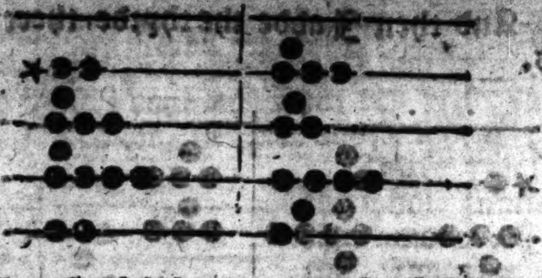
And ſo of more yf you have them.

Scholar. Nowe I thinke beſte that you paſſe forthe to Subtraction, except there be any wayes to examine this maner of Addition, then I thynke that were good to bee knowen nexte.

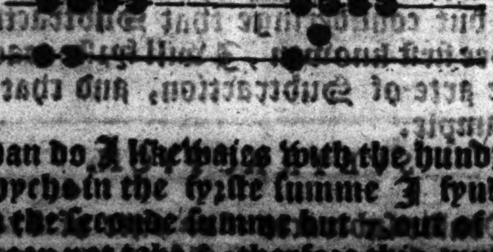
Maſter. There is the ſame profe here that is in the other Addition by the penne, I meane Subtraction, for that only is a ſure waye: but conſideringe that Subtraction muſte bee firſt knowen, I will fyrſte teache you the arte of Subtraction, and that by this example.

SVBTRACTION.

I Woulde ſubtracte 1892 oute of 8746. Theſe ſummes muſt I ſet downe as I did in Addition: but here it is beſte to ſette the leſſer nombre fyrſt, thus.

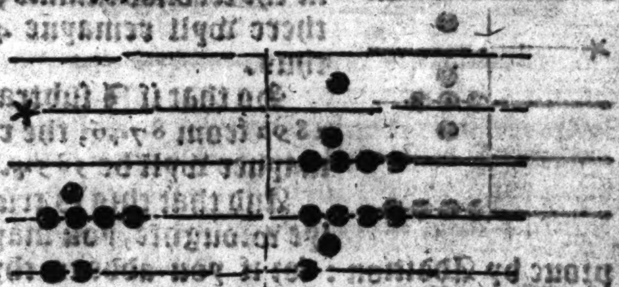


Then ſhall I begyn to ſubtract the greateſt numbres fyrſt (contrary to the vſe of the penne) that is the thouſandes in thys example: therefore I fynde amongeſt the thouſandes 2, for whyche I withdrawe ſo many frome the ſeconde ſumme (where are 8) and ſo remayneth there 6, as this exaumple ſheweth.



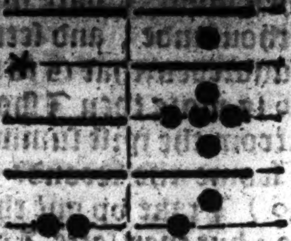
Than do I lykewiſe with the hundredes, of whyche in the fyrſte ſumme I fynde 8, and in the ſeconde ſumme but 7, out of whiche I can not take 8, therfore this muſt I doo: I muſt looke howe muche my ſumme differeth from 10, whiche I fynde here to be 2.

2, then muſte I bate for my ſumme of 800 one thouſande, and ſette downe the exceſſe of hundredes, that is to ſaye, 2, for ſo muche 1000 is more then I ſhould take vp. Therefore from the fyrſt ſumme I take that 800, and frome the ſeconde ſumme (where are 6000) I take vp one thouſande, and leaue 5000: but then ſette I downe the 200 vnto the 700, that are there already, and make them 900, thus.



Then come I to the articles of tennes, where in the fyrſte ſumme I fynd 90, and in the ſeconde ſumme but onlye 40: Now conſiderynge that 90 can not be bated from 40, I looke howe muche that 90 doothe dyffer from the nexte ſumme aboue it, that is 100, or elles (whiche is all to one effecte) I looke howe muche 9 dooth differe from 10, and I fynde it to be 1, then in the ſteed of that 90, I doo take frome the ſeconde ſumme 100: but conſideringe that it is 10 to muche, I ſette downe 1 in the nexte lyne beneth for it,

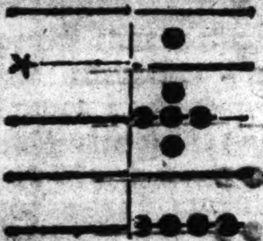
as



as you ſee heere.

Sauynge that here I haue ſet one counter in the ſpace inſteed of 5 in yͤ nexte lyne.

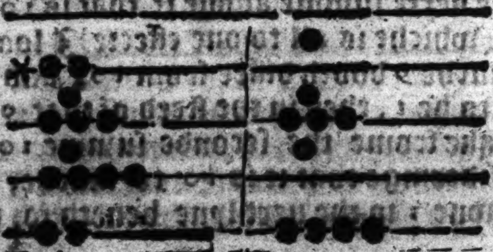
And thus haue I ſubtracted all ſaue twoo, whiche I muſte bate from the 6 in the ſeconde ſumme, & there wyll remayne 4, thus.



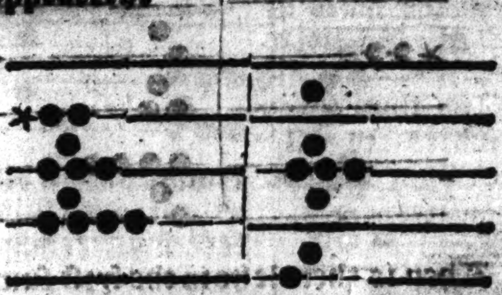
So that if I ſubtract 2892 from 8746, the remayner wyll be 5854.

And that this is truely wroughte, you maye proue by Addition: for if you adde to thys remayner the ſame ſumme that you did ſubtract, than wyll the former ſumme 8746, amount agayne.

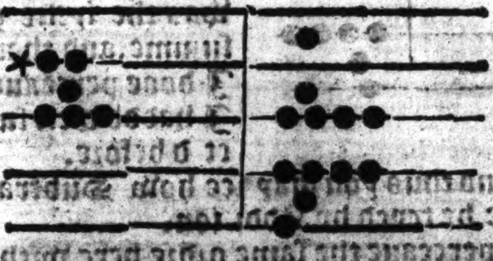
S. That will I proue: and fyrſt I ſette the ſum that was ſubtracted, whiche was 2892, and then the remainer 5854, thus.



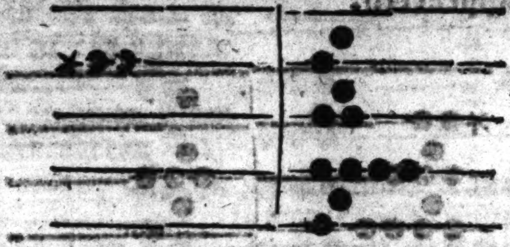
Then do I adde fyrſte the 2 to 4, whiche maketh 6: ſo take I vp 5 of thoſe counters, and in their ſteede I ſette 1 in the ſpace, as heere appeareth.



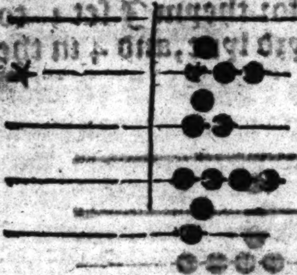
Then do I adde the 90 next aboue to the 50, and it maketh 140, therefore I take vp thoſe 6 counters, and for theym I ſet 1 to the hundredes in the thyrd lyne, and 4 in the ſecond lyne, thus.



Then do I come to the hundredes, of whiche I fynd 8 in the fyrſt ſumme, and 9 in the ſecond, that maketh 1700, therefore I take vp thoſe 9 coũters, and in their ſteed I ſet 1 in the fourth line, and 1 in the ſpace next beneath, and 1 in the thyrd line, as you ſee here.



Then is there lefte in the fyrſt ſumme but only 2000, which I ſhal take vp frō thence, and ſet in the ſame lyne in the ſecond ſumme to the one that is ther alreadye: & then wyll the whole ſumme appear as you may well ſee, to be 8746, which was the fyrſte groſſe ſumme, and therefore I dooe perceaue that I hadde well ſubtracted before.



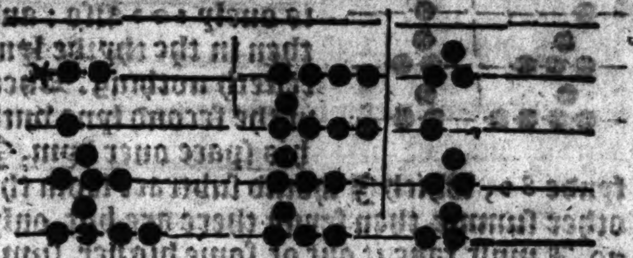
And thus you may ſee how Subtraction maye be tryed by Addition.

S. I perceaue the ſame ordre here with counters, that I learned of you in fygures.

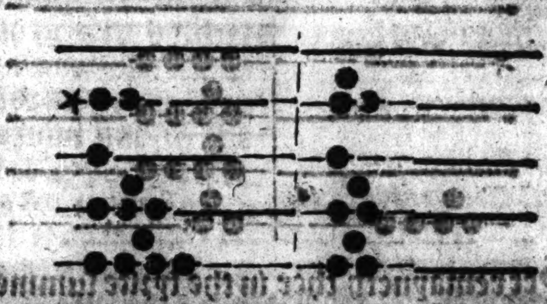
M. Then let me ſee, how can you trye Addition by Subtraction?

S. I wyll [illegible] this example

of Addition, where I haue added 2189 to 4988, and the whole ſumme appeareth to bee 7177.

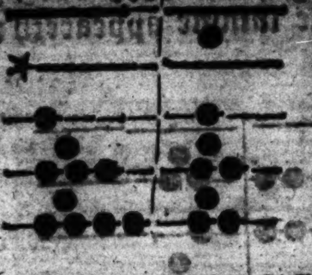


Now to trye whether that ſumme be wel added or no, I wyll ſubtract one of the fyrſt two ſummes from the thyrde, and if I haue wel doone, the remayner wilbe lyke that other ſumme: as for example: I will ſubtract the fyrſte ſumme from the thyrde, whyche I ſette thus in their ordre.

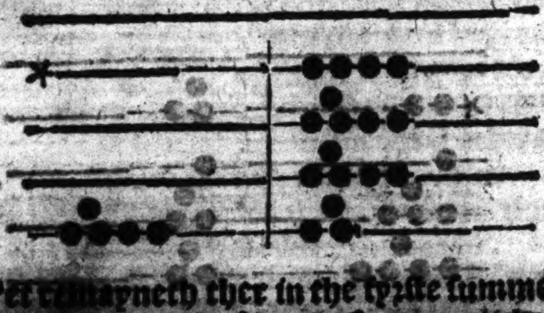


Then doo I ſubtracte 2000 of the fyrſte ſumme from the ſecond ſumme, and there remayneth there 5020, thus.

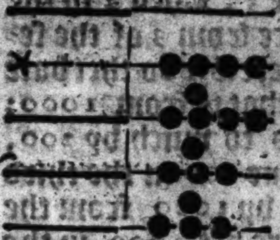
Then



Then in the third lyne I ſubtracte the 200 of the fyrſte ſumme from the ſecond ſum, where is onely 200 alſo: and then in the thyrde lyne reſteth nothing. Then in the ſecond lyne with his ſpace ouer hym, I fynde 80, which I ſhould ſubtract from the other ſumme, then ſeyng there are but onely 70, I muſt take it out of ſome higher ſum, whiche is heere onely 5000; therfore I take vp 5000; and ſeeyng that it is to muche by 4920, I ſet downe ſo manye in the ſeconde roum, which with the 70 beyng there alreadye, doo make 4990, and then the ſummes dooth ſtande thus.



Yet remayneth ther in the fyrſte ſumme 9 to be bated from the ſeconde ſumme, where in that place of vnitees dooth appeare onlye 7: then muſt I bate a hygher ſumme, that is to ſay 10, but ſeyng that 10 is more then 9 (whiche I ſhould abate) by 1, therfore ſhal

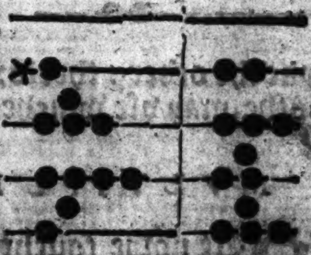


I take vp one counter from the ſeconde lyne, and ſet down the ſame in the fyrſte or loweſt lyne, as you ſee heere.

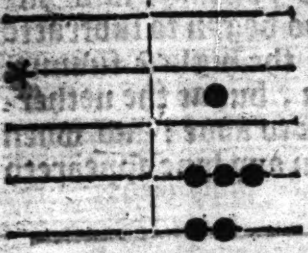
And ſo haue I ended this woorke, and the ſumme appereth to be the ſame, whiche was the ſeconde ſumme of myne addition, and therefore I perceaue, I haue well doone.

An o
way of a
dition.

M. To ſtande longer about this it is but folly, excepte that this you maye alſo vnderſtande, that many do begyn to ſubtracte with counters, not at the higheſt ſumme, as I haue taughte you, but at the nethermoſte, as they doo vſe to adde: and when the ſum to be abated in any lyne appeareth greater then the other, then doo they borowe one of the next hygher rome, as for example. If they ſhoulde abate 1846 frome 2378, they ſet the ſummes thus.



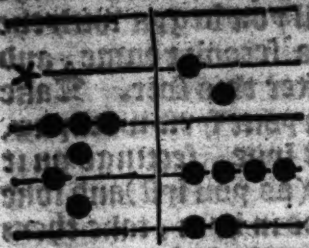
Fyrſt they take 6 whiche is in the lower line, and his ſpace from 8 in the ſame rommes in the ſecond ſumme, and yet there remaineth 2 counters in the loweſt line. The in the seconde line muſte 4 be ſubtract from 7, and ſo remaineth there 3. Then 2 in the thyrde lyne, and his ſpace from 3 of the ſecond ſum can not be, therefore do thei bate it from a hygher rome, that is from 1000; and becauſe that 1000 is to much by 200, therfore muſt I ſet downe 200 in the third lyne, after I haue taken vp 1000 from the fourth lyne. Then is there yet 1000 in the fourth lyne of the firſte ſumme, whiche if I withdrawe from the ſeconde ſumme, then dooth all the fygures ſtande in this ordre.



So that (as you ſee) it differeth not greatly whether you beegynne Subtraction at the hygher lynes, or at the lower. Howe be it, as ſome menne lyke the one waye beſte, ſo ſome lyke the other: therfore you now knowyng bothe, may vſe whiche you lyſte.

MVLTIPLICATION.

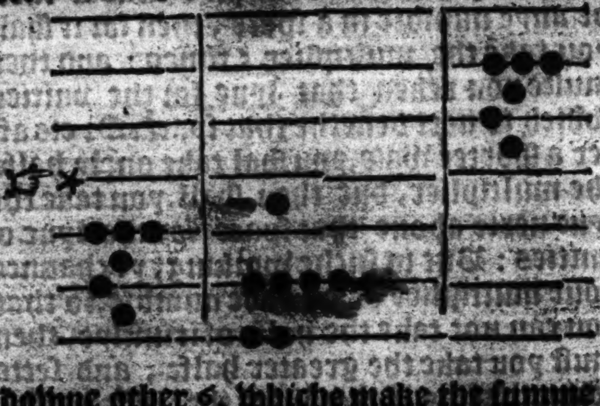
BUt nowe touchynge Multiplication: you ſhall ſette your numbres in twoo roomes (as you dyd in thoſe two other kindes) but ſo that the multiplier be ſet in the firſt roume. Then ſhall you begyn with the higheſt numbres of the ſecond roume, and multiply them fyrſt after this ſort. Take that ouermoſte lyne in your fyrſt workyng, as if it were the loweſt lyne, ſettinge on it ſome moueable marke (as you lyſt) and loke how many counters be in hym, take them vp, & for them ſet downe the hole multiplier ſo many tymes as you toke vp counters: rekenyng (I ſay) that lyne for the vnities. And when you haue ſo done with the higheſt numbre, then come to the nexte lyne beneth, & doo euen ſo with it, and ſo with the next, till you haue doone all. And if there be anye numbre in a ſpace, then for it ſhall you take the multiplier 5 times: and then muſte you reken that lyne for the vnities which is next benethe that ſpace. Or els after a ſhorter waie, you ſhal take onely halfe the multiplier, but then ſhall you take the lyne nexte aboue that ſpace for the lyne of vnities: But in ſuche workyng, yf chaunce your multiplier be an odde numbre, ſo that you can not take the halfe of it iuſtlye, then muſt you take the greater halfe, and ſette down that, as if ỷ it were the iuſte halfe: and farther you ſhall ſet one counter in the ſpace beneth that lyne, whiche you recken for the lyne of vnities, or els onlye remoue forward the ſame that is to be multiplyed.



S. If you ſet forth an exaumple hereto, I thynke I ſhall perceaue you.

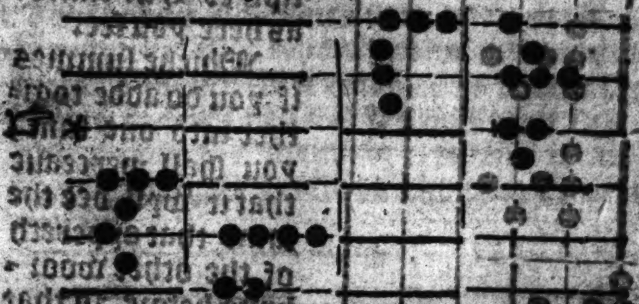
Ma. Take this exãple: I would multiplye 1542 by 365, therfore I ſet the nũbres thus.

Then fyrſte I begyn at the 1000 in the higheſt roume, as if it were the fyrſt place, & I take it vp, ſettyng downe for it ſo often (that is ones) the multiplyer, which is 365, thus as you ſee here: wher for the one counter taken vp frõ the fourth lyne, I haue ſet downe other 6, whiche make the ſumme of the multiplyer, reckenyng that fourth line as if it were the fyrſt, which thyng I haue marked by the hand ſet at the begynnyng of the ſame.

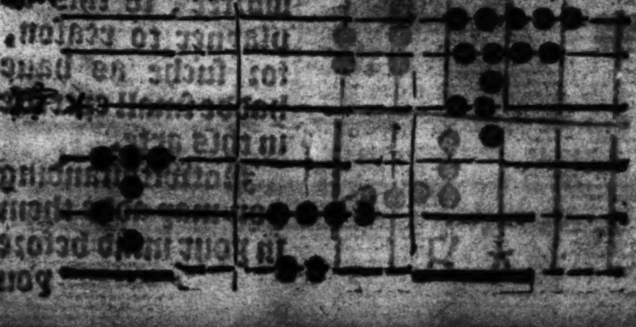


S. I perceiue this wel, for in dede this ſum ỹ you haue ſet doun is 365000: for ſo much doth amount of 1000, multiplied by 365.

M. Well, thẽ to go forth in the next ſpace I fynd one counter, whiche I remoue forwarde but take not vp, but doo (as in ſuche caſe I muſt) ſet downe the greater halfe of my multiplyer (ſeyng it is an odde numbre) whiche is 182, and here I do ſtyll let that fourth place ſtand, as if it were the fyrſte: as in this forme you ſee.



where I haue ſet this multiplication wyth other, but for the eaſe of your vnderſtãding I haue ſet a litle lyne betwene them: Nowe ſhuld they bothe in one ſumme ſtand thus.



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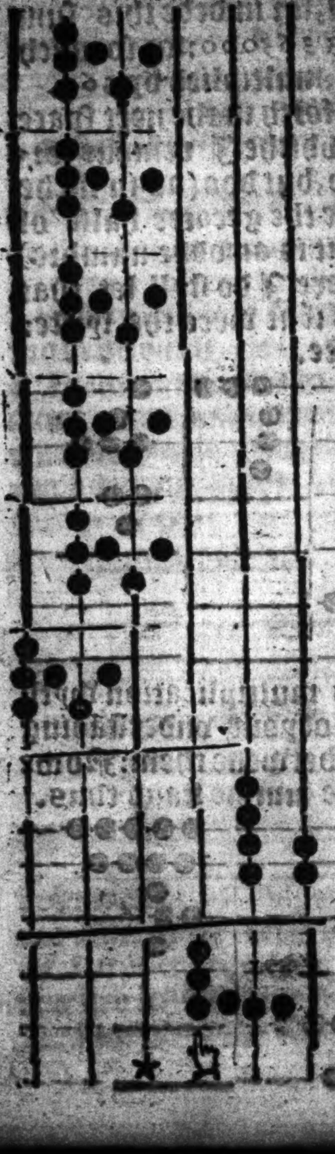
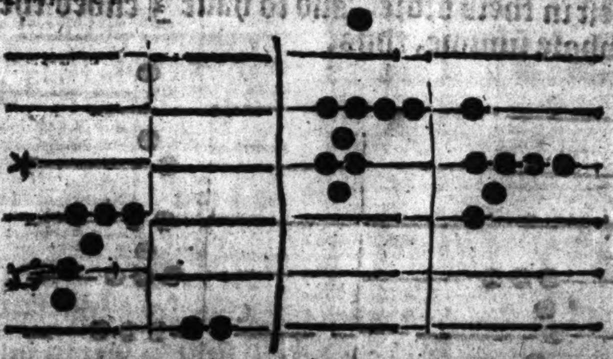


How be it, an other fourme to multiply ſuche counters in ſpace, is this: Fyrſt to remooue the finger to the lyne next beneth that ſpace, and than to take vp that counter, and to ſette down the multiplyer fyue times: as here you ſee.

Whiche ſummes if you do adde togither into one ſum, you ſhall perceaue that it wyll bee the ſame that appereth of the other woorkyng before. So that bothe ſortes are to one intente: but as the other is more ſhorter, ſo this is playner to reaſon, for ſuche as haue hadde ſmall exerciſe in this arte.

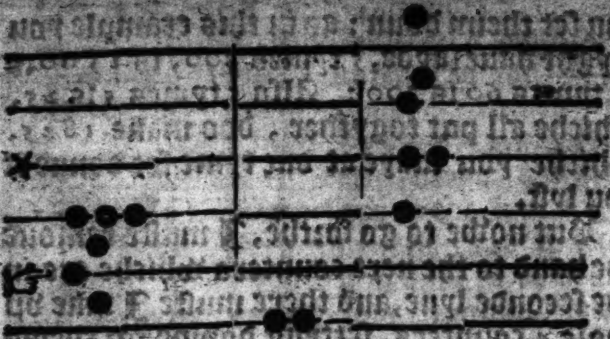
Notwithſtanding you may adde them in your mind before you ſet theim doun: as in this example you myght haue ſayde, 5 tymes 300, is 1500. & 5 tymes 60 is 300: Alſo 5 tymes 5 is 25, whiche all put togyther, doo make 1825, whiche you maye at one tyme ſet doune if you lyſt.

But nowe to go forthe, I muſte remoue the hand to the next counters whiche are in the ſeconde lyne, and there muſte I take vp thoſe 4 counters, ſettyng downe for theym my multiplier 4 tymes: which thyng other I maye doo at 4 tymes ſeuerallye, or els I may gather that whole ſumme in my mind fyrſte, and then ſet it downe: as to ſaye, 4 tymes 300 is 1200: 4 tymes 60 are 240: and 4 tymes 5 make 20, that is in all, 1460, that ſhall I ſette downe alſo, as here you ſee.

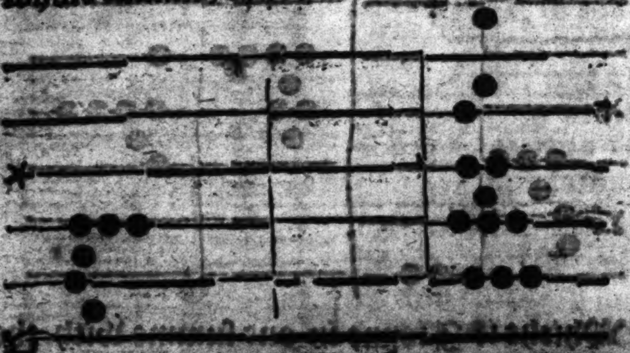


Whiche if I ioyne in one ſumme with the former numbres, it will appeare thus.

Then



Then to end this multiplication, I remoue the fynger to the loweſt lyne, where at onlye 2, them do I take vp, and in their ſtede do I ſet downe twiſe 365, that is 730, for which I ſet one in the ſpace aboue the third lyne for 500, and 2 more in the thyrde lyne with that one that is ther all ready, and the reſt in their ordre, and ſo haue I ended the whole ſumme, thus.



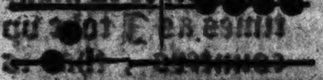
Whereby you ſee, that 1542 (whiche is the numbre of yeares ſythe Chriſtes incarnation) beyng multiplyed by 365 (whiche is the numbre of dayes in one yeare) dooth amount vnto 562830. which declareth the numbre of daies ſyth Chriſtes incarnation vnto the ende of 1542 yeares, (beſide 385 dayes and 12 houres for Leape yeares.)

The ſum of the daies ſyth Chriſtes incarnation.

Sch. Nowe wyll I proue by an other exaple, as this: 40 labourers (after 6 d the day for eche man) haue wrought 28 daies: I wolde knowe what their wages dothe amounte vnto.

A queſtio of wagis.

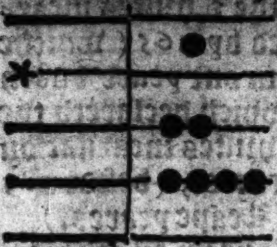
In this caſe muſte I worke doublelye: fyrſte I muſte multiplye the numbre of the labourers by the wages of a manne for one day, ſo will the charge of one daye amount. Then ſecondarily ſhall I multiplye ŷ charge of one daye by the whole noumbre of dayes, and ſo wyll the whole ſumme appeare: fyrſt therfore I ſhall ſette the ſummes thus.



Where in the fyrſte place is the multiplier (that is, one days wages for one man) and in the ſecond ſpace is ſet the numbre of the workmen to be multiplyed. Than ſaye I, 6 tymes 4 (reckenyng that

ſecond

ſeconde lyne as the lyne of unities) maketh 24, for whyche ſumme I ſhuld ſet 2 coūters in the thyrde lyne, and 4 in the ſeconde, therfore do I ſette 2 in the thyrd lyne, and let the 4 ſtande ſtill in the ſeconde lyne, thus.



The ſum
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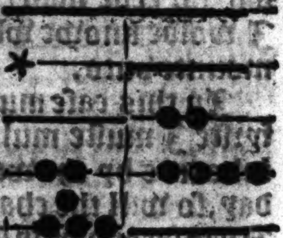
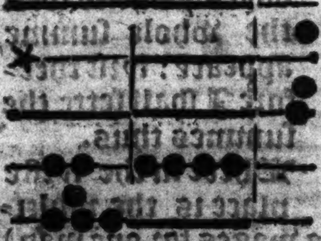


So apeareth ẏ whole dayes wages to bee 240 d, that is 20 s.

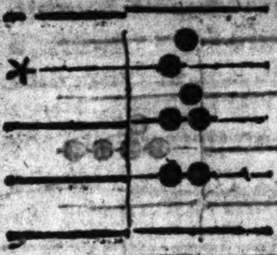
A quart
wages.

Then doo I multiplye agayne the ſame ſumme by the numbre of daies, and fyrſte I ſette the numbres thus.



Then bycauſe there ar coūters in diuers lines, I ſhall begyn with the higheſt, & take theym vp, ſettyng for theym the multiplyer ſo many times as I toke vp counters, that is twiſe. then will the ſumme ſtand thus.

Then come I to the ſeconde lyne, & take vppe thoſe 4 counters, ſettyng for theym the



the multiplyer fower tymes, ſo wylle the whole ſumme appeare thus.

So is the whole wages of 40 workmen for 28 daies (after 6 d eche daye for a man) 6720 d, that is 560 s, or 28 pounde.

Ma. Nowe if you woulde proue Multiplication, the ſureſt waye is by Diuiſion: therefore wyll I ouerpaſſe it tyll I haue taught you the arte of Diuiſion, which you ſhall worke thus.

DIVISION.

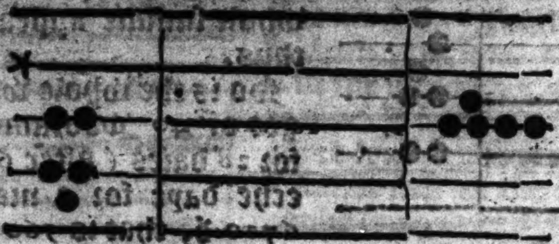
FIrſt ſet doune the Diuiſour, for feare of forgetting, and than ſet the nūbre that ſhalbe diuided at the righte ſyde, ſo farre from the diuiſour, that the quotient maye be ſette betwene theim, as for example.

If 225 ſheepe coſte 45 li, what did euery ſheepe coſte? To knowe thys, I ſhoulde dyuide the whole ſumme, that is 45 li by 225, but that can not be, therfore muſt I firſt reduce that 45 li. into a leſſer denominacion, as into ſhillings: then I multiply 45 by 20, & it is 900: that ſumme ſhall I diuide by the numbre of ſheepe, whiche is 225. theſe two

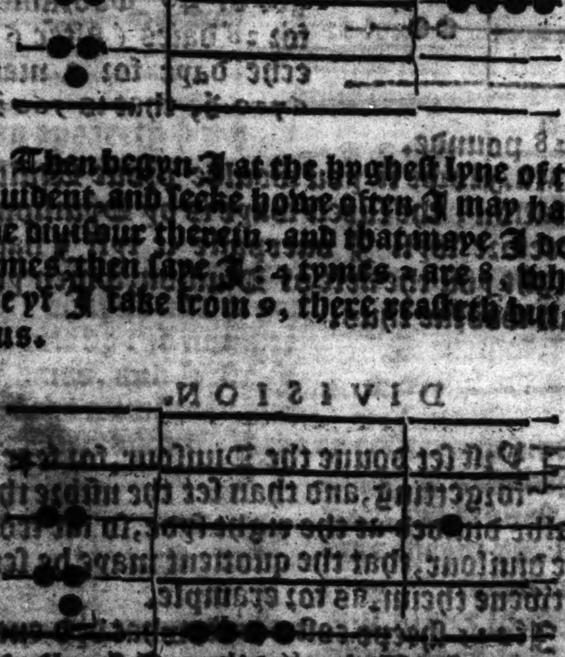
Example of ſheepe

nūbre

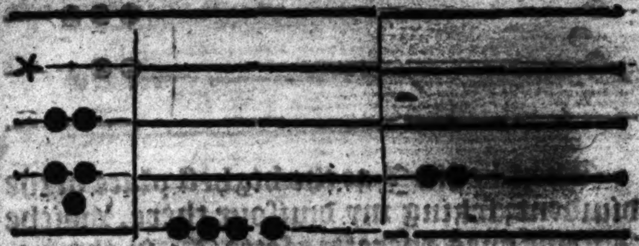
numbres therfore I ſet thus.



Then begyn I at the hygheſt lyne of the dyuident, and ſeeke howe often I may haue the diuiſour therein, and that maye I do 4 tymes, then ſaye I: 4 tymes 2 are 8, whyche yᵗ I take from 9, there reſteth but 1, thus.



And bycauſe I founde the diuiſour 4 tymes in the dyuident, I haue ſette as you ſe, 4 in the middle rowe, which is the place of the quotient: but nowe muſt I take the reſte of the diuiſour as often out of the remaynder: therfore come I to the ſeconde lyne of the dyuiſour, ſaynge: 2 foure tymes make 8, take 8 from 10, and there reſteth 2, thus.



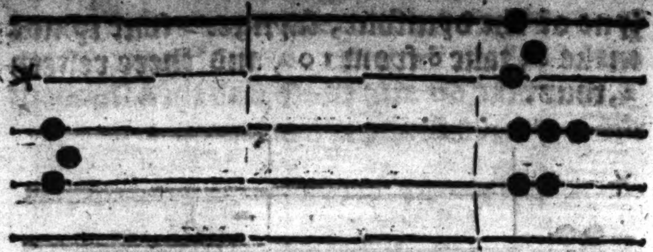
Then come I to the loweſt numbre, whiche is 5, and multiplye it 4 tymes, ſo is it 20, that take I from 20, and there remayneth nothyng, ſo that I ſee my quotient to be 4, whiche are in valew ſhyllinges, for ſo was the diuident: and therby I know, that if 225 ſheepe dyd coſte 45 li, euerye ſheepe coſte 4 s̄.

Scholar. This can I do as you ſhall perceaue by this example. If 160 ſouldiors do ſpende euerye moneth 68 li, what ſpendeth eche man?

Example of ſoldiors wages.

Fyrſt bycauſe I can not dyuide the 68, by 160, therfore I wyll tourne the pounds into pennies by multiplication, ſo ſhal ther be 16320 d.

Nowe muſt I dyuide this ſumme by the numbre of ſouldiors, therfore I ſet theym in ordre thus.

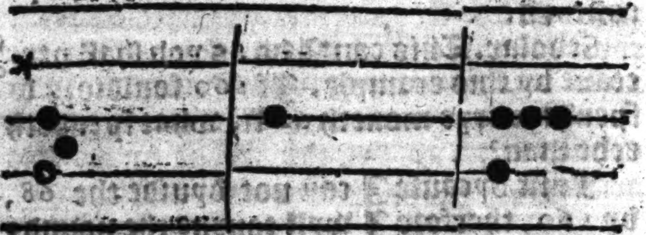


Then begin I at the higheſt place of the diuident, ſeking my diuiſour there, whiche I fynde ones, therefore ſette I 1 in the nether lyne.

Ma. Not in the nether lyne of the whole ſumme, but in the nether line of that worke whiche is the thyrde lyne.

Scholar. So ſtandeth it with reaſon.

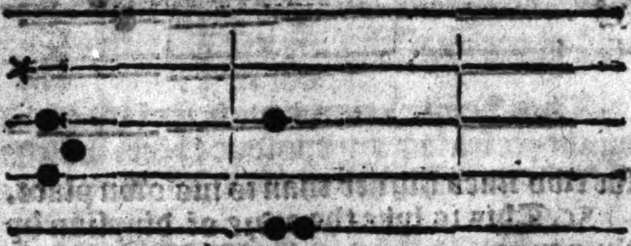
Maſter. Then thus do they ſtande.



Then ſeeke I againe the reſt, how often I may fynde my diuiſour; and I ſee that in the 300 I myght fynde 100 thre times: but than the 60 wil not be ſo often found in 10, therfore I take 2 for my quotiẽt: then take

I

I 100 twyſe from 300, & there reſteth 100; out of which with the 20 (that maketh 120 I may take 60 alſo twiſe, and then ſtandeth the numbres thus.



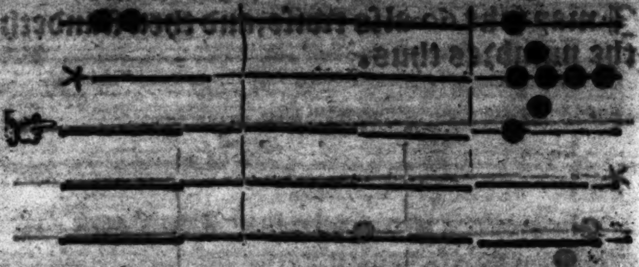
Where I haue ſette the quotient 2 in the loweſt lyne: So is euerye ſowldiors portion 102 d: that is 8 s, 6 d.

Ma. But yet bicauſe you ſhall iuſtly perceaue the reaſon of Diuiſion, it ſhalbe good that you doo ſet your diuiſour ſtyll agaynſt thoſe numbres from whiche you do take it, as by this example I wyll declare.

If the purchaſe of 100 acres of ground dydde coſte 290 pounde, what dydde one acre coſt?

Example of pourchaſe.

Fyrſte wyll I tourne the poundes into pennies, ſo wyll there be 69600 pence. Then in ſettynge downe theſe numbres, I ſhall doo thus. Fyrſte ſette the dyuident on the right hande as it oughte, and then the diuiſor on the left hand againſt thoſe nūbres from which I intend to take him firſt, as here you ſee, where I haue ſet the dyuiſ

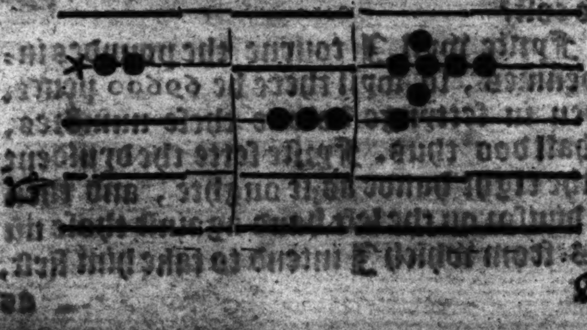


for two lines higher than is his own place.

Sc. This is lyke the oꝛdꝛe of diuyſion by the penne.

M. Truthe you ſay, and nowe muſt I ſet the quotient of this worke in the third lyne, foꝛ that is the lyne of vnities in reſpecte to the diuiſour in this worke.

Then I ſeeke how often the diuiſoꝛ may be founde in the diuidence, and that I fynde 3 tymes, thē ſet I 3 in the third line foꝛ the quotient, and take awaye that 60000 from the diuident, & farther I do ſet the diuiſour one lyne lower, as you ſee heere.



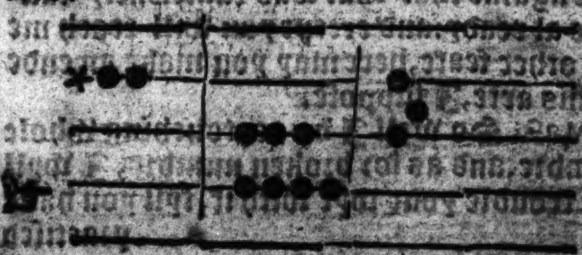
And then ſeeke I howe often the diuiſor will be taken from the number agaynſt it, which will be 4 tymes, and 1 remaynyng.

S. But what if it chaunce that when the diuiſour is ſo remoued, it can not bee ones taken out of the deuident againſt it?

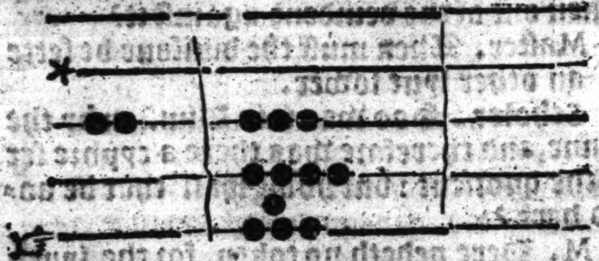
Maſter. Then muſt the diuiſour be ſette in an other lyne lower.

Scholer. So was it in Diuiſion by the penne, and therefore was there a cyphre ſet in the quotient: but howe ſhall that be noted here?

M. Here nedeth no token, for the lynes do repreſent the places: onelye looke that you ſet your quotient in that place, which ſtandeth for vnities in reſpect of the diuiſor. But nowe to returne to the example, I find the diuiſor 4 times in the diuident, and 1 remaining, for 4 times 2 make 8, which I take from 9, and there reſteth 1, as this figure followynge ſheweth: and in the middle ſpace for the quotient I ſet 4 in the ſecond line, which is in this worke the place of vnites.



Then remoue I the diuiſour to the next lower lyne, and ſeeke how often I may haue it in the diuident, whiche I may doo here & tymes fiue, and nothinge remayne, as in this fourme.



Where you may ſee, that the hole quotient is 348 d, that is 29 s, wherby I know that ſo muche coſte the purchaſe of one acre,

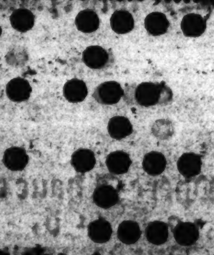
S. Now reſteth the proues of Multiplication, and alſo of Diuiſion.

M. Their beſt proues are eche one by the other, for Multiplication is proued by Diuiſion, and Diuiſion by Multiplication, as in the worke by the penne you learned.

S. If that be all, you ſhal not nede to repete againe that that was ſufficiently taughte al ready: and except you will teach me any other feate, here may you make an ende of this arte, I ſuppoſe.

Maſt. So wyll I do as touching whole numbre, and as for broken numbre, I wyll not trouble your wyt with it, tyll you haue practiſed this ſo well, that you bee full perfecte, ſo that you nede not to doubte in anye pointe that I haue taught you, & than may I boldely inſtruct you in the arte of Fractions or Broken numbre: wherin I will alſo ſhewe you the reaſons of all that you haue nowe learned. But yet before I make an ende, I will ſhewe you the order of commõ caſting, wherin are both pennies, ſhillinges, and poundes, proceding by no grounded reaſon, but onely by a receaued forme, and that diuerſely of diuers men: for Marchauntes vſe one fourme, and Auditours an other.

MARCHAVNTES VSE.



BUt fyrſt for marchaũtes forme, marke this example here, in whiche I haue expreſſed thys ſumme 193 li, 19 ſ, 11 d. Soo that you maye ſee, that the loweſt line ſerueth for pennies, ỹ next aboue for ſhillinges, the thirde for poundes, and the fourthe for ſcores of poundes.

And farther you maye ſee that the ſpace betwene pens & ſhyllinges may receaue but one coũter (as al other ſpaces likewaies do) and that one ſtandeth in that place for 6 d.

Lykewaies betwene the ſhillinges and the poundes, one counter ſtandeth for 10 ſ.

And betwene the poundes and 10 li, one counter ſtandeth for 10 poundes.

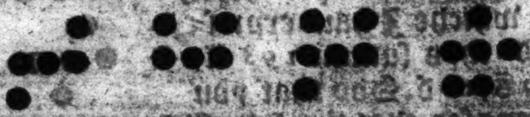
But beſide thoſe you may ſee at the lefte ſide of ſhillinges, that one counter ſtandeth alone, and betokeneth 5 ſ.

So againſt the poūdes, that one counter ſtandeth for 5 li. And againſt the 10 poūdes, the one counter ſtandeth for 5 ſcore poundes, that is 100 li. ſo that euery ſide counter is 5 times ſo muche as one of them againſt whiche he ſtandeth.

AUDITOURS ACCOMPT.

Auditours accompt.

Now for the accompt of Auditors take this example.



Where I haue expreſſed the ſame ſumme 198 li, 19 ſ, 11 d.

But here you ſee the pennies ſtande towardes the right hand, and the other encreaſing orderly towardes the lefte hand.

Agayne you maye ſee that Auditours wil make a croſſe (yea and more) for pennies, ſhillinges, and all other valewes, yf

they

theyr ſummes extende therto. Also you ſe, that they ſet one counter at the righte ende of eche rowe, which ſo ſet there ſtandeth for 5 of that rowme: and on the lefte corner of the rowe it ſtandeth for 10 of ye ſame rowe.

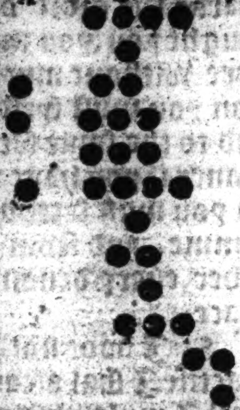
But now if you would adde other ſubtracte after anye of both thoſe ſortes, if you marke the ordre of the other feate which I taught you, you maye eaſily doo the ſame here without much teachyng: for in Addition you muſt fyrſt ſet downe one ſumme, and to the ſame ſet the other orderly, & lyke maner if you haue many: but in Subtraction you muſt ſette downe fyrſte the greateſt ſumme, and from it muſte you abate that other, euery denomination from hys dewe place.

S. I done not doubt but with a lytell practiſe I ſhal attaine theſe both: but howe ſhall I multiply and diuide after theſe formes?

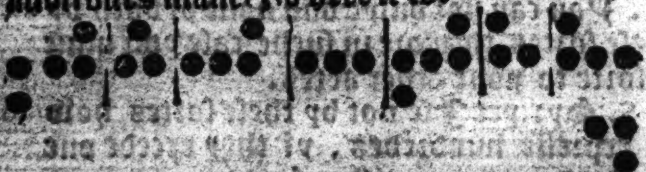
M. You can not dulye doo none of both by theſe ſortes, therfore in ſuche caſe you muſt reſorte to your other artes.

S. Syr, yet I ſe not by theſe ſortes how to expreſſe hundredes, yf they excede one hundred, nother yet thouſandes.

M. They that vſe ſuche acomptes that it excede 100 in one ſumme, they ſette noo at the left hande of the ſcores of poundes, but they ſet al the hundreds in an other for-farther rowe, and 500 at the left hand thereof, and the thouſandes they ſet in a farther rowe yet, and at the left ſyde therof they ſet the 5000, and in the ſpace ouer they ſet the 10000, and in a higher rowe 20000, whiche al I haue expreſſed in thys example, whiche is 978,69 li 12 s, 9 d. ob. q. for I hadde not tolde you before, where nother howe you ſhould ſet downe farthinges, whych (as you ſee here) muſte be ſet in a voyde ſpace ſydelynge beneth the pennies: for q one counter, for ob, 2 counters, for ob q. 3 counters, and more there canne not be, for 4 farthinges do make 1 d, whiche muſte be in his dewe place.



And if you deſire the ſame ſumme after auditours maner, ſo here it is.



But in this thing you ſhal take this for ſufficient, and the reſt you ſhall obſerue as you maie ſe by the working of eche ſort: for the diuers wittes of men haue inuented diuers and ſondry waies, almoſt innumerable.

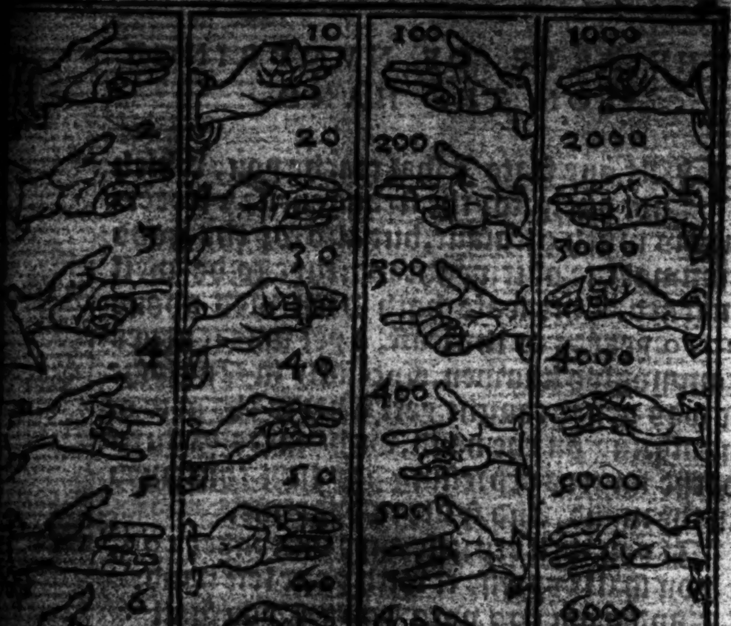
The

THE ARTE OF NVMBRINGE ON THE HAND.

BUt one feate I ſhal teache you, which not onely for the ſtrangeneſſe & ſecretnes is much pleaſant, but alſo for the good commoditie of it, ryght worthye to be well marked. This feate hath been uſed aboue 2000 yeares at the leaſt, & yet was it neuer commonlye knowen, eſpecially in Englyſhe it was neuer taught yet. This is the arte of numbring on the hand, with diuers geſturs of the fingers expreſſing any ſum conceiued in the mynd. And fyrſte to begynne.

If you will expreſſe anye ſum vnder 100,
you ſhall expreſſe it with your left hand:
and from 100 vnto 10000, you ſhall
expreſſe it with your ryght hand,
as here orderlye by thys
Table folowinge
you may per-
ceaue.

Here foloweth the table of the
arte of the hande.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900
9000

1 In whiche (as you maye ſee) 1 is expreſſed 1
by the lytle fynger of the left hand, cloſelye
and hard crooked.

2 is declared by lyke bowinge of the weddyng 2
fynger (which is the nexte to the lytle
fynger) togyther with the lytle fynger.

3 is ſignified by the myddle fynger bowed 3
in like manner with thoſe other two.

4 is declared by the bowinge of the myddle 4
fynger, and the ryng fynger or weddyng
fynger, with the other all ſtretched forth.

5 is repreſented by the myddle fynger onely 5
bowed.

And 6 by the wedding fynger onely crooked: 6
and thys you may marke in theſe a certayne
order. But nowe 7, 8, and 9, are expreſſed
with the boowyng of the ſame fyngers,
as are 1, 2, and 3, but after an other
fourme.

For 7 is declared by the boowyng of the 7
lytle fynger, as is 1, ſaue that for 1 the fynger
is claſped in, harde and rounde, but for
to expreſſe 7, you ſhall boowe the myddle
ioynt of the lytle fynger only, and holde
the other ioyntes ſtreight.

Scho. If you wyll giue me leaue to expreſſe
it after my rude manner, thus I underſtand
your meanyng, that 1 is expreſſed by crookinge
in the lytle fynger, lyke the head of a
biſhops bacle: and 7 is declared by the ſame
fynger bowed lyke a gybbet.

Maſt. So I perceaue, you vnderſtand it.

8 Then to expreſſe 8, you ſhall bowe after the ſame manner both the lyttle fyngar and the ryng fynger.

9 And if you bowe likewyſe with theim the myddle fynger, than dothe it betoken 9.

10 Now to expreſſe 10, you ſhall bowe youre forefynger rounde, and ſet the ende of it on the higheſt ioynt of the thombe.

20 And for to expreſſe 20, you muſt ſette your fyngers ſtraight, & the ende of your thombe to the particion of the formoſte and myddell fynger.

30 30 is repreſented by the ioyning togyther of the headdes of the formoſte fynger and the thombe.

40 40 is declared by ſettinge of the thombe croſſewaies on the formoſte fynger.

50 50 is ſignified by ryght ſtretching forthe of the fyngers ioynctly, and applying of the thumbes ende to the partition of the mydle fynger, and the rynge fynger or weddynge fynger.

60 60 is fourmed by bendynge of the thombe crooked, and croſſynge it wyth the forefynger.

70 70 is expreſſed by the bowing of the formoſte fynger, and ſettynge the ende of the thombe betweene the 2 formoſte or higheſt ioyntes of it.

80 80 is expreſſed by ſettynge of the formoſte

moſte fynger croſſewayes on the thombe, ſo that 80 differeth thus from 40: that for 80, the forefynger is ſette croſſewaies on the thombe, and for 40 the thombe is ſet croſſe ouer the forefinger.

90 is ſignified by bendynge the forefynger, and ſetting the ende of it in the innermoſte ioynct of the thombe, that is euen at the fote of it. And thus ar all the numbres ended vnder 100. 90

Scholer. In dede theſe be all the numbres from 1 to 19, and then all the tenthes wyth in 100, but thys teached me not how to expreſſe 11, 12, 13, &c. 21, 22, 23, &c. & ſuch like. 11, 12, 13, 21 23.

Maſt. You can lyttell vnderſtand, yf you can not doo that without teachinge. What is 11? is it not 10 and 1? then expreſſe 10 as you were taught, and 1 alſo, and that is 11: and for 12 expreſſe 10 and 2: for 23 ſette 20 and 3: and ſo for 68 you muſte make 60 and there to 8: and ſo of all other ſortes.

But nowe if you would repreſent 100, other any numbre aboue it, you muſt do that with the ryght hand, after this manner.

You muſt expreſſe 100 in the ryght hand with the litle finger, ſo bowed as you did expreſſe 1 in the lefte hand. 100

And as you expreſſed 2 in the lefte hand, the ſame faſhion in the ryght hande doothe declare 200. 200

The fourme of 3 in the ryght hand ſtandeth for 300.

The fourme of 4, for 400.

Lykewayes the fourme of 5, for 500.

The forme of 6, for 600. And to be ſhorter: loke how you dyd expreſſe ſingle vnites and tenthes in the lefte hande, ſo muſte you expreſſe vnites and tenthes of hundredes, in the right hand.

Scholler. I vnderſtand you thus: that yf I woulde repreſent 900, I muſte ſo fourme the fyngers of my ryght hand to expreſſe 9, And as in my lefte hand I expreſſed 10, ſo in my ryght hand muſt I expreſſe 1000.

And ſo the fourme of euerye tenth in the lefte hande, ſerueth to expreſſe like numbre of thouſandes, ſo the fourme of 40 ſtandeth for 4000.

The fourme of 80 for 8000.

And the fourme of 90 (which is the greateſt) for 9000, and aboue that I can not expreſſe any numbre.

Maſter. No, not with one finger, how be it, with diuers fingers you maye expreſſe 9999, & all at one time, and that lacketh but 1 of 10000. So that vnder tenne thouſand you may by your fingers expreſſe any ſomme. And this ſhal ſuffice for Numeration on the fyngers. And as for Addition, Subtractiō, Multiplication, and Diuiſion (whiche yet were neuer taught by any man as farre as

I do know) I wil inſtruct you after ẏ tretiſe of Fractiōs: and now for this time fare well. and loke that you ceaſe not to practiſe that you haue learned.

S. Syr, with moſt harty mynde I thanke you, both for your good learning and alſo your good counſell, which (GOD willing) I truſte for to folowe.

FINIS.

THE SECOND PART

OF ARITHMETIKE TOVchynge Fractions, brefely ſette forthe.

Scholar.

ALBE IT I PERceyue your manifold buſineſſe dooth to occupie or rather oppreſſe you, that you can not as yet completely end that treatiſe of Fractions Arithmeticall, whiche you haue prepared, wherin not only ſundry workes of Geometry, Muſike, and Aſtronomie be largely ſet foorth, but alſo dyuers concluſions and naturall workes touchyng myxtures of metals, and compoſitions of medicines, with other ſtraunge exaumples, yet in the meane ſeaſon I can not ſtaie my earneſt deſire, but importunely craue of you ſome brefe preparation towarde the vſe of Fractions, wherby at the leaſt I may be able to vnderſtand the common workes of them, and the vulgar vſe of thoſe rules, whiche without them can not well be wrought.

Maſter. If my leyſer were as great as my wyll is good, you ſhuld not nede to uſe any importunate crauing for the attaynyng of that thinge, wherby I may be perſwaded that I ſhall any waies profit the common wealth, or healpe the honeſte ſtudyes of any good membres in the ſame: wherfore whyle myne attendance wyll permyt me to walke and talke, I am well wyllynge to helpe you as I may.

Therefore fyrſt to begyn with explicatiō of this name Fraction, what take you it to bee?

What a Fraction is.

Scholar. Mary ſyr I thynke a Fraction (as I haue herd it often named) to be a broken numbre, that is to ſaye, to be noo hole numbre, but a part of a numbre.

Maſter. A Fraction in dede is a broken numbre, and ſad conſequentlye the parte of an other numbre: but that muſt be underſtand of ſuche an other numbre, as can not be diuided into any other partes then fractions. for although I maye take the thirde part of 60, or the fourth part of it, and ſo of other partes diuerſly, yet thoſe partes bee not properly, nor ought not to be called Fractiones, becauſe they maye be expreſſed by hole numbres: for the third parte of it is 20: the fourthe part is 15: the twelfth part is 5. and ſo forth of other partes, which all be hole numbres.

Where-

Wherfore properly a Fraction expreſſeth the partes or part only of an vnitie, that is to ſay, that the numbre whiche is the whole or entier ſumme of any fraction, maye not be greater then one, & therfore it foloweth, that no one fraction alone can be ſo greate, that it ſhall make 1: as by examples I will declare, as ſoone as I haue taughte you to knowe the forme howe a fraction is expreſſed or repreſented in wrytyng.

NVMERATION.

BUt firſt to begin with the expreſſynge of a fraction, whiche is the Numeration of it, you muſt vnderſtand, that a fraction is repreſented by 2 numbres, ſet one ouer the other, and a lyne drawen betweene them, as thus, $\frac{1}{3}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{10}{17}$. whiche foure fractions you muſt pronounce thus: $\frac{1}{3}$, one third parte: $\frac{3}{4}$, three quarters: $\frac{2}{5}$, two fyft partes: $\frac{10}{17}$, ten ſeuenteene partes.

Sch. I vnderſtand the forme of theyr expreſſion and pronunciation, but their meanyng or valuation ſemeth more obſcure: yet I thynke that by the two firſt fractions I vnderſtand the valuation of the two latter fractions, and ſo conſequently of other.

Ma. Declare them than, that I may perceaue your takyng of them.

Sc. $\frac{2}{5}$ betokeneth twoo fyfte partes, that is to ſaye, if one bee diuided into 5 partes, that fraction doth expreſſe iiii. of them: ſo $\frac{7}{10}$ doth ſignifie, that if one be diuided into x. partes, I muſt take 7 of them. And this I gather of the ii. fyrſte examples. for $\frac{1}{3}$, that ys one thyrde part, doth eaſily declare, that if any one thyng be dyuided into iii. partes, I muſte take but one of them. ſo $\frac{3}{4}$, that is iii. quarters, doth declare that one beyng diuided into iiii. quarters, I muſte take for this fraction iii of thoſe quarters.

If there bee no more difficultye in their Numeration, then I pray you go forwarde to their Addition and Subtraction, and ſo to the other kindes of workes: for I underſtand that the ſame kyndes of workes be in Fractions, that be in whole numbres.

Ma. There are the ſame kynde of works in both, albeit the ordre of them is dyuers, as I wil anon declare: But yet more in Numeration before we leaue it. You muſte underſtand, that thoſe two numbres which expreſſe a fraction, haue ſeuerall names, the uppermoſt which is aboue the lyne, is called the Numerator, and thother beneth the line is called the Denominator.

umerator

enomina

S. And what is the reaſon of their dyuers names? for in mine opinion both be numerators, ſeyng both they do expreſſe the numeration of the fraction.

M. You are deceiued: for one only (which is the ouermoſt) dooth expreſſe the numeration, and the denominator doth declare the number of partes into whiche the unitie is diuided. as in this example, when I ſay: Diuide a pounde waight of golde betwene iiii. men, ſo that the fyrſt man ſhall haue $\frac{2}{15}$, the ſecond $\frac{3}{15}$ the thyrd $\frac{4}{15}$, and the fourth $\frac{6}{15}$. Nowe do you perceyue that by the denominator (whiche is one in all foure fractions) it is intended, that the pound waight ſhuld bee diuided into ſo many partes, I meane 15, and by the iiii. ſeuerall numeratours is lymited the dyuers portion that eche man ſhall haue, that is, ẏ when the whole is parted into 15, the fyrſt mā ſhal haue 2 of thoſe 15 partes: the ſecond man 3 of theym: the thirde man 4: and the fourthe man 6. And ſo may you ſe the ſeueral offices (as it were) of thoſe two numbres, I meane of the Numerator and the Denominator.

And herby you perceaue, that a man can haue no more partes of any thyng, then it was diuided into: nother yet aptly ſo many. ſo that it were vnaptly ſayd: You ſhall haue $\frac{15}{15}$: that is, xv fyftene partes of any thyng, ſeyng it were better ſaid: You ſhall haue the whole thynge.

S. So dooth it appeare reaſonablye, for the labour is vayne to diuide anye thynge and then to applye the Diuiſion to no vſe. And much leſſe reaſonable wer it to ſay $\frac{16}{15}$.

for

for if the whole bee diuided into 12 partes only, it is not poſſible to take 16 of them, that is to ſaye, more then altogither.

M. This is true, touching the proper and apte vſe of the name of a fraction: yet improperly, after a vulgare acceptation (for eaſyneſſe in worke) bothe thoſe formes be called Fractions, bycauſe they bee written lyke fractions, although they bee none in deede, for $\frac{12}{12}$, and generally all ſuche other wher the Numerator and Denominator be equal, ar not fractions, but the whole thing with all his partes. And ſo $\frac{16}{12}$ is not to bee called a fraction, but a myxte numbre of a whole numbre and a fraction, for it is as muche as $1\,\frac{4}{12}$ that is one whole one, and 4 twelue partes, as ſhall bee declared in Reduction. Therfore they do abuſe the names that call them fractions, where the Numerator is other equall or greater then the Denominator.

Sch. But is there any nedefull cauſe why they ſhoulde ſo abuſe the name?

M. There is cauſe why they ſhall ſometimes for eaſineſſe in worke write ſome nūbres after that forme lyke fractions, but they neded not to cal them fractions, but as they be whole numbres, or myxte numbres (that is whole numbres with fractions) wrytten lyke fractions.

Nowe muſte you vnderſtande, that as no

fractio

fraction properly can be greater then 1, ſo in ſmalneſſe vnder one the nature of fractions doth extende infinitely; as the nature of whole numbres is to increaſe aboue one infinitely, ſo that not onlye one maye be dyuided into infinite fractions or partes, but alſo euery fraction may be diuided into infinite fractions or partes, whiche commonly be called Fractions of fractions, and they bee expreſſed diuerſlye: As for example: $\frac{3}{4}\ \frac{2}{3}\ \frac{1}{2}$, that is three quarters of two thyrdes partes of one halfe parte. Whereby is ſignified, that if one be diuided into two halues, and the one halfe into three partes, and two of thoſe thre partes be dyuided ioyntly into four quarters, this fraction doth repreſent thre of thoſe quarters.

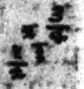
S. I praye you let me proue by an example in common money, whether I do rightly vnderſtande you or no. One crowne whiche I take for an vnitie, dooth containe 60 pennies, therfore the halfe of it is 30 pence: $\frac{2}{3}$ of that halfe is 20 pence, whereof $\frac{3}{4}$ is 15 pence, ſo than 15 pence is $\frac{3}{4}\ \frac{2}{3}\ \frac{1}{2}$ of a crowne. And ſo 3 pence is $\frac{3}{4}\ \frac{2}{3}\ \frac{1}{2}$ of a ſhillyng.

M. You perceaue this well ynough, but how happened that you found no doubte in the forme of writyng theſe fractions? ſeing the two latter fractiōs haue no line betwen their numbres as the firſt hath?

S. Bicauſe I had forgotten (as ſcholers oft tymes do) that that was told me before: but I pray you, expreſſe the reaſon therof.

M. This form is but voluntary, and therfore hath none other reaſon then the will of the diuiſor, whiche forme many do folow. Some other do make lynes betwene euery fraction, and adde wordes of diſtinction after this forte, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$. whiche forme is good alſo.

Some other expreſſe them thus in ſlope forme, to dyſtincte them from ſeuerall fractions of one whole numbre. for yf they were ſette in one right line thus, $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$, than ought it to bee pronounced, thre quarters, and two thyrde partes and an halfe. whiche maketh almoſt two whole vnites, lacking but one xij part. And ſo is it nothinge agreeable with the other Fraction of fractions. wherfore it is a greate ouerſyghte in certaine learned men, whiche do expreſſe them ſo confuſedly wyth ſuche ſeuerall fractions, that a man can not knowe the one from the other.



Therfore ſumme menne (as Stifelius) doo expreſſe, withoute a line, noumbres of proportion beeinge applyed to addition or ſubtraction, bicauſe they muſt bee taken as twoo, where the line in fractions maketh theym to bee taken for one, for of the Numeratour and Denominatour is made one numbre.

ſortes
of fra
ctions.

S. Than I perceaue there be thre ſeuerall varieties in fractions: Fyrſt whan one only fraction is ſette for one numbre, as $\frac{4}{5}$, that is four fifte partes: The ſecond, is when ther be ſet two or more ſeuerall fractions of one numbre, as $\frac{4}{9}\ \frac{2}{5}$, that is iiij nynth partes & two fyft partes: The thyrd ſorte is fractiōs of fractions, as $\frac{4}{9}\ \frac{2}{5}$, that is iiij nynthe partes of two fyfte partes.

M. You haue ſayd well, if you vnderſtand well your owne wordes.

S. If it ſhall pleaſe you I wyll by an exāple in the partes of an olde engliſh Angell expreſſe my meanyng.

M. Let me heare you.

S. The olde engliſh Angell dyd contayn vij. ſ̃, vi d, that is 90 d. Now $\frac{4}{5}$ of it is 72 d. And of the ſame 90 d, yf I take $\frac{4}{9}\ \frac{2}{5}$, that is foure nynthe partes, and two fyfte partes. $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, whiche bothe make 76. but yf I take $\frac{4}{9}\ \frac{2}{5}$, that is foure nynth partes of two fyfte partes, ſeyng $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 wyll yelde but 16, for $\frac{1}{9}$ of 36 is but 4, and that taken foure tymes, maketh 16.

M. This is plainly expreſſed & truly, & hereby (I doubt not) but you doo perceaue, that as great a difference as is betwene 16 and 76, ſo much difference is betwene theſe two fractions $\frac{4}{9}\ \frac{2}{5}$, and $\frac{4}{9}\ \frac{2}{5}$.

e ordre workes ractiōs.

And nowe that you vnderſtand theſe varieties, I wyll proceede to the reſte of the workes: fyrſt admoniſhynge you that there is an other ordre to be folowed in fractions then there was in whole numbres. for in whole numbres this was the ordre, Numeration, Addition, Subtraction, Multiplyplication, Diuiſion and Reduction. but in fractions (to folowe the ſame aptneſſe in procedyng from the eaſyeſt workes to the harder) we muſte vſe this ordre of the workes, Numeration, Multiplication, Diuiſion, Reduction, Addition, and Subtractiō.

Sch. That Multiplication and Diuiſion ſhould go togither, and Subtraction to folow Addition, natural ordre doth perſwade: but why multiplication ſhould be fyrſt in ordre here nexte to Numeratiō, & Reductiō in the myddle, I deſyre to vnderſtand yͤ reaſon.

M. As in the arte of whole numbres ordre woulde reaſonablye begyn with the eaſieſt, and ſo go forwarde by degrees to the hardeſt, euen ſo reaſon teacheth in Fractions the lyke ordre. And conſidering that Addition or Subtraction of Fractions can very ſeldome be wrought without Multiplication and Reduction: and contrary wayes Multiplication and Reduction maye bee wroughte without this forme of Addition or Subtraction, therfore was it ordrely required, that Multiplication and Reductiō ſhould go before Addition and Subtractiõ. And the ſame reaſon ſerueth for the placing of Multiplication before Reduction.

S. Then if Multiplication be the eaſieſt, I pray you declare the forme of it, fyrſt by rule, and then by example.

Maſter. Your requeſt is good.

MVLTIPLICATION.

Therefore when any two fractions bee propoſed to be multiplyed togyther, the Numerator of the one muſt be multiplied by the Numerator of the other: and the ſumme that amounteth thereof, muſt be ſet for a newe numerator: likewaies the Denominator of the one muſt be multiplyed by ỹ denominator of ỹ other, and that that amoũteth ſhalbe ſet for the common denominator, and this new thyrd Fraction expreſſeth the multiplication of the two fyrſt fractions propoſed, whereof take this example $\frac{3}{5}$ multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$.

S. I perceaue then that 3 beynge the numeratour of the firſt fraction, is multiplied by 5, beyng the numeratour of the ſeconde fraction, whereof amounteth 15, the Numerator of the thyrde fraction. And ſo likewaies 5 beyng denominator of the firſt Fraction, is multiplyed by 12 the Denominator of the ſeconde fraction, whereof amounteth 60 the new denominatour, ſo that I perceaue how the work is done. but I do not perceue how $\frac{15}{60}$ is greater then $\frac{1}{5}$. For if I ſhal vſe my former manner of examination by the partes of ſome coyne, I ſe that $\frac{3}{5}$ of a croune is 36 d, and $\frac{5}{12}$ of a croune, is 25 d, wherof the one multiplyed by the other, doth make 900 d, whiche is 15 crounes. but by your multiplication there amounteth $\frac{15}{60}$, which is but 15 d, and that is muche leſſer then any of bothe the fyrſte fractions.

M. That difference is betwene multiplication in whole numbres, and multiplication in broken numbers, that in whole numbres the ſum that amounteth is greater the bothe the other wherof it came: but in fractions it is contrary wayes: for the ſum that amounteth is leſſer then any of the other two fractions, wherof it came.

S. I deſire muche to vnderſtande the reaſon therof.

M. Althoughe I purpoſed to reſerue the reaſons of works Arithmeticall for the perfect boke of Arithmetik, yet I wil ſhew you this, bicauſe of the ſtrangnes of the worke.

You ſee in whole numbres, that of twoo numbres beeyng multiplyed togither, is made the thyrde numbre: which third numbre doth beare the ſame proportion to the noumbre multiplyed, that the multiplyer doth beare to an vnitie. And ſo in fractions, the thirde noumbre whiche amounteth of multiplication, beareth the ſame proportiō to eche of the twoo fyrſte fractions, that the other of thoſe two fractions doth beare to an vnitie.

Scholar. Syr I vnderſtand your wordes thus: Whan 40 is multiplyed by 12, there doothe amounte 480, whyche 480 doothe contayne 40 ſo mannye tymes in it, as 12 doth containe vnities: that is to ſay, twelue tymes. And ſo it appeareth that 480 doothe contayne 12 ſo mannye tymes alſo, as 40 dooth contayne vnities, that is 40 tymes. But nowe I ſee not howe the third numbre in this exaumple of fractions can contayne anye of the twoo former (as it happened in whole numbres) ſeyng it is leſſer thē eyther of them.

Ma. No meruaile if you can not ſee that thyng whiche is not poſſible to bee ſeene of anye man, howe the thyrde numbre in multiplication of fractions ſhoulde bee greater than any of the two former fractions, but yet this may you ſee (which I ſayd) that the thyrde noumbre in fractions ſo multiplied, dooth beare the ſame proportion to anye of the two former fractions, that the other of thoſe ij. fractions doth beare to an vnitie, as in your exaumple $\frac{3}{4}$ beyng multiplied by $\frac{1}{5}$, doth make $\frac{3}{20}$. Nowe ſaye I, that $\frac{3}{20}$ dothe beare the ſame proportion to $\frac{3}{4}$, that $\frac{1}{5}$

doth beare to an vnitie, as you may in your owne forme of examination by coyne try it. For in an olde Angell are 180 halfe pence, whiche I ſette for the intiere vnitie, whoſe partes accordynge to the Fractions aforeſayde, are theſe. for $\frac{15}{60}$ ſett 45 ob. for $\frac{3}{5}$, take 108 ob. and for $\frac{5}{12}$ put 75 ob. Nowe dothe 45 beare the ſame proportion to 108, that 75 doth beare to 180. for 45 is $\frac{5}{12}$ of 108, & ſo is 75 alſo $\frac{5}{12}$ of 180.

But theſe reaſons may bee better reſerued tyll an other tyme, when the knowledge of proportions in due ordre ſhall be taught. yet in the meane ſeaſon I wyll ſhewe you how it cometh to paſſe, that in fractions the thyrde ſumme muſt nedes be leſſer then any of the other two.

Conſydre thus, that when a Fraction is propoſed as in the former example $\frac{3}{5}$, if it bee multiplyed by more then one, it wyll make more then one entiere numbre. As yf I multiplye thys $\frac{3}{5}$ by 5, that is to ſaye, yf I take it fyue tymes, it wyll make three entiere vnities. exaumple in a crowne: $\frac{3}{5}$ of it maketh 3 ſ, which yf I take fyue tymes, it will amount to 15 ſhyllinges. that is thre entiere crownes. ſo if I take the ſame $\frac{3}{5}$ but twyſe, it will yelde 6 ſ, that is one entiere crowne and $\frac{1}{5}$. Nowe if I take it but ones, it can not be more then it was before, that is 3 ſ. And yf I take it leſſe then ones, it can not be ſo muche as it was before. Then ſeeyng that a Fraction is leſſe then one, yf I multiply a Fraction by an other fraction, it foloweth, that I do take that fyrſte fraction leſſe then ones, and therefore the ſumme that amounteth muſte needes bee leſſe then the fyrſte Fraction.

Scholar. Syr, I thanke you muche for this reaſon: And I truſte, I dooe perceaue the thinge, as by exaumple of this ſame fraction $\frac{3}{4}$ I wyll expreſſe. Yf I take $\frac{3}{4}$ of a crowne ones, that is to ſaye, yf I multiplie $\frac{3}{4}$ by 1, it wyll be as it was before, but 3 s. So yf I dooe multiplye it by $\frac{1}{2}$, that ys, yf I take it but halfe one tyme, than wyll yt be but halfe ſo much: lykewayes, yf I multiplye it by $\frac{1}{3}$, that is, if I take it but the thyrd part of ones, yt wyll yelde but 12 pence, that ys the thirde parte of the fyrſte Fraction. And ſo to make an ende. Yf I take it but the twelfthe parte of ones, that is, yf I dooe multiplye it by $\frac{1}{12}$, it yt wyl yelde but the twelfthe parte of the fyrſt fraction, whyche is but 3 pence. And ſo it foloweth, that yf $\frac{1}{12}$ make 3 pence, than $\frac{5}{12}$ muſte needes make fyue tymes ſo muche, that is 15 pence, which was the ſumme that hath geuen the occaſion of all this doubte.

Maſter. Thenne I perceaue youe haue ſufficient vnderſtandynge in this ſorte of Multiplication for this tyme, wherefore

I

I wyll omytte that I myght ſaye more of Multiplycation, tyll we come to Reduction, and wyll paſſe to the other woorkes, and fyrſte to Dyuyſion, whoſe place foloweth Multyplycation, bothe by naturall order, and alſo in eaſyneſſe of woorke.

DIVISION.

WHen ſo euer two Fractions bee proponed, that the one ſhoulde bee diuyded by the other, I muſte ſet downe fyrſte the fraction that ſhall bee dyuyded (whyche is called the Dyuydende) and thenne after yt the other, whych is the Dyuiſour. Then ſhall I multyply the numeratour of the dyuidend by the denominatour of the dyuiſor, and that whych amounteth, I muſt put for a newe numerator. Agayne I ſhall multiply the denominatour of the diuidende by the numeratour of the diuiſour, and the number that amounteth thereof I muſt put for the newe denominatour. And this thyrde fraction is the quotient of the ſayd diuiſion.

Scholer. This ſeemeth eaſye in forme, as by example, thus: If I woulde dyuyde $\frac{5}{8}$ by $\frac{3}{4}$, fyrſt I muſte multiply 5 (being the numeratour of the diuidend by 4, which is the denominatour of the diuiſoure, and thereof riſeth 20: then I multiply 8 (beyng the denominatour of the diuidende) by 2, beynge numeratour in the diuiſor, and ſo ryſeth 16, the whiche I muſte make in a thyrde fraction thus [illegible].

Maſter. Me ſeemeth you are quycker in vnderſtandyng now, thenne you were when I taught you the arte of hole numbres: but that is no merueile, for the more knowledge that any man getteth, the redyer ſhall hee fynde his wytte, and quicker in vnderſtanding, but yet of ii. thinges I will admoniſhe you, whiche you myght haue obſerued here for eaſe of worke and lightneſſe of vnderſtandinge the nature of the Quotient.

When ſoeuer you do diuide one fraction by an other, other they bee bothe equall togyther, other elles the one is greater thenne the other: yf they bee equalle, theyr Quotient ſhall bee ſuche, that the Numeratoure and the denominatour of it ſhall be equal alſo. And if the two firſt fractions be vnequal, their quotient ſhall declare the ſame, by the vnequalitie of the numerator and denominator, as in theſe examples followinge ſhall appeare.

Fyrſte of equall fractions: [illegible] and [illegible] be equal togither: and if the one be diuided by the other, the quotient wyll bee $\frac{108}{108}$, as you maye perceaue by that rule aforeſaide.

Now in vnequall fractions, as [illegible] and [illegible], the quotient will be [illegible]: where the numerator

so

is greater then the denominatoꝛ.

S. I ſee it is ſo, but I ſee not the reaſon why it ſhoulde be ſo.

M. The reaſon is this. When any fractiō is dyuided by an other, the quotient declareth what proportion the diuidend beareth to the diuiſor. So $\frac{1}{2}$ dyuided by $\frac{1}{4}$ maketh $\frac{2}{1}$, which muſt bee counted, not twoo, but twyſe; declaryng that $\frac{1}{4}$ is contayned twiſe in $\frac{1}{2}$.

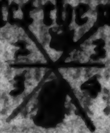
Note. And note this, that the Numerator in the Quotient repreſenteth the Diuidende, and the denominator repreſenteth the diuiſour. And this is alwaies true, whether the greater fraction be dyuided by the leſſer, or the leſſer by the greater. But this proportion will not bee exactly knowen, tyll you haue learned the art of Proportions: notwithſtandinge ſomewhat of it will I declare in the next rule of Reduction. But now for the eaſy remembraunce of the quotient in diuiſiō, as ſone as you haue ſet downe your ij. fractions, the one againſt the other, then make a lyne for the quotient: and as ſoone as you haue multiplyed the numeratoꝛ of the diuidende, by the denominatoꝛ of the dyuiſour, ſet the numbꝛe that amounteth ouer the ſaid lyne, and then multiply the other two numbꝛes, and ſet their totall vnder the ſame line.

S. I perceyue you wold not haue me truſt to memory till I wer better expert, leaſt of-

tentymes

ſentymes I happen by myſle remembrance to be abuſed. This example I take for that declaration:

Yf I woulde diuide $\frac{2}{3}$ by $\frac{3}{4}$, I muſt ſet the nūbres one agaynſt the other, (as here doth appeare) & then make an other line for the quotient in ſom good diſtaunce, where I maye ſet the numbres of the quotient, as ſone as anye of them is multiplied: Soo then as ſone as I haue multiplied 2 by 4, which maketh 8, I ſhall ſet that 8 ouer that lyne, thus, $\frac{8}{\ }$, and then multiplye 3 by 3, whiche yeldeth 9: & that 9 muſt I ſet vnder the ſame lyne, and then wyll the whole quotient appeare thus, $\frac{8}{9}$. Wherby appereth (as I remembre youre wordes) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$, as 8 is to 9. but howe may I perceaue that?



M. Although you ſhal better perceaue it by the ryle of Reduction, yet this example may be declared in common coyne, as in a common ſhyllyng of xii. pens, of whiche $\frac{2}{3}$ maketh 8 d, and $\frac{3}{4}$ doth make 9 pens, and ſo you may eaſyly ſee that their propoſitions do agree. And if you had taken this example before, when you tooke the example of $\frac{4}{5}$ and $\frac{2}{3}$, your quotient wold appere (as thys doth) more eaſyer to vnderſtande, where as that quotient being $\frac{10}{12}$, is not an eaſy proportion for you to perceaue, beinge yet lytle acquainted with proportions: whereof to gyue you ſome taſte, I wyl enter to the rule of Reduction, in which alſo I wyll declare other workes bothe of Multiplycation, and alſo of Diuiſion, whiche now I muſt for a time omit, as thinges that do need the helpe of Reduction.

REDVCTION.

varieties f Reduction.

THerfore wil I nowe declare the diuerſities of Reduction of Fractions, whiche commonly hath v. varities.

1 Fyrſt, whe ther be ſundry Fractions of one inteer vnitie, thei muſt be reduced to one denomination, & alſo into one fraction.

2 Secondaryly, when there be proponed Fractions of fractions, they muſt be reduced lykewiſe into one fractiõ, for other waies they can not be broughte into one denomination.

3 Thyrdly, when an Impropre fraction is proponed, that is to ſay, a Fraction in forme which in dede is greater then an vnitie, it muſt be reduced into apt forme, expreſſynge the vnitie or vnities of it, and the propre Fraction diſtinctly. And ſome times alſo it ſhall be nedefull to conuert ſuch a myxt nũbre of vnities with fractions into the forme of a fraction, that is, into an Impropre fraction, whych 2 formes I eſteme but as one, bycauſe they worke on one kynde of numbre.

bre.

4 Forthly, there happeneth ſomtymes fractions to be wrytten in great numbres, whiche might be written in leſſer numbres, therfore is there a meane to reduce ſuche greate numbres into theyr ſmalleſt tearmes.

5 Fyftly, whan any fraction betokeneth the partes of a whole thynge, which hath by cōmon partition certayne partes, but none of lyke denomination with that fraction, then may you reduce the ſayde fraction into an other, whoſe denomination ſhall expreſſe the common partes of that whole thynge.

Sco. This diſtinction in doctrine delyteth me much, but more with hope then preſente fruite. for as yet I do not vnderſtand ſcarſelye the varieties, and much leſſe the practiſe and vſe of their workes.

Ma. Reduction is an orderly alteration of numbres oute of one forme into an other. which is neuer doone orderly, but for ſome nedefull vſe, as in euerye of the ſaid ſeuerall varieties I wyll diſtinctly declare.

The ſorte o ductio

Fyrſt therefore, when two or more ſeuerall fractions of any vnitie bee proponed, as for example, $\frac{6}{3}$ and $\frac{4}{7}$: bycauſe it is hard to tell what portion of the intiere numbre theſe two fractions doo expreſſe, therefore was Reduction deuiſed to be a meane wherby theſe ſeueral fractions might be brought into one denomination and fraction.

And in theſe fractions this is the arte for bryngyng them to one denomination.

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minatiõ.

Multiply fyrſt the denominatours togither, and the totall thereof you ſhall ſette twyſe downe vnder two ſeuerall lynes for two newe denominatours, or rather for one cõmon denominatour: Then multiply the numeratour of the fyrſte fraction, by the denominatour of the ſecond, and ſet the totall therof for the numerator ouer the fyrſt lyne. Lykewyſe multiplye the numerator of the ſecond fraction by the denominator of the fyrſte, and ſet that totall ouer the ſeconde lyne for the numeratour of that fraction, & ſo are thoſe two fyrſte fractions of ſeuerall denominatiõs brougt to one denomination.

Scho. If I vnderſtand you, as I thinke I doo, my example ſhall declare the ſame. The fractions whiche you proponed were theſe $\frac{3}{16}$. & $\frac{4}{6}$, whoſe denominatours (being 16 and 6) I multiplye together, and there amounteth 96, whych I ſet vnder ii. lines thus, $\frac{}{96}$ $\frac{}{96}$. Then I multiplye the numerator of the fyrſt Fraction by the denominatour of the ſecond, ſayinge: 3 into 6, maketh 18: that ſet I ouer the fyrſt line for a newe numeratour, and it wyl be thus. $\frac{18}{96}$ Lykewayes I multiply the numeratour of the ſecond Fraction by the denominatour of the fyrſt, ſaying: 4 tymes 16 maketh 64, that I ſet for the ſecond numerafor, and the fraction wyll appeare thus: $\frac{64}{96}$ ſo that both Fractions brought to one domination, muſt ſtand thus $\frac{18}{96}$ and $\frac{64}{96}$.

Maiſter. You haue done well.

Sco. I beſeeke you, lette me examyne it after my accuſtomed fourme, by common partes of coyne.

Maiſter. Go to.

5. A newe Angel accompted at eight ſhillinges, containeth 96 pens, whereof $\frac{1}{16}$ that is the xvj. part, is vj pence, and $\frac{3}{16}$ is 18 pence, that is $\frac{18}{96}$. Agayne $\frac{1}{6}$ of the ſame Angel, is 16 pens, ſo that $\frac{4}{6}$ maketh 64 d, that is $\frac{64}{96}$. And ſo I fynde the ſummes to agree with thother before.

M. So haue you nowe the arte to bringe ſuche ij Fractions into one denominatyon. And if there be more then ij, then muſt you multiply all the denominators togyther, & ſet the total therof ſo many times downe as ther be fractions, & then to get for eche one a newe numeratour. Multiply the numerator of the fyrſt, by the denominator of the ſecond, and the totall therof multiplye by the denominator of the third, and ſo forth if ther be more. Lykewyſe multiply the numerator of the ſecond, by yͤ denominator of the fyrſt, and the total therof by the denominator of the thyrd. And in the ſame ſort multiply the numerator of ye third into the denominator of the fyrſt: & ye total therof into the denominator of the ſecond, & ſo forth if ther were mo. So theſe 3 fractiõs $\frac{2}{5}$ $\frac{3}{4}$ $\frac{2}{3}$ doth make by reduction theſe other 3 fractions of one denominatiõ $\frac{24}{60}$ $\frac{45}{60}$ $\frac{40}{60}$. Al which you may bryng into one fractiõ by adding the numeratours togither, & putting that total for the commõ numerator, reſeruing ſtil that ſame commõ denominator. And thoſe 3 fractiõs make one improper fraction, thus. $\frac{109}{60}$

Schaler. Al this I perceaue, and alſo ye this laſt fraction is more then an vnitie, & therfore you did cal it an Improper fractiõ.

M. Ther be certaine other formes of working in this Reductiõ, which I wyl bryefely touche alſo, to giue you an occaſion of exerciſe your wit therein.

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arietie
f this re-
uction.

The fyrſt varietie is this. Whẽ you haue made and written downe your common denominator (as I haue taught before) then to get a numerator for the firſt, do thus. Diuide the common denominator by the denominator of ye firſt fraction, and the quotient multiplied by the numerator of the ſame, yeldeth a newe numerator for ye fyrſt newe fraction: So likewiſe do with the ſecond & ye thyrd, & with all ye reſidue, if ther be more.

Sch. That wil I proue in your laſte exãple of theſe 3 fractions $\frac{2}{5}$ $\frac{3}{4}$ $\frac{2}{3}$. When the denominatours be multiplied, they make 60, for 5 into 4 maketh 20, and 20 by 3 yeldeth 60, that I ſette downe 3 tymes, thus $\frac{\ }{60}$ $\frac{\ }{60}$ $\frac{\ }{60}$, then to haue a numerator for the firſt, I muſt diuide 60 by 5, (the denominator of the fyrſt) & the quotiēt is 12, whych I muſt multyply by 2 (the numeratour of the fyrſt, and that maketh 24, and ſo haue I for the fyrſt fraction $\frac{24}{60}$.

The ſeco
varietye

Lykewiſe for the ſecond fractiō I diuide 60 by 4 & there commeth 15, which I multiply by 3, and ſo haue I 45, and the ſecond fractiō $\frac{45}{60}$. Then for the thirde in like ſort will com $\frac{40}{60}$.

Ma. An other way is this. If it happē ſo that the leſſer denominator can by any multiplication make the greater, then note the multiplier, and by it multiplye the numerator ouer that leſſer denominatoure, and for the leſſer Denomynatoure putte the greater, as thus in theſe two fractions, $\frac{5}{12}$ $\frac{2}{3}$, 3 beyng ye leſſer denominatour multyplied by 4 wyl make 12, whych is the greter denominatour: therefore by the ſame 4 I do multyply 2, whiche is numeratour ouer 3, and that maketh 8; vnder whyche I put 12 being the greter denominator, whiche is alſo made by multiplicatiō of 4 into 3, and ſoo haue I theſe two fractions $\frac{5}{12}$; $\frac{8}{12}$ thus ſhortly reduced without alteryng the one fraction.

Scholer. This I vnderſtand.

Maſter. Then marke thys thyrder waye: Yf the denominatours do not happē ſo, that one by multiplication may make the other, then loke whether they both may be partes of any other one numbre, as in $\frac{5}{12}$ and $\frac{7}{18}$, althoughe the leſſer taketh but twyſe her to great to make 18, yet they both may be partes vnto 36; therfore loke howe many times 12 is in 36, and that quotient being multyplied by the numeratour ouer 12, the totall ſhal be put in ſteed of ỷ numerator ouer 12, and for 12 put 36, thus, $\frac{15}{36}$. So lykewayes looke how often is 18 in 36, and bycauſe yt is twyſe, therefore by 2 multiply 7, whyche is ouer 18, and it wyll be 14: ſet that for the numerator, and in ſteed of 18 put 36, & then ſhal your fractions reduced ſtande thus $\frac{15}{36}$, $\frac{14}{36}$ in ſteede of $\frac{5}{12}$ and $\frac{7}{18}$.

thrd
aye.

And if you wil proue whether you haue wrought well or no, that may be proued by Reduction of them agayne to theyr former denominations, whiche arte ſhall be taught in the fourth kynd of Reduction, wher greater termes of fractiōs be reduced into ſmaller in numbre, but not ſmaller in proportiō. And if in ſuche Reduction the ſame termes or numbres come agayne that were before, than is the woorke good, els not.

Scholer. Syr I heare youre wordes, but I do not vnderſtand many of theym, which it may pleaſe you to declare.

Mayſter. With a good wyll, whan conuenient place ſerued, but that muſt be in thee ſayd iiij kynde of Reduction. In the meane ſeaſon I wyl declare the ſecond forme of Reduction, whiche teacheth how to reduce fractions of fractions into one fraction, and ſo to one Denomination.

When Fractions of fractions be proponed, you ſhall multiply the numerators of eche into other, and ſet the total for the newe numeratour, than multiplye all the denominatours lyke wayes, and take their total for the newe denominator, and ſo are they ſpedyly reduced.

Reductio
fractions
fractions
to one fra
ction and
Denomi
tion.

Sholer. If that be all, then I vnderſtand it al redy, as by this example I wil declare. Theſe bee the fractions, $\frac{3}{4}$ $\frac{2}{3}$ $\frac{6}{7}$ $\frac{7}{9}$, whiche I woulde reduce to one denomination. Therefore begyn I with the numeratours, and multiply them altogyther, ſaying: 3 into 2 maketh 6, and 6 by 6 maketh 36, whyche multiplied by 7 yeldeth 252, that I ſet ouer a lyne for the numeratour, thus: $\frac{253}{}$

Then I multiplie the denominatours 4 by 3, maketh 12, and that by 7 bringeth 84, whiche multyplyed by 9, yeldeth 756, the newe denominatour.

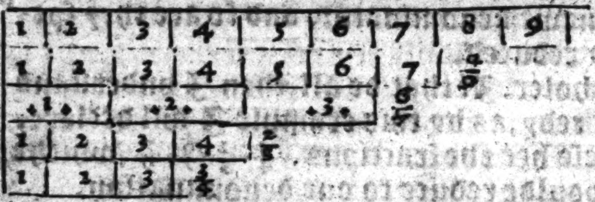
And ſo the whole reduced fraction is this $\frac{252}{756}$

Which is to hard a fraction for me to vnderſtande yet.

Maſter. You thinke ſo, and no meruaile, but anon you ſhall learne to iudge it eaſily. for this fractiõ is no more in dede then $\frac{1}{3}$, although it be in greater termes, and therfore more ſtraunger and more obſcure.

And this ſuffiſeth for this Reduction, ſaue that I wyll ſhewe you by a figure of meſure the iuſt rate and reaſon of thys kynde of fractions, and alſo the dewe vnderſtanding of their Reduction.

The intiere meaſure parted into 9.



Here you ſee the longeſt meaſure, whyche ſtandeth for the hole and intiere quantitie fyrſt parted into 9 diuiſions, whereof 7 are ſeuered by the ſecond meaſure, and thereof agayne are parted out 6. And that 6 beyng diſtincte into 3 partes, 2 of theim are parted by the fourth meaſure, of which fourthe meaſure beyng diuided into 4 partes, the loweſt meaſure doothe containe $\frac{3}{4}$, ſo that the ſame $\frac{3}{4}$ muſte bee truely named, not $\frac{3}{4}$ of the hole meaſure, but in deede is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$; or as I would rather expreſſe it, $\frac{3}{4}$ $\frac{2}{3}$ $\frac{6}{7}$ $\frac{7}{9}$.

Scholer. This example is ſo ſenſible, that I can not chooſe but ſee it. And farthermoꝛe I ſee alſo, that the ſame fraction is equall to $\frac{3}{9}$ of the intire meaſure, as yͤ lynes whyche runne vp and downe doo expꝛeſſely ſet foꝛthe. Alſo I ſee here, yͭ $\frac{2}{3}$ $\frac{6}{7}$ $\frac{7}{9}$ is equal to $\frac{4}{9}$. And farther yet that $\frac{6}{7}$ $\frac{7}{9}$ is equal to $\frac{6}{9}$.

Maſt. I am glad that you ſee it ſo well, not doubtinge but you wyll gather greater lyghte of knowweledge hereby.

The thir
foꝛme of
Reductio

But nowe it is tyme that we come to yͤ thyꝛd foꝛme of Reduction, which teacheth of Impꝛopꝛe fractions, that is to ſay, Mixt numbꝛes of vnities and fractions, although they appeare like fractions, as this $\frac{26}{5}$, whiche doothe include 5 vnites wholy, and $\frac{1}{5}$ ouer. Wherfoꝛe fyꝛſt you ſhall knowe them be that the numeratoꝛ is greater than the Denominatoꝛ.

Scholer. In dede ſyꝛ that appeareth reaſonable, that if the numeratoꝛ doo expꝛeſſe moꝛe partes to be taken of any vnitie, than the denominatoꝛ doth ſignifie that vnitie to be diuided into, it muſt nede folow, yͭ ſuche a fraction impoꝛteth moꝛe than yͤ hole, yͭ is to ſaie, thee whole with certaine partes ouer. But what Reduction is there in it?

Maiſt. Ther be ij. ſeuerall kindes of Reduction, concerning ſuche fractions. Some tymes it ſhall bee needefull to conuert thoſe fractions into the vnities & the proper fraction that will remayne. And ſometimes cōtrary wayes, it ſhall bee mete to reduce mixt numbres, that is vnities written with fractions, into the forme of one ſymple Fraction, and ſo be there ij. wayes.

Sco. What is the meane of the fyrſt way to tourne Impropre fractions into vnities with theyre propre fractyons?

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M. That is thus. Youre numerator beynge greater then the denominatour, muſt be diuided by the ſame denominatour, and the quotient therof expreſſeth the vnities, ỹ remayner ſhall be put for the numerator of the Fraction that reſteth, and the denominatour muſt be the ſame that was before.

Scho. For example, I take $\frac{17}{5}$. And diuidyng 17 by 5, the quotient wyll be 3, and there wil remaine 2.

Ma. That muſt you write thus 3 $\frac{2}{5}$, wher (you ſee) I haue written 3 without any line, as intier numbres ought to be wrytten, and the 2 that remayned, I haue ſette ouer the former denominator with a line, as a propre Fraction. And thys numbre dothe ſignifye nowe 3 vnities, and $\frac{2}{5}$ of one.

Scho. Then yf I woulde by vnities here vnderſtand crownes, ſo it were 3 crownes, and $\frac{2}{5}$, that is 2 s.

Ma. Euen ſo, and therefore $\frac{17}{5}$ dyd ſignifie the ſame. But this happeneth ſomtimes that when the reduction is ſo wrought, ther remayneth nothinge. And then is it not a Myxte numbre, but a ſymple intier numbre repreſented lyke a Fraction.

Sco. As $\frac{15}{5}$ wyll make 3 iuſte, and $\frac{18}{3}$ wyll make euen 6. This I wyll remembre. But now what is the ſeconde fourme of Reduction that you ſpake of for theſe ſortes of fractions?

Maiſter. Whenſoeuer you haue anye of theſe two ſortes of numbres, that is to ſay, hole numbres with out fractions, or whole numbres with fractions, and you woulde turne them into the fourme of a Fractyon, you muſte multiplye the whole number by that denominator whych you wyll haue to remayne ſtyl, and to the total therof adde the numeratour whiche you haue all ready, and all that ſhall you ſet for the newe numeratour, kepyng ſtyll the former denominatour: as if you haue $6\frac{3}{4}$, whych you woulde conuert into an improper fraction, you muſt multiplye 6 by 4, whereof commeth 24, and thereto adde the numeratour whyche is 3, and ſo haue you 27 for the numeratour, and 4 ſtyll for the denominatour.

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Sco. Then is $\frac{27}{4}$ equall to $6\frac{3}{4}$.

Maiſter. Euen iuſte, and ſo backewarde (as appeareth by the former Reductyon) $6\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of theſe reductions may be the proofe of the others worke.

Scho. This I perceaue, but nowe if you woulde tourne whole noumbres withoute Fractions into any fraction, I ſee not how that maye be done, bycauſe there is no denominatour to make the multiplication by.

Ma. That was wel marked, but this you knowe, that no man intendeth to tourne any whole numbre into a fraction, but he hathe in his minde that denominatour by whiche the multiplication muſte be made: for the profe whereof I ſet downe 7, whiche is a whole numbre. And if you will haue thys numbre conuerted into anye certayne Fraction, wyll me to doo it.

Scholer. I praye you reduce 7 into a Fractyon.

Mayſter. Then you care not what the Fraction bee, ſoo it bee ſome fraction.

Scholer. No I paſſe not for the ſorte of the fraction.

Maiſter. Then howe canne you thynke that you require me to doo anye thyng certayne, whan you leaue mee to doe as I lyſte? And ſeynge you ſtande at that ſtaye, whether thynke you that I muſte fyrſte intende in mynde what Fractyon I wyll make of it, before I canne dooe it in dede?

Scholer. Els you ſhoulde doo ignorauntlye.

Maiſter. Then I wyll lymyt my ſelfe (ſeeyng you wyll not) to tourne it into quarters. And therefore I multyplye 7 by 4, (which is the denomination of quarters) and there amounteth 28 to be ſet for the Numeratour, and the 4 muſte be ſette for the denominatour, and the fraction wil bee thus $\frac{28}{4}$.

Scholer. In dede I perceue this to be reaſonable, for without muche trial I underſtand that $\frac{28}{4}$ of any thing, doeth make 7. And ſo then if I woulde turne 8 into fifte partes, it wyll make $\frac{40}{5}$, whyche is al one with eight. for 8 crownes tourned into fift partes, (that is into ſhillynges) will make 40 ſhillinges, that is $\frac{40}{5}$ of a crowne.

Mai. Seinge you underſtand nowe theſe three kindes of Reduction, I wil declare unto you the fourth kynd. that is when fractions be written in greter tearmes then they nede, howe they maye be broughte to leſſer termes.

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kynde of
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Scholer. To wryte anye fraction in greter tearmes than nedethe, ſemethe to be a faulte, and ſo this rule ſemethe to amende that faulte.

Mayſter. It were a faulte to doo anye thynge without nede, whych after muſt be redreſſed: but in this caſe it is not ſo, nother dyd I ſaye abſolutelye (as you doe) that it nedethe not to expreſſe ſuche fractyons in ſo great tearmes, but that the Fractions doe not neede, I meane for theyr valewe to be vnderſtande, but yet it maye be needefull for the eaſe of thoſe woorkes whereto they be applied, as for example: In the firſt kynde of reduction this was youre owne example: $\frac{5}{16}$ and $\frac{4}{6}$, whyche whan you wolude reduce, you were faine to tourne theym firſt into one denomination, and ſo appeared they thus, $\frac{30}{96}$ and $\frac{64}{96}$, where the Fractions (for theyr owne vnderſtandynge) needed not to bee tourned out of ſmaller tearmes into greater, but yet the eaſyneſſe of woorking needed it.

Scholer. Syr I vnderſtande nowe not onely the difference of this ned (for the Fractions myghte better bee vnderſtanded as Fractions ſeuerall, eche in his valewe, whē they were in leſſer Termes, although they could not ſo well be reduced) but alſo I vnderſtand what you meane by greater Tearmes & leſſer Tearmes whereof before I was in doubt, for I ſee you call the Numerator and Denominatour the tearmes of the fraction.

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action,

Maſter. I am glade you vnderſtande it ſo well. Nowe than whenne you woulde valew any Fractions, bycauſe that maye beſt bee doone whenne the Tearmes are moſte ſmalleſt, you ſhall reduce theym to ẏ ſmalleſt that you canne, whiche thynge you may

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maye doo thus: Diuide the greateſt of anye ſuche two termes by the leſſer, and if anye thynge remayne, by that remayner dyuyde the laſte diuiſour, and if any thing remayne, by that diuide the laſt diuiſour (which was before the remayner of the fyrſt diuiſion) and ſo continue ſtyll, tyll nothing do remayne in the diuiſion, and then marke your laſte diuiſour, for it is the numbre that wyll eaſily reduce your fraction, if you dyuide bothe the numerator and the denominatour by the ſame numbre, and put for the numeratour the quotient of his diuiſion, and for the denominatour alſo his quotient, that riſeth by his diuiſion.

S. I take for example $\frac{18}{96}$, and bycauſe 96 is the greater numbre, I diuide it by 18, & the quotient is 5, and there reſteth 6: what ſhall I do with this quotient?

M. Nothyng in this worke: but nowe ſeyng there remaineth ſomwhat, by that remayner muſte you diuide the laſte diuiſour.

S. Then I ſhall dyuide 18 (whiche was the laſte diuiſour) by 6 that was the remayner, ſo is the quotient 3, and nothing reſteth.

M. As for the quotient, omitte hym yet: but bycauſe there doth remayne nothyng, therfore is 6 (which was your laſte diuiſour) that numbre by whiche you may reduce the fraction proponed.

S. Then as you taught me, I muſt diuide

the numeratour 18 by 6, & the quotiente is 3, which I muſt put for the numeratour ouer a lyne, thus $\frac{3}{}$, and then by the ſayd 6, muſte I diuide alſo the denominator 96, & the quotient will be 16, whiche I muſt take for the denominator, and ſo is the fraction $\frac{3}{16}$. And ſo me thinketh this rule doth proue yͤ worke of the fyrſte reduction.

Ma. That is true, if the fyrſte reduction wer made of fractiōs in their leaſt termes, and els not without ſome help, as the ſecōd numbre in that place will declare.

S. The ſecond numbre was $\frac{4}{6}$, which was tourned into $\frac{64}{96}$, by that rule. Now if I ſhal by thys rule reduce it againe into the leaſte termes, I muſt diuide 96 by 64, and there reſteth 32, by which 32 I dyuide 64, & ther remayneth nothing: wherefore I muſt take that 32 for the diuiſour, to reduce the ſayde fraction. Then doo I dyuide 64 by 32, and the quotiente is 2, whiche I ſette for my numeratoure. Agayne I dyuide 96 by 32, and the quotient wyll bee 3, and ſo haue I but $\frac{2}{3}$.

M. Muſe not at the matter, for you haue done well inough: but you thinke you haue not the fraction that you looked for, that is $\frac{4}{6}$, yet haue you one equal to it, as by the partes of a ſhillyng you may prooue.

S. Truth it is, for eche of them will bringe forth 8 pence, ſo that $\frac{8}{12}$ and $\frac{4}{6}$ and $\frac{2}{3}$, bee all thre equall. And nowe I perceaue that bycause ¼ was not writte in the leaſt tearmes that it mighte bee, therefore this reduction brought forth not it, but that other whiche is written in the leaſt tearmes. Nowe vnderſtand I this rule well. But is there anye other way to worke this Reduction?

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Ma. Yes, but fyrſte note this, that if you finde no ſuche dyuiſor to reduce the fraction tyll you come to 1, bycauſe 1 doth make no diuiſion, therefore that fraction is alreadye in his leaſte tearmes, as by $\frac{71}{100}$ you maye proue, & ſo of $\frac{31}{32}$, and many other lyke. But now to your queſtion, yf you can withoute that diuiſion by memory eſpye the greateſt numbre that may dyuide exactly both tearmes of your fraction proponed, then needs you not to vſe that diuiſion. as in this fraction $\frac{36}{48}$, I ſee that 12 is the greateſt numbre that can dyuide them both: and therfore without any worke by memory only, I turne that into $\frac{3}{4}$, but this abilitie in knowledg is gotten by exerciſe.

An other waye.

Yet one other way of eaſye reduction by thys kinde ther is, when your fraction hath anye cyphers in the fyrſte places of bothe tearmes, then maye you by caſting away the cyphres, make a bryef reduction, as thus $\frac{100}{400}$, here take away the cyphres, and it wyll be $\frac{1}{4}$, whych is the ſame in valew with $\frac{100}{400}$.

S. And ſo if I haue $\frac{190}{270}$ it wyll be $\frac{19}{27}$.

M. You are deceyued: for you take awaye more cyphers from the numerator, thā you do take from the denominator, whiche you maye not doo.

S. I confeſſe my faulte, which came of to much haſte. I was more gladder of the rule then wiſe in vſyng it: but nowe I vnderſtād it, I truſt.

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M. Then may I go in hand with the fifte or laſte kynd of Reduction, which teacheth how to turne any fractiō proponed into any other denomination that you lyke: or into any partes of common coyne, waightes or meaſures, or ſuche lyke.

For declaration whereof, fyrſt you ſhall marke whether your fraction bee a ſimple fraction, other els a fraction of ſundry partes, I meane of more termes then two. And if your fraction be a fraction of fractions, or other wayes compound, you muſt reduce it to one ſimple fraction. And then marke well the denomination of that other fractiō into whiche you would tourne this. For by that denominatour you muſte multiply the numerator of your fyrſt fraction, and the totall thereof ſhall you diuide by the denominatour of your fyrſt fraction, & that quotient ſhall be the numeratour to the denominator proponed: as for exāmple I haue thys fraction, $\frac{3}{4}$ which I would tourn into tenth partes, therfore I multiplye this 10 by 3, that is the numerator of my fraction, and there ryſeth 30, which I diuide by 5, and the quotient is 6, which muſt be the numeratour to 10, and ſo $\frac{3}{5}$ wyll be $\frac{6}{10}$.

Scho. Thys is eaſy inough to do.

Ma. Then ſhall you ſee an other example of the ſame fraction that is not ſo eaſy: as if I woulde tourne $\frac{3}{5}$ into 8th partes. proue you that worke.

Scho. I muſt multiply 8 by 3, and there amounteth 24, which I diuyde by 5, and the quotient is 4, then is the new fraction $\frac{4}{8}$.

Maſter. And ſee you nothing doubtful in this worke?

S. I ſe, that when 24 was dyuided by 5, there remained 4, which I dyd not paſſe of, becauſe ye ſpake nothing of any remayner but only of the quotient.

M. Why, I ſuppoſe you remember what I ſayde to you in diuiſion of whole numbers, that you ſhoulde not paſſe of the remayner there, but only note it as a ſumme that coulde not be diuided without knowledge of fractions. Wherfore I muſt make this, that in all diuiſions of whole numbers, when there is any remayner, you ſhall ſet it ouer a line as a numeratour, and ſet the diuiſor for the denominatour, and that fraction doth make the diuiſion complete, and is part of the quotient: as if I would diuide 48 by 5, the quotient will be 9 $\frac{3}{5}$: ſo in your former worke

when

when 24 was diuided by 5, the quotient ſhould bee 4 $\frac{4}{5}$, and ſo the newe fraction ſhoulde bee thus. [illegible] that is $\frac{4}{5}$ of the entiere numbre, and $\frac{5}{8}$ 1, whyche you maye proue by exaumple of ſome coyne.

Schollar. Then I take a Crowne, whoſe $\frac{5}{8}$ is 3 s. Nowe yf I would proue whether 3 s bee, I ſhall haue a combrouſe worke to do. $\frac{4}{5}$—$\frac{5}{8}$

M. In deed for whole pennies your exaple is troubleſome: yet turnyng the crown into halfpennies, it is eaſy inoughe.

S. Nowe will I do it.

M. Fyrſte let me tell you an eaſy way how to fynde any numbre, that will eaſily be diuided into ſuche partes as you deſyre, which way is this. Set downe the partes that you deſyre, and then by one of them multiply al the other, the totall whereof ſhall containe all the partes propoſed: As if I woulde haue a numbre that may be diuided into 4 5 6 and 7 partes, by 4 multiply 5, and there ryſeth 20: then multiply 20 by 6, and it wyll make 120: whych multiplied by 7, wyll yeld 840. and ſo of any other numbres.

Sc. Then in our former example where is mention but of 5 partes & 8 partes, I ſhall only multiply 5 by 8, which maketh 40. and that numbre will ſerue.

Maſter. So will it.

S. Then what is $\frac{3}{5}$ of 40?

M. Proue by the ſame rule which you confeſſe eaſy inough: 3 times 40, is 120, whych beyng diuided by 5, maketh 24: $\frac{4}{5}\,\frac{3}{4}$
and that in lyke,

Nowe to knowe whether be equall to 24, fyrſt I ſee by the ſame rule, that $\frac{1}{2}$ is 20, and $\frac{1}{4}$ is 5, of which 5 I muſt take $\frac{4}{5}$:
and that by the ſame rule is 4. $\frac{4}{5}\,\frac{1}{4}$
So that I ſee nowe, that is equall to $\frac{1}{10}$.

Maſt. And by the waye note this forme of fraction howe it is written, that is to ſaye, both the numeratour and his fraction aboue the line. Although I knowe it may be written other waies, as thus $\frac{3}{4}$ and $\frac{4}{5}$ / $\frac{1}{2}$, but I accompt the other waye more apte a greate deale.

And ſo may you expreſſe by an other waye then is before mentioned, al fractions of fractions, as thus. $\frac{1}{4}$
that is $\frac{1}{4}$ of $\frac{1}{3}$, and ſo of other. $\frac{1}{3}$
but I remitte theſe fourmes to the arbitrement of euery wiſe artes man, to vſe as he thinketh moſt apte and ready.

But nowe one example more for this rule, and then ſhall we ende it. If I haue $\frac{7}{15}$ of a Soueraygn (accompting the Soueraygn 20 ſhillinges) how many ſhyllinges is that $\frac{7}{15}$?

Sco. I muſte multiplye 7 by 20, and that maketh 140, which I ſhall dyuide by 15,

and

and the quotient wyll be 9 [illegible], or in the leſſer termes [illegible].

M. That is 9 [illegible] and [illegible]. And thus for this tyme ſhal ſuffiſe for Reduction, ſaue that I muſt now repeate a lyttle touching Multiplication and Diuiſion, and ſo go forward.

MULTIPLICATION.

IN Multiplication, it happeneth ſome tyme þ there be hole nūbres to be multiplyed with fractions. And may be in two ſortes: for other the whole nūbre is ſeuerall from the fraction, and is the multiplyer: or els, the whole numbres ioyned with one or both of the fractions, and ſo maketh a mixt numbre therof. If it be in the fyrſt ſort, then needeth there no reduction, but only multiply the numeratour of the fraction by that whole numbre, and the totall therof ſet for the new numerator, reſeruyng ſtyll the former denominator.

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S. I vnderſtand you thus. If I haue $\frac{6}{21}$ to be multiplyed by 16, then muſt I multiply that 16 with 6, whiche is the numerator, wherof commeth 96, and that muſt I ſet for the new numerator, keping ſtill 21 for the denominator, and ſo the fraction will be $\frac{96}{21}$ that is 4 $\frac{4}{7}$.

M. And in this ſort of worke you maye abbrege the labour thus: If it happen the denominator [illegible] ſuche a numbre, as may euenly be diuided by the ſayde whole numbre propoſed, then dyuide it thereby, and ſette the quotient of that diuiſion for the [illegible] [illegible] ſtyll the [illegible] tor, and [illegible] the multiplycation wrought as before. Then if I haue this example $\frac{9}{10}$ to bee multiplyed by 5. And bycauſe I wyll diuide 10 by 5, therfore I take the quotient of that diuiſion, whiche is 2, and ſet it in ſtede of 10, and ſo the whole will bee $\frac{9}{2}$, that is 4 $\frac{1}{2}$, [illegible] [illegible] is [illegible] which $\frac{9}{2}$ that would haue folowed by the other ſort of worke.

Scholer. I perceaue it very well.

Maſter. Now then for the other ſorte where the numbre is myxt, take this waye: fyrſt reduce the ſayde whole numbre and fraction into one fraction improper (as I taught you in Reduction) and then multiplie them together, as if they were proper fractions.

S. 15 $\frac{1}{2}$ beyng ſet to bee multiplyed by [illegible] fyrſt I muſt reduce the whole numbre, by multiplying it by 2, and [illegible] [illegible] [illegible] the numeratour, and [illegible] will be $\frac{31}{2}$, which now I ſhall multiply [illegible] and it wyll be [illegible]

M. [illegible] and ſo may you [illegible]

Multiply-

How to multiply myxte numbres.

of multiplication maye be wrought without Reductiõ, yet ſom cannot, as namely Mixed numbers.

And yet one note more will I tell you of Multiplication, before we leaue it: That is, when ſo euer you would multiplye any fraction by 2, which commonly is called Duplation, you may do it not only by doubling the numeratour, but alſo by partynge the denominatour into halfe, if he be euen. Duplation

Sc. Then if I woulde double $\frac{5}{12}$, I maye chooſe whether I wyll make it, $\frac{10}{12}$, or els $\frac{5}{6}$. And in deed I ſee that al is one, but that the diuyding of the denominatour ſeemeth the better waye to make ſmaller termes of the fraction, and ſo they ſhall neede the leſſe reduction.

Ma. It is ſo: and now I ſhall not neede to tell you that Multiplication is proued by Diuiſion, and Diuiſion lykewaies by Multiplication, but the lyke workes that I ſhewed you in Multiplicatiõ, will I ſhew you in Diuiſion alſo.

Diuiſion. To diuide whole nũ by a fraction.

When any whole numbre ſhall be dyuided by a fraction, you muſte multiplye the ſayd whole numbre with the denominator of the fraction, and ſette the totall thereof for the newe numerator, and for the denominatour ſet the numeratour of the fraction.

Sch. Then 20 diuided by $\frac{4}{9}$ will make 45.

Ma. Euen ſo. But if you would dyuide

the fraction by the whole noumbre, then multiplye the denominatour by the ſame whole noumbre, and ſet that totall for the denominatour, without changing the numeratour.

To diuide a fraction by a whole noumbre.

Scholar. Then to dyuide $\frac{20}{27}$ by 4, it wyll bee $\frac{20}{108}$.

Ma. You ſaye well. And by the ſame example you giue me occaſiō to remembre an other brief waye to doo the ſame. for if you had diuided the ſaid numerator by 4. & ſet the quotient for the numerator, keping ſtyll the old denominator, it wold haue been not only as well doone, but alſo in a fraction of leſſer termes.

An other brief way.

S. I geſſe it to be euen ſo, by ȳ like worke that you taught me in multiplication. And for prooue therof $\frac{20}{27}$ beyng the diuidend, and 4 the diuiſor, I diuide the numerator 20 by 4, and the quotient is 5, whiche I ſet for 20 ouer 27, thus $\frac{5}{27}$. And I ſee that it is all one with $\frac{20}{108}$, as by dyuiding both theſe termes by 4, and ſo reducing them to their leaſt denomination, I maye eaſily prooue.

M. You cōceiue it wel. And if there be mixt numbres (other one or both) you muſt firſt reduce that mixt numb. into an improper fractiō. And then work as you haue lerned.

S. That was ſufficiently taught in Multiplication. Therefore I praye you go forward to ſome other thing.

M. Then take this note yet for Diuiſion.

If

If the denominators be lyke, then dyuide the numerators as if it were in whole nũbres, & the quotient, whether it be fraction, whole nũbre or myxt, is a good quotient for that diuision. And generally, if one of the numerators maye iustly diuide the other, by that quotient multiplye the denominator of the lesser numerator, & set it that doth amount, in the rowme of the same denominator, and then for a numerator to it, set the denominator of the other fraction.

[illegible] when I would diuide [illegible] by [illegible], I se that [illegible] wil dyuide [illegible] the quotient wyl be 4, by whiche I must multiplye the other 4 that is the denominator vnder [illegible], and then is it [illegible] whiche I set in the denominator, and [illegible] I must sette [illegible] the other denominator, and so is it thus [illegible]

[illegible] And so is [illegible] in steede of [illegible], whyche would haue rysen by the common worke.

Then haue [illegible] (which is to diuide by 2) [illegible] be euen, set the halfe of it in his place without [illegible] haue you done. And if the numerator [illegible] then double the denominator.

[illegible] I may [illegible] diate [illegible], I must make it [illegible]

[illegible] I haue [illegible] you [illegible] sufficient knowledge in Reduction, Multiplication, and Diuiſion. And therefore will I go in hande with Addition and Subtraction, whiche now will appeare eaſy inoughe.

ADDITION.

When ſo euer you haue any fractions to be added, you muſte conſider whether they be of one denomination or not. And yf they be of one denomination, then adde the numerators together, and ſet that amounteth for the numerator ouer the common denominator, and ſo haue you done. The reaſon is, bycauſe that ſuche differ lyttle in Addition or Subtraction frome the vſage of vulgar denominations, where the denominatours bee no numbers, as 4 pence and 4 pence, make 8 pence, where the denomination is not altered. But and yf the fractions bee not of one denomination, or annye of theym bee myxte of whole numbers and fractions, then muſte you fyrſt reduce them to one denomination, and after adde them. And if they bee manye, then adde fyrſt two of theim, and to the ſumme that doothe amount of that Addition, adde the thyrde, and then the fourthe, and ſo foorth, yf you haue ſo many.

Scholar. Thys ſeemeth eaſye ynoughe, nowe that I haue already learned to multiply tiplye and to Reduce, withoute whiche two, I could neuer haue wrought this. And therfore nowe I ſee good reaſon, why you dyd place Multiplication and Reduction before Addition.

Ma. It is well conſidered, but yet refuſe not to expreſſe your underſtandynge of it by an example.

Scholar. Then woulde I adde fyrſte $\frac{7}{12}$ with $\frac{5}{12}$, and bycauſe the denominatours are lyke (and ſo needeth no Reduction) I adde 7 to 5, whyche maketh 12, and then is my ſumme $\frac{12}{12}$, that is in ſmaller numbres $\frac{1}{1}$.

And if I haue many numbres to bee added, as here $\frac{3}{4}$ $\frac{4}{5}$ $\frac{9}{10}$, fyrſte I muſte reduce them (bicauſe they haue dyuers denominatours) into one denomination, & then wyll they bee thus.

$$\frac{150}{200} \quad \frac{160}{200} \quad \frac{180}{200}$$

Or in leſſer tearmes $\frac{15}{20}$ $\frac{16}{20}$ $\frac{18}{20}$, whych by addition do make $\frac{49}{20}$, that is 2 $\frac{9}{20}$.

Maſter. Nowe maye wee go to Subtraction.

Subtraction hathe the ſame preceptes that Addition had. for if the denominatours be lyke, then muſte you ſubtracte the one numeratour from the other, and the reſt is to be ſet ouer the common denominator, and ſo is your ſubtraction ended: but and yf you haue manye fractions to bee ſubtracted out of many, then muſt you reduce them to one denomination, and into twoo ſeuerall fractions, that is, all that muſt bee ſubtracted into one fraction, and the reſidue into an other fraction, and then worke as I ſaid before.

Scholar. For the fyrſte exaumple I take $\frac{5}{12}$ to be ſubtracted out of $\frac{7}{12}$, and the reſte wyll be $\frac{2}{12}$ or $\frac{1}{6}$.

For an other exaumple I take $\frac{1}{2}$ to bee ſubtrahed out of $\frac{7}{12}$, whyche I muſte fyrſte reduce, and it will bee thus, $\frac{24}{48}$ and $\frac{28}{48}$. Then doo I ſubtracte 24, oute of 28, and there reſteth 4, whiche I ſet ouer the common denominatour for a remayner, thus. $\frac{4}{48}$, that is $\frac{1}{12}$.

Now for the thyrd example, I take $\frac{3}{4}$ and $\frac{5}{6}$ to be ſubtrahed from $\frac{7}{8}$ & $\frac{9}{10}$. And bicauſe their denominatours bee dyuers, I doo reduce them thus, $\frac{1440}{1920}$, $\frac{1600}{1920}$, $\frac{1680}{1920}$, $\frac{1728}{1920}$.

Then doo I adde the twoo fyrſte, and they make $\frac{3040}{1920}$. Alſo I adde the two laſte and they yelde $\frac{3408}{1512}$. Then do I ſubtracte 3040 out of 3408, and there reſteth 368, ſo is the remainder $\frac{368}{1512}$, that is in ſmaller termes $\frac{46}{189}$. And thus haue I doone wyth Subtraction, except you haue any more to teache me.

M. Nowe one example more of two fractions of diuers denominations.

S. I take theſe two fractions, $\frac{7}{8}$ and $\frac{9}{24}$, whiche beyng reduced, wyll ſtande thus, $\frac{168}{192}$, and $\frac{72}{192}$. Nowe would I ſubtracte 168 oute of 72, but I can not.

M. Then may you perceyue that you miſtoke the fractions: for you can neuer ſubtracte the greater out of the leſſer, althoughe you may adde, multiply or diuide the greater with the leſſer. And albeit that $\frac{7}{8}$ hath bothe his termes leſſer then $\frac{9}{24}$, yet is $\frac{9}{24}$ the leſſer fraction: for generally if you multiply the numeratours and denominatours of two fractions croſſe wayes, that fraction is the greateſt, of whoſe numeratour commeth the greateſt ſumme: as in this example, 7 multiplied by 24, maketh 168, and 9 beyng multiplied by 8, yeldeth but 72, therfore is the fyrſt fraction $\frac{7}{8}$ the greater of theſe two: ſo canne you not ſubtract it out of a leſſer fraction.

The greateſt of two fractions.

S. But and you ſhoulde ſubtracte a fraction out of a whole numbre, what wold you doe?

M. Firſte I wold reduce that whole nũbre into a fraction of the ſame denomination, that my fraction is, & then worke by Subtraction.

Maſter. So maye you doo, but it is eaſyer much yf your fraction be a propre fraction, that is to ſaye, leſſe then an vnitie, to take an vnitie from the whole numbre, and then tourne it into an Impropre fraction, & ſo worke your Subtraction. As yf I wold ſubtracte $\frac{2}{3}$ from 4, I maye take one from 4 and tourne it into $\frac{3}{3}$, from whych yf I abate $\frac{2}{3}$, there wyll remayne 3 $\frac{1}{3}$. And if the fyrſt fraction be an impropre fraction, then maye I take ſoo manye vnities from the whole numbre, that they may make an impropre fraction greater then that fyrſt, and then worke the ſubtraction. As yf there bee proponed $\frac{10}{3}$ to be ſubtrahed frō 6, bicauſe $\frac{10}{3}$ is more then 3, and not 4, I muſt take 4 from 6, and tourne them into thyrdes, thus, $\frac{12}{3}$, than abate $\frac{10}{3}$, and there reſteth $\frac{2}{3}$, ſoo the whole remayner is 2 $\frac{2}{3}$. And thus wyll I make an ende of the workes of common fractions for this tyme, not doubtinge, but you can applie them both vnto the rules of Progreſſiō, and alſo vnto the Golden rule, without any other teachyng then you haue learned before, which might ſeme tedyous to repete, ſaue that in ſome ſpeciall diuerſities, which be peculier to fractions, I can not ouerpaſſe, but inſtructe you ſomewhat by the waye.

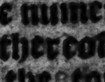
The Golden rule.

Wherefore as touchyng the Golden rule for the placing of the iij. numbres proponed in the queſtion, wherby to find yͤ third and for yͤ forme of their worke, with other like notes, I referre you to that whych you haue already learned.

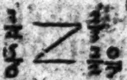
But thys eaſy fourme of workinge by fractions ſhall you note, that yf your thre numbres be fractions for an apte worke and certayne, multiply the numerator of the firſte noumbre in the queſtion, by the denominatour of the ſecond, and all that agayne multiplye by the denominatour of the third numbre. And the totall thereof ſhall you kepe for to bee the diuiſor: Then multiply the denominator of the firſt numbre by the numeratour of the ſecond, and the whole thereof by the numerator of the thirde, and the totall therof ſhall be your diuidend. Nowe dyuide this dyuidend by the dyuiſour whiche you found out before, and that numbre ſhalbe the iiij. numbre of the queſtion, which you ſeke for: As in thys example, If $\frac{1}{4}$ of a yarde of veluet coſte $\frac{2}{3}$ of a Soueraigne, (eſtemed at 02 ſhillinges) what ſhall $\frac{1}{6}$ coſt.

uestion
Veluet.

Scho. If it pleaſe you to lette me make the aunſwere, I wold firſt place theſe thre numbres, as I lerned in whole numbres, thus.



And then according to your new rule, I muſt multiply 3 being numeratour in the fyrſt nombre, by thre the denominatour of the ſecond, and therof commeth 9, which I multiply agayne by 6, the denominatour of the third numbre, and ſo haue I 54, which I kepe for the diuiſor. then multiplye I 4 the denominator of the fyrſte, by 2, the numeratour of the ſecond, and there ryſeth 8, which agayne I multiply by 5, the numeratour of the third, and it maketh 40. then muſt I diuide 40 by 54, and it wyll be $\frac{40}{54}$ ẏ is $\frac{20}{27}$, in leſſer termes, and then the figure wyl ſtande thus.



But what that is in mony, I can not tell, except I ſhall worke it by Reduction, as you taught me.

Ma. It forceth not now, you may reduce it when you lyſt, but it were diſorderlye done here to mingle diuers workes togyther, wher we do not ſeke the valew of the thing in common monye, but in an apte numbre, which you haue wel done. And therfore wil I yet ſhew you an other like way of eaſines in work, how you may chaũg your iii. fractions into 3 whole numbres, by whiche you ſhal worke as yf the queſtion were proponed in whole numbers. The fyrſt numbre you ſhall fynde, as I taughte youe,

The Golden rule.

Wherefore as touching the Golden rule for the placing of the iij. numbres propoſed in the queſtion, wherby to find yͤ third and for yͤ forme of their worke, with other like notes, I referre you to that whych you haue already learned.

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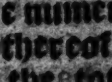
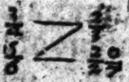
ueſtion Veluet. Scho. If it pleaſe you to lette me make the aunſwere, I wold firſt place theſe thre numbres, as I lerned in whole numbres, thus.



And then accordyng to your new rule, I muſt multiply 3 being numeratour in the fyrſt nombre, by thre the denominatour of the ſecond, and therof commeth 9, which I multiply agayne by 6, the denominatour of the third numbre, and ſo haue I 54, which I kepe for the diuiſor. then multiplye I 4 the denominator of the fyrſte, by 2, the numeratour of the ſecond, and there ryſeth 8, which agayne I multiply by 5, the numeratour of the third, and it maketh 40. then muſt I diuide 40 by 54, and it wyll be $\frac{40}{54}$ ꝧ is $\frac{20}{27}$, in leſſer termes, and then the figure wyl ſtande thus.

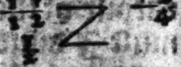
But what that is in mony, I can not tell, except I ſhall worke it by Reduction, as you taught me.



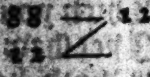
Mai. It forceth not now, you may reduce it when you lyſt, but it were diſorderlye done here to mingle diuers workes togyther, wher we do not ſeke the valew of the thing in common monye, but in an apte nombre, which you haue wel done. And therfore wyl I yet ſhew you an other like way of eaſines in work, how you may chaūg your iii. fractions into 3 whole numbres, by whiche you ſhal worke as yf the queſtion were proponed in whole numbers. The fyrſt numbre you ſhall fynde, as I taughte you, now to find the dyuiſor. for the ſecond numbre take the numeratour of the ſecond fraction: and for the third numbre take that, ẏ ryſeth of the multiplication of the denominatour of the firſt, by the numeratour of the thyrd, and then worke your queſtiō.

queſtiō of Syluer.

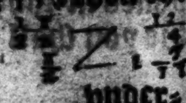
Sc. For exāple herof, I put thys queſtiō, If $\frac{11}{12}$ of 1 li waight of ſyluer, be worth $\frac{12}{4}$ of a ſoueraign, what is $\frac{1}{2}$ of 1 li. weight worth? For the aunſwere, fyrſte I place ẏ fractiōs in order thus.



Then to tourne theſe fractiōs into whole numbres, I multiply 11 which is the numeratour of the firſt, by 4, (the denominatour of the ſecond) and there cometh 44, which I multyplye by 2 the denominatour of the third, and it amounteth 88, whiche I ſet for the diuiſor in the fyrſt place. Then in the ſeconde place I ſette 12, which is numeratour in the ſecond fraction, and in the third place I ſet the ſumme that amounteth of 12, beyng the denominatour in ẏ fyrſt nombre, multiplied by 1, being numerator in ẏ third numbre. & ſo ẏ figure wyl ſtand as here you ſe.



Thē to worke it forth, I multiply 12 by 12, and there amounteth 144, which I diuide by 88, and the quotient wil be $1\frac{56}{88}$, or in leſſer tearmes. $1\frac{7}{11}$, & then the figure wil ſtand thus.



M. Theſe ij. formes now you

vnderſtand wel ynoughe: And as for any other, at this time I will not repete. onelye this ſhall you marke for the proofe of thys rule, whether your worke be well wrought or no. Multiplye the fyrſte numbre by the forth, and note what amounteth: then multyplye the ſecond by the thyrde, and marke what amounteth alſo. Nowe yͭ thoſe ii. numbres ſo amountyng be equall, then is your worke well done, ells you haue erred. And this ſhal ſuffice for the former rule, but in the Backer rule, thys ſhall you note for eaſe of worke, that you multyplye the numeratour of the fyrſt by the numeratour of the ſecond, and the whole thereof by the denominatour of the thyrd, and that amounteth thereof, ſhall be the diuidende. Then multiplye the denominatour of the firſte, by yͤ denominator of the ſecõd, & that whole by the numeratour of the third, and that riſeth therof ſhal be the diuiſor. Example of this: I dyd lend my friend $\frac{3}{4}$ of a Porteguiſe vii. monethes, vpon promiſe that he ſhould do as much for me agayne. and when I ſhould borowe of him, he could lend me but $\frac{1}{2}$ of a porteguiſe. nowe I demaund howe longe time muſt I keepe his money in iuſt recompence of my lone, accompting 13 monethes in the yeare?

The proofe of the Golden rule.

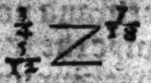
The Backer rule.

A queſtion of lone.

Scho. The firſt noumbre muſt be yͤ firſt monye borowed, that is $\frac{3}{4}$ of the Porteguiſe

W.iij. guiſe

guiſe: the ſecond numbre ẏ 7 monethes ẏ is $\frac{7}{13}$ of a yeare, and ẏ third numbre the mony that was lente in recompence. that is $\frac{5}{12}$ of a Porteguiſe, then I ſet the noumbres thus.



Then (as you taught me) I multiply 3 (beinge numeratour in the firſt numbre) by 7 the numeratour of the ſecond numbre, and it makethe 21, which I multiply by 12 the denominatour of the third, and ſo haue I 252 for the diuidende: then I multiply 4 the denomynatour of the fyrſt, by 13 the denominatour of the ſeconde, and it yeldeth 52, whiche I multiply againe by 5, the numeratour of the third, and it will make 260, that is the diuiſour. Then muſt I diuide 252, by 260, ſo it will be in the ſmalleſt Fraction $\frac{63}{65}$ of a yeare.

Statute of aſſiſe of breade & ale

Maſter. And this doo you ſee ſome eaſe, in working better then to multiply and diuide tediouſlye ſoo manye fractions. An other queſtion yet wyll I propoſe, to the intent you may ſee therby the reaſon of the ſtatute of aſſiſe of breade and ale, whiche in all ſtatute bokes in French, Latyn, & Engliſhe, is muche corrupted for want of knowledge in this arte, for the right vnderſtanding wherof I propone this queſtion.

queſtion.

When the price of a quarter of wheat is 2.ſ. & the farthing white lofe ſhal way 68 ſ. then

I demaund, what ſhall ſuche a loſe waye, when the quarter of wheate is ſold for 3.s?

Scho. This queſtion muſt be wrought, as it is proponed in whole noumbers and not in fractions.

Ma. You ſeme to ſay reaſonable, how be it in that ſtatute of aſſiſe, the rate is made by the proportion of partes in a poūd weight Troye, els could it not be a ſtatute of anye long continuaunce, ſeing the ſhillinges doo chaung often, as al other monies doo: but this ſtatute well underſtand, is a continuall rule for euer, as I wyll anone declare by a newe table of aſſiſe, conuertinge the ſhyllynges into unces and parte of unces. Therefore here by a ſhillyng you muſte underſtand $\frac{1}{20}$ of a pound waighte, and ſoo by pens $\frac{1}{20}$ of a unce, wherefore although you mighte worke this queſtion proponed by whole numbre well ynough for that tyme when the Statute was made, yet to applye it to our time, and to make it ſerue for all times generallye, it is beſt to worke it by fractions, ſetting for ij. ſhillinges $\frac{2}{20}$: and for 68 ſhillinges, $\frac{68}{20}$: and ſo for three ſhyllinges $\frac{3}{20}$ and then will the figure of the queſtyon ſtand thus

$$\begin{array}{ccc} \frac{2}{20} & \text{Z} & \frac{68}{20} \\ \frac{3}{20} & & \end{array}$$

In which queſtion bycauſe all the denominatours be lyke, you ſhall worke onelye with the numeratours.

Note w
a Shyll
is.

Scoler. Then I ſhall multiply 68 by 2, whereof commeth 136, whiche if I dyuyde by 3, the quotient wyll be $45\frac{1}{3}$: but how ſhal I make a fraction of that to ſtand with the other?

Ma. Haue you ſoo ſone forgotten what was taught you ſo latelye? Thys is hys fourme.

$$\frac{45\frac{1}{3}}{20}$$

Scho. I remembre it now, & then it ſignifieth 45 twenty partes, and the third deale of one twenty part.

Ma. So is it, and that maketh in ſhyllynges, 45 ſhillinges, foure pens. Whereby you may note one greate errour in the ſtatute bokes which haue conſtantly 48 ſhyllynges in that aſſiſe. And by thys rule, yf you examin the ſtatute, you ſhall fynd many ſummes falſe: wherefore for the true vnderſtanding of that ſtatute and ſuch like as I haue made mention of it, and ſomwhat recogniſed it, ſo do I wyſhe that all gentlemen and other ſtudentes of the lawes, wold not neglecte this arte of Arithmetike as vnneedfull to their ſtudies. Wherefore to encourage them therto, and to gratifye bothe them and al other in general, I wil here exhibit a table of that part of the ſtatute in ij columnes, and in a third column I will adde the correction of thoſe errours which haue crept into it.

Here foloweth the Table.

When a quarter of wheate ys solde for

The price of a quarter of wheat.		A weight of a farthing white lofe by the statute bokes.			The correction by iuste Assise.		
ſ.	d.	li.	ſ.	d.	li.	ſ.	d.
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4 1/4	2	14	4 4/5
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	10 2/7
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 2/3
5	0	1	8	2 1/2	1	7	2 2/5
5	6	1	4	8 1/4	1	4	6 8/11
6	0	1	2	8	1	2	8
6	6	0	19	11	1	0	11 1/13
7	0	0	19	1	0	19	5 1/7
7	6	0	18	1 1/2	0	18	1 3/5
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 1/4	0	15	1 1/3
9	6	0	14	4 3/4	0	14	3 15/19
10	0	0	13	7 1/2	0	13	7 1/5
10	6	0	12	11 1/2	0	12	11 3/7
11	0	0	12	4 1/2	0	12	4 4/11
11	6	0	11	10	0	11	9 21/23
12	0	0	11	4	0	11	4

In yͤ comon bookes ther is no farther rate of aſſiſe made than vnto 12 s̄ the quarter of wheat, but in an anciēt copie of 200 yeares ould (which I haue) ther is added yͤ rate of aſſiſe vnto 20 s̄ yͤ quarter, but yet was yͤ aſſiſe alſo either wrong caſt at yͤ fyrſt penning or els corrupt ſith yͤ time for lack of iuſt knoledg in yͤ rule of proportion, whiche I will adde her alſo to gratify ſuche as be ſtudious in this ſorte of knowledge, and deſire to vnderſtand truth exactly.

The price of a quarer of whæat.		The wheight of the farthing white loſe by the ſtatute bookes.		The correction of the errours.	
s̄.	d̄.	s̄.	d̄.	s̄.	d̄.
12	6	11	?	10	10 $\frac{14}{25}$
13	0	15	0 $\frac{1}{2}$	10	5 $\frac{7}{13}$
13	6	10	1 $\frac{1}{2}$	10	0 $\frac{24}{27}$
14	0	9	7	9	8 $\frac{4}{7}$
14	6	9	2 $\frac{1}{2}$	9	4 $\frac{7}{20}$
15	0	9	1 $\frac{1}{2}$	9	0 $\frac{4}{5}$
15	6	9	1 $\frac{1}{4}$	8	9 $\frac{9}{31}$
16	0	9	0	8	6 0
16	6	8	6	8	2 $\frac{10}{11}$
17	0	8	3	8	0 0
17	6	7	10	7	9 $\frac{9}{35}$
18	0	7	6	7	6 $\frac{2}{3}$
18	6	7	3	7	4 $\frac{8}{37}$
19	0	7	2	7	1 $\frac{17}{19}$
19	6	5	10	6	11 $\frac{9}{13}$
20	0	5	6	6	9 $\frac{1}{2}$

These .ii. tables I haue ſet ſeueral, bicauſe no man ſhould thinke that I would either adde or take away from any lawe, thoſe partes which might of right ſeme eyther ſuperfluous other diminute, but yet I maye not be ſo curious as to neglect manifeſt errours, which is not onely my parte, but euery good ſubiects duetie with ſobrietie to correct. And for auoyding of offence, I haue rather done it in this priuate booke, rather then in any booke of the ſtatutes ſelfe, truſtinge that all men wil take it in good part.

Scho. I would wyſhe ſo, but I dare not hope ſo, ſith neuer good man that woulde reforme errour, could eſcape the venimous tungues of enuious detractors, whiche bycauſe they either can not or liſt not do anye good theim ſelues, do delite to barke at yͤ doyngs of other, but I beſeke you to ſtaye nothing for their peruerſe behauiour.

M. I conſyder many thinges that ſome may obiecte, wherunto I am not vnprouided of iuſt aunſweres, but I wyll not ſeme ſo haſty to make the aunſwers before I heare their obiectiõs, but as I truſt that mẽ are of a better nature, and more gratfull now then ſome hath ben in times paſſed, as I haue done in the ſtatute of aſſiſe for brede in rate of ſhillinges, ſo wyll I ſet forthe the like table in poundes & vnces, and the partes therof, that it may be eaſily applied to al tymes:

but I meane not by this to alter any worde of the ſtatute (being ſo good an ordinaunce and of ſo great continuaunce) but onlye to make it as a kynd of expoſition & declaration of the ſaid ſtatute, truſting that thereby the ſtatute may be better vnderſtande, and conſequently better put in execution. And here you ſhal note that I haue accompted ẏ ſhyllynges after the rate of lx.ſ. to the poūd weight, bycauſe I eſteme it the moſte apte rate for our tyme. Wherfore if in the fyrſte columne you find the price of wheate, directly agaynſte it in the ſeconde colume you may fynd the weight of the farthing white lofe, in this our tyme. and if you double that numbre (as I haue done in the thyrde colume) then haue you the weight of the halfe peny white lofe. and ſo in the fourth colume is ſet the weyght of the peny white lofe. It nedeth not to tell you that that the ſyghte dooth teſtifye, howe that euerye colume is parted into three ſmaller pillers, whereof the fyrſt colume hath theſe iii. titles, Poundes, Shillings, and Penies: the other three columes, haue eche of them theſe three titles, Poundes, Vnces, & Penye weightes. And as in the fyrſt colume xii. pence make a ſhyllynge, and xx.ſ. maketh a pound, ſo in the other iii. columes xx. peny weight maketh an vnce, and xii. vnces doo make a pounde.

pound
weighte.

The

The price of a quarter of wheate			The weyght of the same thing whyte lofe.		
l	s	d	li.	Unces	D. w.
0	3	0	6	9 ½	2
0	4	6	4	6 ½	5
0	6	0	3	4 ¼	1
0	7	6	2	8 ½	1 ¾
0	9	0	2	3	4
0	10	6	1	11 ½	1 ¾
0	12	0	1	8 ¼	3
0	13	6	1	6	3 ½
0	15	0	1	4 ¼	1 ½
0	16	6	1	2 ¾	1 ½ ¼
0	18	0	1	1 ½	2
0	19	6	1	0 ½	1 ½ ½
1	1	0	0	11 ½	3 ½
1	2	6	0	10 ¾	3 ¼
1	4	0	0	10	4
1	5	6	0	9 ½	2
1	7	0	0	9	1 ½
1	8	6	0	8 ½	1 ½
1	10	0	0	8	3 ¼
1	11	6	0	7 ¾	0 ½
1	13	0	0	7 ¼	3 ¼
1	14	6	0	7	1 ¾
1	16	0	0	6 ¾	1
1	17	6	0	6 ½	0 ½
1	19	0	0	6 ¼	0

The weyght of the halfpenny white lofe.			The weight of the pennye white lofe.		
li.	vncz	d.w.	li.	vncz	d.w.
13	7	4	27	1/4	3
9	0 3/4	1	18	1 1/2	2
6	9 1/2	2	13	7	4
5	5 1/4	0 3/5	10	10 1/2	1 1/3
4	6 1/4	3	9	0 3/4	1
3	10 1/2	2 4/7	7	0 1/2	0 1/7
3	4 3/4	1	6	9 1/2	2
3	0 1/4	0 1/3	6	0 1/2	0 2/3
2	8 1/2	2 4/5	5	5 1/2	0 3/5
2	5 1/2	3 5/11	4	11 1/2	1 10/11
2	3	4	4	6 1/4	3
2	1	2 2/13	4	2	4 4/13
1	11 1/4	1 2/7	3	10 1/2	2 4/7
1	9 1/4	0 1/3	3	7 1/2	0 2/5
1	8 1/4	2	3	4 3/4	1
1	7	4	3	2 1/4	3
1	6	2 2/3	3	0 1/4	0 1/3
1	5	3 11/19	2	10 1/4	2 9/13
1	4 1/4	1 2/3	2	8 1/2	2 4/5
1	3 1/2	0 6/7	2	7	1 5/7
1	2 3/4	1 8/11	2	5 1/2	3 5/11
1	2	3 10/23	2	4 1/4	2 [illegible]
1	1 1/2	2	2	3	4
1	1	1 10/75	2	2	6/25
1	0 1/2	1 4/11	2	1	2 [illegible]

Scho. Syr I doo thanke you moſt hartilye for this, not onely in myne owne name, and in the name of all ſtudentes, but alſo in the name of the hole commons, to whō the reſtitution of this aſſiſe (I truſt) ſhall bringe reſtitution of the weight in breade, whiche longe time hath bene abuſed. And yf you know any lyke thinges more, wherein you wold vouchſafe to declare the errours and ſet forth the truthe, you can not but obtaine great thankes of all good harted men that loue the common wealth.

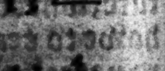
M. I haue ſundry thinges to declare, but I haue reſerued them for a pryuate boke by it ſelfe, yet notwithſtanding bicauſe the ſtatute of the rate of meaſuringe of ground is ſo common that it toucheth al men, and yet no more common then nedeful, but ſo much corrupt, that it is to far out of al good rate, not onely in the Engliſhe bokes of ſtatutes, commonly printed, but alſo in the latin bokes, and in the French alſo, for I haue read of eche ſorte, and conferred them diligently. I wyll giue you a table for the reſtitution of thoſe errours, as may ſuffice for this preſente time. And firſte will I propoſe one queſtion to you, touching the vſe of that ſtatute, whereby you may perceaue the order howe to examine the whole ſtatute, and euery parcell thereof, and the queſtion is this. When the aker of grounde dooth containe

A queſtio
of meaſure
of Groun

foure perches in bredthe, than muſte it conteyne 40 perches in length. Thenne doo I demaund of you, how much ſhall the length of an aker bee, when there is in the bredth of it 13 perches? But before you ſhall anſwere to this queſtion, I will declare unto you an other ſtatute, which is the grounde of that former ſtatute. And that ſtatute is this. It is ordeined, that 3 barly cornes, dry and round, ſhall make up the meaſure of an ynche. xii. ynches ſhall make a foote, and iii. fote ſhall make a yarde (the common Engliſhe bookes haue an elne) fyue yardes and a halfe ſhall make a perche, and forty perches in lengthe, and iiii. in breadthe, ſhall make an aker: this is that ſtatute, whereby you may perceaue, that the intente of the ſtatute is, that an aker ſhould conteine 160 perches. Nowe let me here your anſwere to the queſtion.

A ſtatute of meaſures.

Scholer. As I perceue by the wordes of that ſtatute, a perche to be $\frac{1}{160}$ of an aker, ſo could I make thoſe numbres all in fractions, and ſo worke the queſtion: but ſeyng I maye do it alſo in hole noumbres, I take yͭ forme for yͤ moſt eaſy, therfore thus I ſet yͤ queſtiō in forme.



then do I multiply 40 by 4, it maketh 160, whiche I diuide by 13, and the quotient is 12 $\frac{4}{13}$.

Mai. Nowe turne that $\frac{4}{13}$ into the common

mon partes of a perche, as they bee named in the former ſtatute: howbeit, it ſhall be beſt to take one of the leaſt partes in denomination for auoydyng of muche labour, as feet, wherof the perche contayneth 16 $\frac{1}{2}$.

S. Than to tourne $\frac{4}{13}$ into feete, I multiplye 16 $\frac{1}{2}$ by 4, and it maketh 66, whiche I muſt diuide by 13, and the quotient is 5 $\frac{1}{13}$.

Ma. So I fynde that if the acre holde in bredth xiij. perches, it ſhal contain in length 12 perches, 5 foote, and $\frac{1}{13}$ of a foot, which is not fully an ynche, for the ynche is $\frac{1}{12}$ of a foote: But heere all the ſtatute bookes in latine and engliſhe (that I haue ſeene) doo note it to be 13 perches, 5 foot, and 1 ynch: which maketh aboue xiij. perches to manye in the acre. ſo that I woulde haue thought the errour to haue crepte into the prynted bokes by the great negligence that printers in our tyme do vſe, ſaue that in written copies of greate antiquitie, I doo fynde the ſame. Yet haue I one frenche copye, which hath 12 perches $\frac{1}{4}$, and one foote, and that myſſeth very lyttle of the truthe.

S. Then I ſe it is true that I haue often heard ſay, that the trueſt copies of the Statutes, be the frenche copies.

M. That is often true, but not generally, as I haue by conference tried dyuerſly: but in this ſtatute the frenche boke is moſt corrupt in all other places lightly.

But nowe to perfourme my promiſe, I wyll ſet forth the table for meaſuring of an aker of ground only by ſuche partes as the ſtatute doth mention, bycauſe at thys tyme I doo of purpoſe write it for the better vnderſtanding of the ſtatute, and herafter with other thynges I intende to ſet foorth thys ſame more at large.

In this table folowing I haue not done as in the other ſtatute before, compared my reſtitution with the faultes crept into the ſtatute, but only haue written that trewe meaſure, whiche the equitie of the ſtatute dooth pretende. For it were to vyle to iudge of ſo noble princes and woorthy concellours as haue auctoriſed and ſet foorth this ſtatute, that they wold make one aker in any forme greater then an other, but euerye one to be iuſte and equall with eche other, whyche is the grounde alſo of my woorke, and hereby may all menne perceaue howe needefull Arithmetike is vnto the ſtudentes of the law. But nowe I thinke beſte to make an ende of theſe matters for this preſent tyme, ſithe the table hath in it none obſcuritie, that I ſhoulde neede to declare.

The breadthe	The lengthe of the aker		The breadthe	The lengthe of the aker	
perches	perches	feete.	perch	perch	feete.
10	16	0	28	5	11 $\frac{11}{14}$
11	14	9	29	5	8 $\frac{31}{58}$
12	13	5 $\frac{1}{2}$	30	5	5 $\frac{1}{2}$
13	12	5 $\frac{1}{13}$	31	5	2 $\frac{41}{62}$
14	11	7 $\frac{1}{14}$	32	5	0
15	10	11	33	4	14
16	10	0	34	4	11 $\frac{11}{17}$
17	9	6 $\frac{27}{34}$	35	4	9 $\frac{3}{7}$
18	8	14 $\frac{2}{3}$	36	4	7 $\frac{1}{3}$
19	8	6 $\frac{18}{19}$	37	4	5 $\frac{13}{37}$
20	8	0	38	4	3 $\frac{9}{19}$
21	7	10 $\frac{3}{14}$	39	4	1 $\frac{9}{13}$
22	7	4 $\frac{1}{2}$	40	4	0
23	6	15 $\frac{18}{23}$	41	3	14 $\frac{73}{82}$
24	6	11	42	3	13 $\frac{5}{14}$
25	6	6 $\frac{3}{5}$	43	3	11 $\frac{77}{86}$
26	6	2 $\frac{7}{13}$	44	3	10 $\frac{1}{2}$
27	5	15 $\frac{5}{18}$	45	3	9 $\frac{1}{6}$

Schol. In dede ſyr, I vnderſtand the table (as I thinke) by thoſe other which you ſette foorth before. For in the firſt columne is ſette the perches of the bredth of any A‑

ker, and then in the ij columnes folowinge appeareth howe manny perches and howe many foote the ſame acre muſt haue for his lengthe.

M. You take it well: howe be it to ſpeake exactly of breadth and length, the fyrſte columne dooth ſomtime betoken the breadth, and ſometyme the length. for proprelye the longeſt ſyde of any ſquare doth lymitte his lengthe, and the ſhorter ſyde dooth betoken the breadth: yet it is no greate abuſe in ſuch tables, where a man can not well chaunge the tytle, to let the name remaine, although the proportions of the numbres do chaung. for ſtill by the fyrſt columne is expreſſed the meaſure of the one ſyde, and by the twoo other pyllers in one columne, is ſet forth the meaſure of the other ſide. And thys ſhall be ſufficient now for the vſe of the Goldē rule.

Now ſomwhat will I touch certayn other rules, whiche for their ſeuerall names may ſeeme diuers rules and diſtincte from thys, but in deed they are but branches of it: yet bicauſe they haue not onlye ſeuerall woorkinges in appearaunce, but alſo pleaſant in vſe, I wyl gyue you a taſte of eche of them.

As for the rule of Felowſhip both ſingle & double, with Tyme & without time, I ſhall need to ſay little more than I haue already ſayd in teaching the woorkes of whole numbres, yet an example or two will we haue to refreſhe the remembrance of the ſame, and to declare certayne propre vſes and applications of it, as this for one. Foure men gette a bootye or price in tyme of warre, the priſe is in valewe of mony 8190 li, and bicauſe the men be not of lyke degree, therfore their ſhares may not be equall, but the chiefeſt perſon will haue of the bootye the thyrde parte, and the tenthe parte ouer: the ſeconde will haue a quarter and the tenthe part ouer: the third will haue the ſyxt part: and ſo there is left for the fourth man a verye ſmall portion, but ſuche is his lot, (whether he be pleaſed or wroth) he muſt be content with one xx. parte of the pray. Now I demaunde of you, what ſhall euery manne haue to his ſhare?

A que
of vneq
ſocie

S. You muſt be fayne to anſwere to your owne queſtion, els it is not lyke to bee aunſwered at this tyme.

M. The forme to vnderſtand the ſolution of this queſtion, and all ſuche lyke, is thys: Reduce all the denominators into one nūbre by multiplication; excepte that anye of them be partes of ſome other of theym, for all ſuch partes you may ouerpaſſe, and take for them all thoſe noumbres, whoſe partes they bee, as in this example the ſhares bee theſe, $\frac{1}{3}$ $\frac{1}{10}$, $\frac{1}{4}$ $\frac{1}{10}$, $\frac{1}{6}$, $\frac{1}{20}$, yf I multiply al the denominatours togither, begynninge wyth 3, & ſo go on vnto 20, it will make 144000,

but

but coſideryng that 3 is a part of 6, I ſhall omit that 3, & likewaies 10 which is a parte of 20, I may ouerpaſſe alſo, and then is ther but 3 denominatours to multiply, that is 4, 6, and 20, whiche make 240, whiche ſumme I take foꝛ my wooꝛke, bicauſe all the denominatours will be found in it. Then I take ſuch partes of it as the queſtion impoꝛteth, that is foꝛ the fyꝛſte man $\frac{1}{3}$ $\frac{1}{10}$: the $\frac{1}{3}$ is 80: the $\frac{1}{10}$ is 24: which I putte in one ſumme foꝛ the fyꝛſt mans ſhare, and it maketh 104. Then foꝛ the ſecond mannes ſhare I take $\frac{1}{4}$, whiche is 60, and $\frac{1}{10}$ which is 24, and that maketh in the whole 84. Now foꝛ the third man which muſt haue $\frac{1}{6}$, I take 40. And foꝛ the fourth man ther remayneth but 12, whiche is $\frac{1}{20}$ of the whole ſum, ſo yͭ if the whole pꝛay had been but 240 li, thē were the queſtion anſwered: but bicauſe the ſum was of greater valew, by this meanes now ſhall I know the partition of it: I muſt ſet my nūbꝛes by the order of yͤ Golden rule, putting in the fyꝛſt place the nūbꝛe that I found by multiplying the denominatours, and in the ſecond place the ſum of the booty. And loke what pꝛopoꝛtion is betwene that firſt nūbꝛe & that ſecond, the ſam pꝛopoꝛtion ſhalbe betwene the partes of the firſt number and the partes of the ſecond, comparing eche to his like. therfore I muſt put in the third place, one of the partes oꝛ ſhares, & then worke by the

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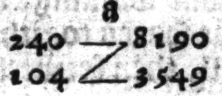
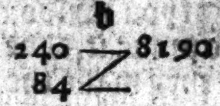
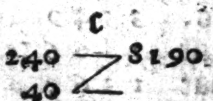
this
le.

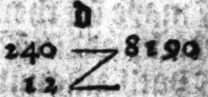
yͤ former rule of proportiō or Golden rule. And bicauſe I haue 4 ſeuerall partes of the fyrſt numbre, by which I woulde fynde out 4 like partes of the ſecond numbre, therfore muſt I make 4 ſeuerall figures.

S. Now I truſt I can anſwer to your queſtion, as by your fauour I will prooue.









And to trye it, I ſette the 4 fygures thus, marked with a, b, c, d, to ſhewe their ordre. And then in eche of theym I multiply yͤ ſecōd nūbre by yͤ third, & diuid their total by yͤ firſt, & ſo amounteth yͤ forth ſũe which I ſeke for; for if I do multiply 8190 by 104, it maketh 851760, which beyng diuided by 240, maketh in the quotient 3549 for the fyrſt mannes portion. And ſo woorkinge with the other three fygures, I fynde for the ſeconde manne 2866 ½, and for the thyrde manne 1365. And then for the fourth manne 409 ½. And ſo is euerye mannes ſhare ſette foorthe in the fygure heere anexed.

a
240 Z 8190
104 Z 3549

b
240 Z 8190
84 Z 2866 $\frac{1}{2}$

c
240 Z 8190
40 Z 1365

d
240 Z 8190
12 Z 409 $\frac{1}{2}$

And thus I thynke I haue doone well.

M. If you myſdoubt your workinge and lyſte to proue it, adde al the ſhares togither; and if they make the totall, then ſeemeth it well doone.

proofe addiciõ.

S. I maye ſet them thus, and then by additiõ the iuſt ſumme doth amounte, that is 8190, and therefore (as you ſaye) it ſeemeth to bee well wroughte.

3549
2866 $\frac{1}{2}$
1365
409 $\frac{1}{2}$
8190

But I beſeeke you, is there any doubte in this tryall, that you vſe that word, Semeth?

Ma. You may eaſily coniecture, that if you dydde aſſigne the fyrſte mannes ſhare to the laſt, and ſo chaunge all the reſt, that one had an others ſhare, yet would the Addition appear al one, & therfore is not the profe exact.

iuſte profe.

But if you will make a iuſte proofe, for the fyrſte mannes parte take $\frac{1}{3}$ $\frac{1}{10}$ of the whole ſumme, and if it agree with the numbre in the fygure, then it is well doone. And ſo doo for the ſecond, thyrd and fourth ſummes, and this profe faileth not. Now wyll

I

I propounde certayn other queſtions, whiche haue been ſet foorth by certaine learned men, all be it not without ſome ouerſyghte. Whiche queſtions I proteſt hartily I do not repeate to depraue thoſe good men, whoſe labours and ſtudies I much praiſe & greatly delyte in, but only according to my profeſſion, to ſeeke out truth in all thinges, and to remooue all occaſions of errour, as much as in me lieth: and for that cauſe I will only name the queſtions without hurting the authors name. The fyrſte queſtion is this.

A queſti
on of bu
ding.

Four men dyd buyld a houſe, whiſh coſt them 3000 crounes, their ſhares were ſuch that one man ſhould paye $\frac{1}{2}$ of the ſumme, & 6 crounes ouer: the ſecond ſhould pay $\frac{1}{3}$, and 12 crounes ouer: the third man muſt lay out $\frac{2}{3}$, abatyng 8 crounes, and the fourth manne ſhould pay $\frac{1}{4}$, and 20 crounes more. can you anſwere to this queſtion?

S. No in good fayth ſyr, & that you know beſt of any man, for I knowe no more then you haue taught me.

M. Than I dare ſaye you can not doo it, nother yet the beſte learned man that euer dyd propoſe it. for the queſtiō is impoſſible: for declaration wherof I will be bold to vſe fyrſt the repreſentation of the noumbres in their apteſt forme, (althoughe I haue not yet taughte you that manner of woorke) bicauſe it maye appeare plainly that the queſtion is not poſſible, for here I haue ſet the partes, and added them, and they make the whole ſum, 1 $\frac{3}{4}$, and 30 more. Now how is it poſſible to diuyde truely either gaines either charges ſo, that the particulars ſhalbe more thē the totall?

$$\left\{\begin{array}{lll} \frac{1}{2} & + & 6 \\ \frac{1}{3} & + & 12 \\ \frac{2}{3} & - & 8 \\ \frac{1}{4} & + & 20 \\ \hline 1\frac{3}{4} & + & 30 \end{array}\right\}$$

S. It is againſt the forme of profe by addition of the partes.

M. You ſay truthe. And bicauſe you ſhall perceaue it the better, I wyll trye it after the vulgare forme, as in this fygure you ſee, where the $\frac{1}{2}$ with 6 ouer, is 1506: for the totall is as you heard before 3000: the $\frac{1}{3}$ and ẏ 2 more, is 1012: the $\frac{2}{3}$ would be 2000, but then abating 8, it is but 1992, and then laſt of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: whiche all beeynge added in one ſumme, do make 5280, where the totall ſum ſhoulde be but 3000. Whiche ſum if you dyuide by $\frac{4}{3}$, ſo ſhall you haue $\frac{3}{4}$ of it, that is 2250. and thereto adde 30 more, then wyll thoſe 3 ſummes make 5280: whereby you maye ſee howe this forme as well as the other, doothe declare that the particulars in that queſtion would make more then the whole ſumme.

$$\begin{array}{r} 1506 \\ 1012 \\ 1992 \\ 770 \\ \hline 6280 \end{array} \qquad \begin{array}{r} 3000 \\ 2250 \\ 30 \\ \hline 5280 \end{array}$$

ſumme by $\frac{1}{4}$, and 30 more, and therfore ca that queſtion not bee accepted as a poſſible thynge. but yet doo certayne learned menne propunde ſuche queſtions, and anſwere to them. Therfore ſomwhat to ſay to their excuſe, rather of their good meaning then for their doing, I will anone declare what may be ſayd for theyr defence: but in the meane ſeaſon I will propounde the queſtion as it maye be wrought by good poſſibilitie. As yf four men buyld a houſe togither, and it coſt them 3000, and then for the partition they agree thus: that as often as the fyrſte man dothe paye 6 crounes, ſo often the ſeconde ſhall paye 4, the third man 8, and the fourth man 3: Or els thus: that the fyrſte man ſhal pay double ſo much as the iiij. & the ſeconde man ſhall pay $\frac{2}{3}$ of the fyrſt mans charg: the thyrd man ſhall paye double ſo much as the ſeconde: (And theſe twoo waies are to one end) but farther for their agreement it is apointed alſo, that the fyrſt man ſhall giue 6 crounes ouerplus, and the ſeconde 12, and the iiij. ſhall giue 20, but the thyrd man ſhall geue no ouerplus, but ſhall haue 8 crounes abated of his charge. Nowe is the queſtion poſſible to be ſoyled, and this is the waye to doo it. Marke the proportion of the ſeueral charges, and ſet out ſmall numbres in that rate, by which you may reduce the work to the Golden rule, as here in the firſt forme, ỹ

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numbres are already named, 6, 4, 8, 3: and in the ſeconde forme, although thei bee not plainly named, yet thei may be the ſame nūbres, for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6: and againe 8 is double to 4. Now adde theſe togither, and they make 21, Whiche 21 muſte be ſet as the fyrſt nūbre in the Goldē rule. for if it with the ouerplus of eche mannes charge wolde make the totall ſumme of the charges, then were thoſe ſeuerall ſummes the charges of ech mā, beſide his ouerplus: but nowe it is not ſo.

The rule. But yet this is true, that loke what proportion eche of theſe ſeuerall ſummes doth beare to 21, the ſame proportion doothe the iuſte charges of euery man (beſide his ouerplus) beare to the totall of the charges, the ouerplus being deducted: wherfore this mai you note, that before you do apply the total of the charges to the Goldē rule, you muſt deduct the ouerplus which is 6, 12, and 20, that is in the whole 38, but then 8 muſt bee reſtored for the abatement of the thyrd mā, & than remaineth to be deducted 30. Take 30 therfore out of 3000, and there will reſt 2970, which I muſte ſet in the Golden rule for the ſecond ſumme: and for the third ſum I muſt put eche of the ſmall numbres before mentioned, whiche althoughe they bee not the ſeuerall charges, yet they repreſente them in proportiō. And ſo making for euery mans charge a ſeueral queſtion, the figures will be 4, whiche I marke with 4 letters, a.b.c.d.thus.

a.
21 2970
6 848 $\frac{4}{7}$

b.
21 2970
4 565 $\frac{5}{7}$

c.
21 2970
8 1131 $\frac{3}{7}$

d.
21 2970
3 424 $\frac{2}{7}$

Where I haue ſet for brieſnes the ſumme of euery mans charge in the fourth place, preſuppoſyng that you can tell how to trye out that fourth ſumme by ſo many examples as ye haue had.

Scho. As I truſte that I doo vnderſtande this forme, ſo I deſyre much to know what maye bee ſayde for them that myſtooke this queſtion.

M. You ſeeme ſo deſyrous to know thys errour, that you haue forgotten to examine whether this worke be without errour.

S. Me ſemeth this worke to be well done, bicauſe the addition of the iiij. ſeuerall numbres dothe make the totall ſumme of 2970, whiche was to bee diuided into ſuche foure partes.

Mayſt. But then haue you forgotten that the fyrſt manne muſte paye 6 crounes more beſyde thys ſhare, and the ſecond man 12

crounes

crownes more: the thyrd man 8 crownes leſſe: and the fourth man 10 crownes more. for w out theſe your fyrſt totall of 3000 crownes wyll not be made.

Scholler. Then muſte I adde to the firſte mannes ſumme 6 more, and it will be 854 $\frac{4}{7}$, & to the ſeconde ſume I muſt adde 12, and it wyll be 577 $\frac{5}{7}$: from the thirde ſumme I muſt abate 8, and then wyll the ſumme be 1123 $\frac{3}{7}$. then adding unto the iiij. ſumme 20, yt will bee 444 $\frac{2}{7}$, and theſe iiii. ſummes will make 3000, which is the whole charg, as in this exãple it mai appere, wher fyrſte I gather yͤ $\frac{14}{7}$, that maketh 12 & ſo procede I in yͤ addition, to yͤ end.

$$\begin{array}{r} 854\frac{4}{7}\\ 577\frac{5}{7}\\ 1123\frac{3}{7}\\ 444\frac{2}{7}\\ \hline 3000 \end{array}$$

Maſter. Nowe haue you well doone. and this worke in the ſame ſummes is brought of other learned men for the true ſolution of the queſtion, as it was fyrſte proponed, which (as I ſayd) was impoſſible, and now examin it by theſe ſeuerall ſummes, and ſee wether it doo agree with the ſummes in the queſtion poponed.

The fyrſte manne muſte paye $\frac{1}{2}$, and 6 ouer of yͤ totall ſumme, howe thinke you, is 854 $\frac{4}{7}$ the halfe, and 6 more of 3000?

Scholer. No that is it not, for it woulde be 1506: and for the ſeconde man 1012: and for the thyrde man 1992: and for the forthe man 770. whereof not one ſumme agreeth to this worke, but I meruayle that ſo wiſe men could be ſo muche ouer ſene.

Mayſter. It is commonly ſene, that whē men wyll receaue thinges from elder writers, and wyll not examine the thynge, they ſeme rather willing to erre with their ancientes for company, then to be bolde to examine their workes or wrytinges. which ſcrupuloſitie hath ingendred infinite errors in all kyndes of knowledge, and in all ciuile adminiſtration, and in euery kynde of arte. but theſe learned men did not meane anye other thing by this queſtion, then to fynde ſuche numbres as ſhould beare the ſame proportion togyther, as thoſe numbres in the queſtion proponed dyd beare one to an other: whiche thinge you ſhall perceaue more plainly by an other queſtion of their, that is this.

A questio
of a testa
ment.

A man lieng on his death bedde, bequeatheth his goodes (whiche wer worth 3600 crownes) in this ſorte. Bycauſe hys wife was great with chylde, and he yet vncertain whether the chylde wer a male or a female, he made his bequeſt conditionally, that if his wife bare a daughter, then ſhuld the wife haue halfe his goodes, and the daughter $\frac{1}{3}$, but if ſhe were deliuered of a ſonne, thā that ſon ſhuld haue $\frac{1}{2}$ of the goodes, and his wyfe but $\frac{1}{3}$. Now it chanced hir to bring forth both a ſonn and a daughter, the queſtion is, how ſhall they part the goodes agreable to the teſtatour his wyll?

Scholer. If ſomme cunning lawyers had this matter in ſcanning, they woulde determine this teſtament to be quite voyde, and ſo the man to die vnteſtate, bycauſe the teſtament was made vnſufficient, ſyth this condition was not expreſſed in it. and alſo it myght haue chaunced that ſhe ſhould haue broughte foorth neither ſonne nor daughter, as often hath ben ſene, ſo is the wil vnſufficient in that poynt alſo.

Mayſter. Suche ſcanners ſhould ſeeme to cunnyng, and yet not ſo cunning as cruell, for the mynd of the teſtatour is to be taken fauorably, for the ayd of the legataries, whẽ ther ryſeth ſuche doubtes: But let vs trye thys worke not by force of lawe, but by proportion arithmeticall, ſeeyng the teſtatour dyd mynd to prouide for eche ſorte of theim.

Scholler If the ſonne ſhall haue $\frac{2}{3}$ by force of the teſtament, ſo muſt the mother haue $\frac{1}{3}$. Agayn bycauſe ſhe hath a daughter alſo, therfore ought ſhe to haue $\frac{2}{3}$, & the daughter $\frac{1}{3}$: ẏ is bothe waies $\frac{2}{3}$ $\frac{1}{3}$ and $\frac{2}{3}$ $\frac{1}{3}$, whiche cometh to the hole goods, and $\frac{1}{3}$ more. Wherfore it ſemeth alſo vnpoſſible.

Maſter. In this matter the mynde of the teſtator is ſo to be vnderſtand, that ſuche proportion ſhoulde bee betwene the portion of the wyfe and the ſonne, as is betwene ½ and ⅓, that is, the ſonne muſt haue $\frac{3}{2}$, for $\frac{3}{2}$ to his mother, ſo ſhall he haue 3 to 2, that is as much as his mother & halfe as muche more: and the mother muſte haue the lyke rate in compariſon to her doughter. Then muſte I find out 3 noumbres in ſuche proportion, that the firſt maye be as muche as the ſecond, and halfe as much more, that is in proportion *ſeſquialtera*, and the ſecond to the third in the ſame proportion, ſuch numbres be 9, 6, 4.

Scholer. I pray you ſyr, how ſhal I find out thoſe numbres?

Maiſter. That will I gladlye tell you: What ſoeuer the proportion be for anye 3 numbres, multiply the termes of that proportion togither, & the numbre that amounteth ſhalbe the myddle numbre of the 3: the multiply that myddle numbre by the leſſer terme, and diuide that totall by the greter, and the leaſt numbre of the 3 will amount. So if you multiply that mydle numbre by the greater extreme, and diuide that totall by the leſſer extreme, then will the greateſt noumbre of that progreſſion amount.

To find
numbers i
any propo
cion.

Scholer. Then in this example, to fynde the proportion of ½ to ⅓, I muſt diuyde (as you taught me in diuiſion) ½, by ⅓, and the quotient wyl be $\frac{3}{2}$, that is 1 ½. whereby I perceaue that the proportion in this queſti-

P.i. on

on is, as 3 to 2. Therfore (as you taught me euen now) I multiply 3 by 2, and the ſume is 6, which muſt be the middle numbre: the I multiply yͤ middle numbre 6 by 2, whiche is the leaſte tearme, and the ſumme is 12, that doo I diuide by 3, beinge the greater terme, and the quotient is 4, ſo is 4 the leaſt numbre of the 3: then I multiplye 6 by 3, wherof cometh 18, and that I diuide by 2, and ſo haue I 9, which is the greateſt nũbre of the 3.

Ma. An other way yet may you finde the thirde numbre in anye progreſſion, yf you haue 2 of them, for yf the middle numbre be one of them which you haue, then multiply it by it ſelfe (as in this example 6 by 6 maketh 36) and that totall diuide by the other numbre which you haue, and the third numbre will be the quotient.

Sch. Then if I diuide 36 (which cometh of 6 multiplied by it ſelfe) by 4, the quotyent will be 9: and if I diuide 36 by 9, the quotient wil be 4. But what if I know the fyrſte numbre and the third, and woulde haue the middle numbre?

M. Multiply the ii. numbres togither, and in their total you muſt ſeke the roote of that numbre, and it ſhal be the middle numbre: but becauſe as yet you haue not lerned how to extract roots, therfore vſe the firſt forme

whiche

which I haue taught you, till I teache you to extract rootes. And nowe go forewarde with the anſwere to the queſtion.

S. I perceaue then that the ſonne muſte not haue $\frac{1}{2}$ of the goodes, nother the mother $\frac{1}{3}$, nor yet the doughter $\frac{1}{3}$, but yet muſt the goodes be deuided in ſuch proportion, that the ſonne ſhal haue 9 crownes for 6 to his mother: and the mother ſhal haue 6 crounes for euery 4 to the doughter. Then I applye it to the Goldē rule in 3 exāples thus. wher the firſt numbre is the addition of thoſe 3 nūbres, 9, 6, 4, and the thirde is one of thē ſeuerally, the ſecond is the total of the goodes in the teſtament. & then by the worke of yͤ goldē rule, I find out the fourth numbre in euery worke, that is for the ſonne 1705 $\frac{5}{19}$: for the mother 1136 $\frac{16}{19}$ and for the doughter 757 $\frac{17}{19}$, whiche thre ſummes added togither do make the ſūme of the whole goodes, as may be ſene by this example.

19 3600
9

19 3600
6

19 3600
4

1705 $\frac{5}{19}$
1136 $\frac{16}{19}$
757 $\frac{17}{19}$
3600

And this (me thinketh) I doo perceaue yͭ bicauſe in this caſe ther is a neceſſary remedy deuiſed agaīſt an vrgēt incōuenience, therfore thoſe lerned mē thought thei might vſe the like libertie in that other queſtion.

Ma. Your geſſe is good, but they had ſo good reaſon for them in the one, as they haue in the other: as in an other example of theirs it may better appere, yͤ is thys.

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teſtamēt,

A mā left unto his iij. ſonnes 7851 crounes, to be parted in this ſort, that the firſt ſonne ſhould haue $\frac{1}{2}$, the ſeconde ſonne $\frac{1}{3}$, and the third ſonne $\frac{1}{4}$ which is not poſſible, for $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ doth make $\frac{26}{24}$: or $\frac{13}{12}$, that is 1 $\frac{1}{12}$ ſo is it more then the hole: but reduce theſe fractions into one denomination, and they wyll be $\frac{6}{12}$, $\frac{4}{12}$ $\frac{3}{12}$. and ſo maye you parte the goodes in ſuch proportion as theſe 3 numeratours beare togyther: that is, the fyrſt to haue 6 for euery 4 to the ſecond: and the ſecond to haue 4 as often as the thirde hath 3: and ſo their portions will be for the fyrſt 3623 $\frac{7}{13}$: for the ſecond 2415 $\frac{9}{13}$: and for the third 1811 $\frac{10}{13}$ and thoſe iij. ſhares added togither wil make the total ſumme of the hole goodes, as you mai eaſyly ſe in this example.

3623 $\frac{7}{13}$
2415 $\frac{9}{13}$
1811 $\frac{10}{13}$
7851

An other
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on.

An other queſtion is there proponed thus. There is 450 crounes to bee diuided betwene 3 mē, ſo that the fyrſt man muſt haue $\frac{1}{2}$ $\frac{1}{3}$, the ſecond man $\frac{1}{3}$ $\frac{1}{4}$, the thyrd man ſhall haue $\frac{1}{4}$ $\frac{1}{5}$.

Scholer. I meruaile that any manne ſhulde be ſoo ouerſeene to propound that queſtion as a thinge poſſible, ſith $\frac{1}{2}$ $\frac{1}{3}$, $\frac{1}{3}$ $\frac{1}{4}$, $\frac{1}{4}$ $\frac{1}{5}$, do make 1 $\frac{13}{15}$, that is almooſte double the whole ſumme.

But I perceaue it might be thus propoſed, that as often as the firſte man dyd receaue 50 crounes, ſo often the ſeconde man ſhuld receaue 35, and the third man 27, for $\frac{1}{2}$ $\frac{1}{3}$, is equall to $\frac{50}{60}$: and ſo is $\frac{1}{3}$ $\frac{1}{4}$ equal to $\frac{35}{60}$, and $\frac{1}{4}$ $\frac{1}{5}$ is $\frac{27}{60}$, ⁊ ſo working the queſtion, the 3 fygures wyll appeare in thys forme; wherby the fyrſte mans portion is found to be 200 $\frac{50}{56}$: the ſecõd mãs part is 140 $\frac{35}{56}$: the third manns ſhare is 108 $\frac{27}{56}$: which in the whole doth make 450 crounes, that was the whole ſumme to be diuided betwene them.

112 Z 450
50 200 $\frac{50}{56}$

112 Z 450
35 140 $\frac{35}{56}$

112 Z 450
27 108 $\frac{27}{56}$

M. And thus you are (I thinke) ſufficyently inſtructed in the rule of felowſhyp,

The Rule of Alligation.

Nowe wyll I go in hand with the rule of Alligation, which hath his name, for that by it there are diuers parcels of ſundry prices, and ſundry quantities allegate, bounde, or myxed togyther, wherby alſo it might be well called the rule of Myxture, and it hath great vſe in compoſition of medicines,

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dicines, and alſo in myxtures of mettalles, and ſome vſe it hath in myxtures of wines, but I wiſhe it were leſſe vſed therin than it is now a daies. The order of ÿ rule is this: When any ſummes are proponed to be mixed ſet them in order, one ouer an other, and the common numbre wherunto you wyll reduce thẽ, ſet on the left hand: then marke what ſummes be leſſer then that common numbre, and which be greater, and with a draught of your pen, euermore linke .ii. nũbres togither, ſo that one be leſſer then the common numbre, and the other greter then he, for two greater or two ſmaller can not well be linked togyther, and the reaſon is this, that one greater and one ſmaller may be ſo myxed, that they will make the meane or common numbre very well: but .ii. leſſer can neuer make ſoo manye as the common numbre, being taken orderly: nor more can two ſummes greater than the meane, neuer make the meane in due order, as it ſhall appeare better to you hereafter. And as it is of neceſſitie to linke euery ſmaler ones (at the leaſt) with one greater, and euery greater with one ſmaller, ſoo it is at libertie to lynke them oftener then ones, and ſo may there be to one queſtion many ſolutions. When you haue ſo linked them, then marke howe muche eche of the leſſer numbres is ſmaller then the meane or common num-

The reaſon of this rule.

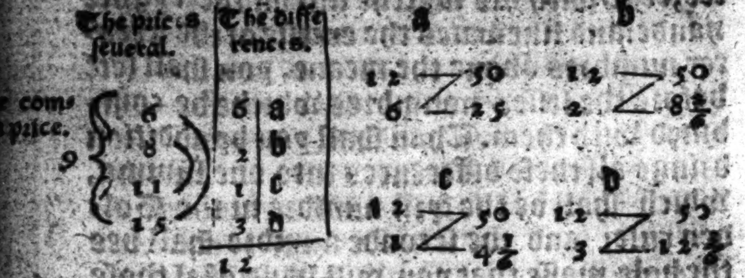
bre, and that difference ſet agaynſt the greater numbres which be linked with thoſe ſmaller, eche with his matche ſtill on the ryghte hande: and likewaies the exceſſe of the greater numbres aboue the meane, you ſhall ſette before the leſſer noumbres whiche be combined with them. Than ſhall you by addition bringe all theſe differences into one ſumme, which ſhall be the fyrſt numbre in the Golden rule: and the ſeconde numbre ſhall bee the hole maſſe that you will haue of al thoſe particulars: the third ſumme ſhalbe eche difference by it ſelfe, and than by theym ſhall be founde the fourthe noumbre, declaryng the iuſte portion of euery particular in that mixtur. As now by this examples I wil make it playne.

A queſtio mixinge wyne.

There is .iiii. ſortes of wyne of ſeuerall prices, one of 6 d a galon, an other of 8 d, the thyrde of 11 d, and the fourth of 15 pens the galon, of all theſe wines woulde I haue a mixture made to the ſumme of 50 gallons, and ſo that the price of eche galon maye be nine pens. Nowe demaunde I howe muche muſte bee taken of euerye ſorte of wyne?

Scholar. If it ſhall pleaſe you to worke the firſt example, that I may marke the applyeng of it to the rule, then I truſte I ſhall bee able not onelye to doe the lyke, but alſo to ſe reaſon in yͤ ordre of the work.

Maiſter. Marke then this forme, and the placinge of euerye kynde of noumbre in it.


The prices ſeueral.
The differences.
e com-
n price.
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4 1/6
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3
50
12 3/6

Here (you ſee) I haue ſet downe the ſeuerall prices which be 6, 8, 11, 15, and haue linked togither 6 with 15, & 8 with 11. the comõ price 9 I haue ſet on the left ſide, and the difference betwene it, and euery particuler price, I haue ſet on yͤ right hand, not againſt the ſumme whoſe difference it is, but againſt the ſumme that it is lynked withal: for the difference of 15 aboue 9 is 6, which I haue ſet not againſt 15, but againſt 6, that is linked with 15, and the difference betwene 6 and 9, (that is 3) I haue ſet againſt 15. So lykewayes the difference betwene 8 and 9 is but 1, that haue I ſet agaynſt 11, and the difference of 11 aboue 9 (which is 2) I haue ſet againſt 8. Then adde I all thoſe 4 differences, and they make 12, which I ſette for the fyrſt numbre in the Golden rule: the ſecond numbre I make 50, which is the ſumme of the galons that I woulde haue, and the thirde ſumme is euery particular difference. Now if you worke by the Golden rule, you ſhall fynde the numbre of galons that ſhall be taken of eche ſorte of wyne: for the better diſtinction wherof, I haue ſet theſe letters, a b, c, d, bothe againſt the numbres for which the workes doo ſerue, and ouer the workes alſo, which ſeuerally ſerue for eche of them. And now if you lyſt to examine the truth of theſe workes, adde thoſe iiij. ſumes togither, and they wil make 50, that is the total which I wold haue, as by this example you maye eaſylye perceue. And for to proue how the prices do agree, do this. Multiplye this totall ſumme 50 by the cōmō price 9, and it wyll make 450: than kepe that ſumme by it ſelfe, and afterward multiplye euery ſeuerall ſumme of galons, by the price belongynge to the ſame galons, and if that ſum do agree with this, whiche you haue kept fyrſt, then is your work well done. As here, 25 is the numbre of galons of 6 d price, multiplye then 25 by 6, and it maketh 150, which you ſhal ſet downe: then multiply $8\frac{1}{3}$ by 8, whiche is the price for that numbre of galons, and it wyll make $66\frac{2}{3}$: ſoo agayne

The pro
of this

25
$8\frac{1}{3}$
$4\frac{1}{6}$
$12\frac{1}{2}$

50

150
$66\frac{2}{3}$
$45\frac{5}{6}$
$187\frac{1}{2}$

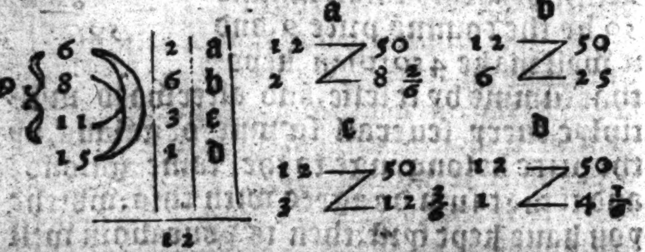
450

againe 4 $\frac{1}{6}$, multiplyed by 11, dothe make 45 $\frac{5}{6}$. And laſte of all 12 $\frac{3}{6}$ multiplied by 15, makethe 187 $\frac{3}{6}$. All theſe added togither dothe make 450, as in the example annexed you may ſe. Wherfore ſeing it doth agre with the former ſumme of 50 multiplied by 9, I may iuſtly affyrme this worke to be good and well doone.

he vari- ion of que- ion.

And now to proue howe you can doo the like, I propounde the ſame queſtion, onely willing you to vſe ſome other forme of combinyng or lynkyng the ſummes.

Scholer. That ſhall I proue with youre fauour, and therfore I combine 8 with 15, and 6 with 11, and then the forme wyll be thus.



Whereby amounteth the ſame ſumme in totall of the differences, as dyd before: and yet now yͤ differences be altered, as the cōbination is chaunged, whereof I vnderſtand the reſon by your former worke. And therefore here appeareth no ſtraunge thing but that now I muſt haue 8 $\frac{2}{6}$ galons, of 6 6, and 25 galons of 8 ð, and 12 galons, and

3

$\frac{3}{6}$ of 11 d, and ſo conſequently 4 galons and $\frac{1}{6}$ of 15 d. ſo that multiplyng 8 $\frac{2}{6}$ by 6, it maketh 50, and then 25 multiplyed by 8, maketh 200. likewaies 12 $\frac{3}{6}$ multiplied by 11, yeldeth 137 $\frac{3}{6}$, and 4 $\frac{1}{6}$ multypplied by 15, maketh 62 $\frac{3}{6}$, whiche iiii. ſummes added in one, will yelde in the totall 450, whiche agreeth with the multiplication of 50 (being the totall ſume of galons) by 9 the common or mean price.

50
200
137 $\frac{3}{6}$
62 $\frac{3}{6}$
450

Mayſter. Seing you conceaue this worke ſo wel, I will propound an other example unto you of more varietie in the alligations or combinynges: As thus.

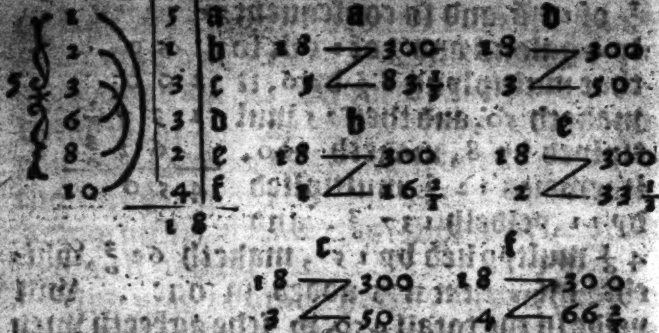
A marchaunt beynge mynded to make a bargaine for ſpices in a myxte maſſe, that is to ſaye, of cloues, nutmigges, ſaffron, peper, ginger, and almoundes, the cloues being at 6 s̃ a pounde, the nutmigges at 8 s̃, ſaffron at 10. s̃. pepper at 3. s̃. ginger at 2. s̃. and almoundes at 1. s̃.

A que
of Spi

Now woulde he haue of eche ſorte ſome, to the valewe of 320 li in the whole, and eche pounde one with an other to beare in price [illegible] s̃. howe muche ſhall hee haue of eche ſorte?

Scholar. That will I trye thus.

Fyrſte I ſette downe thoſe ſyxe ſeuerall prices, and at the lefte hand


5
1
2
3
6
8
10
5 a
1 b
3 c
3 d
2 e
4 f
18
a
18 300
1 83 1/3
d
18 300
3 50
b
18 300
1 16 2/3
e
18 300
2 33 1/3
c
18 300
3 50
f
18 300
4 66 2/3

I ſet the common price 5 ſ. Then I lynke them thus, 1 with 10, 2 with 6, & 3 with 8.

M. I had minded to haue combined thē in more varietie, but I am content to ſe youre owne worke fyrſte, and then more varieties in combination may folow anone.

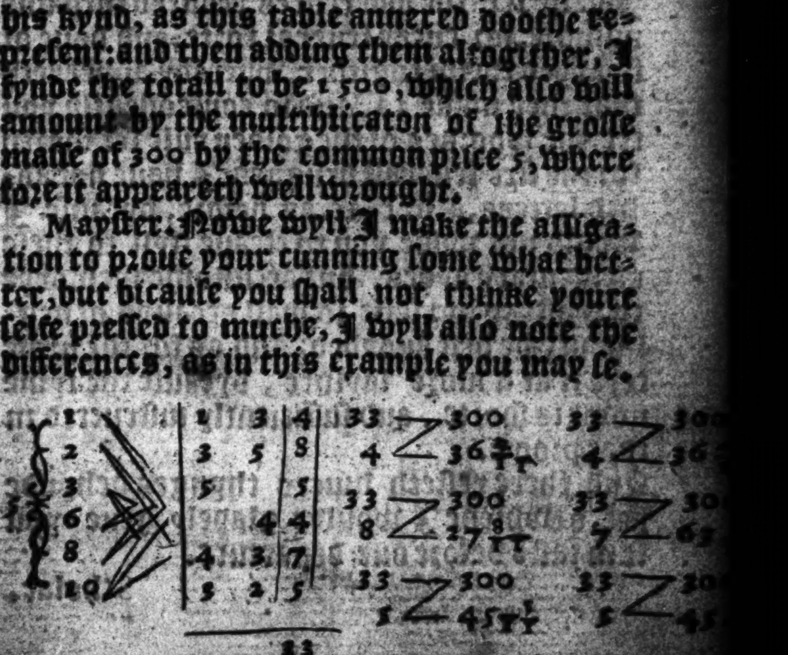
Schol. Then to contineue, as I beganne, I ſeke the difference betwene 1 and 5 (whiche is 4) and that I ſet agaynſt 10: then agaynſt 1 I ſet 5, whiche is the exceſſe of 10 aboue 5: ſo I gather the difference betwene 2 and 5, which is 3, and that I ſet againſt 6, bicauſe it is combined with 2: and likwayes ẏ difference of 6 aboue 5 (which is 1) I ſet agaynſt 2. Then take I the difference of 3 from 5, which is 2, and that I ſet agaynſte 8, and before that 3 I ſet the difference of 8 aboue 5, which is 3. Then gather I al theſe differences by Addition, and they make 18, whiche I ſette for my fyrſte numbre in the golden rule, and ſo appeareth by thoſe workes, that of almondes I muſt take 83 li $\frac{1}{3}$, of gynger 16 li $\frac{2}{3}$, of pepper 50 li, of cloues 50 pounde, of nutmigges 33 pounde $\frac{1}{3}$, and of ſaffron 66 pound $\frac{2}{3}$.

Then for triall hereof, I multiplye euery parcell by his ſeuerall price, as $83\frac{1}{3}$ whiche is the ſume of almones, I multiplie by 1, which is their price.

$83\frac{1}{3}$
$33\frac{1}{3}$
150
300
$266\frac{2}{3}$
$666\frac{2}{3}$
1500

Alſo $16\frac{2}{3}$ the ſumme of ginger, I multyplie by 2, which is the price of it. And ſo eche other in his kynd, as this table annexed doothe repreſent: and then adding them altogither, I fynde the totall to be 1500, which alſo will amount by the multihlicaton of the groſſe maſſe of 300 by the common price 5, wherefore it appeareth well wrought.

Mayſter. Nowe wyll I make the alligation to proue your cunning ſome what better, but bicauſe you ſhall not thinke your ſelfe preſſed to muche, I wyll alſo note the differences, as in this example you may ſe.



5
1
2
3
6
8
10
1 3 4
3 5 8
5 5
4 4
4 3 7
5 2 5
33
33 300
4 $36\frac{4}{11}$
33 300
8 $27\frac{8}{11}$
33 300
5 $45\frac{5}{11}$
33 30
4 36
33 30
7 63
33 30
5 45

Where I haue alligate 1 with 6 and 8, and therfore haue I sett agaynſt 3, both their differences, that is 1 and 3. Likwayes bicauſe 2 is combined with 8 & 10, I ſet before him their differences 3 and 5. Agaynſt 3 I haue ſet onely 5, whiche is the difference of 10, wᵗ whom 3 is combined onlye: likewayes 6 is only alligate to 1 , and therefore is the difference of 1 only ſet agaynſt it; 8 is linked with 1 and 2, & therfore hathe he agaynſt him ther both differences 4 & 3: & 10 is ioined wᵗ 2 & 3, therfore hath he their differences 3 & 2. And bicauſe of eaſe for you, in an other columne I haue ſet the differences reduced into one numbre, for euery ſeueral ſorte, and haue alſo added theim togither, whereby appeareth that they make 23. And ſo conſequently you ſee the woorkes of the golden rule ſet foorth for the ſixe ſeuerall drugges: I haue not added letters, a, b, c, &c. as before, bycauſe I would not wiſhe you to cleaue ſtyll to theſe elementary aydes, but accuſtome memorye to truſte to hyr ſelfe, ſo ſhall occaſion of negligence be beſt auoyded. And as for the profe trye it at a more layſure, bycauſe the tyme nowe is ſhorte, and ſufficiently inſtructed in that proofe.

And there reſteth diuers thynges behynde yet, of whiche I woulde gladely geue you ſom taſte before our departure.

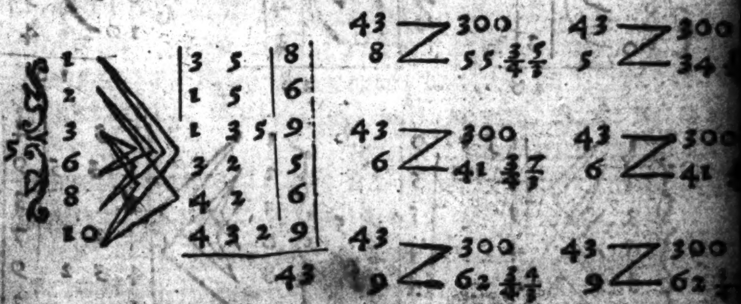
Scholar. But yf it maye pleaſe you to let me ſee all the variations of this queſtion, before you go from it, for me thinketh I could varie it ij. or three wayes more yet.

Maſter. I am content to ſe you make two or thre variations, but I woulde bee loothe to ſtaye to ſee all the variations, for it maye be varied aboue 300 waies, althoughe many of them would not well ſerue to this purpoſe.

Scholar. I thought it impoſſible to make ſo many variations.

Maſter. Meruaile not there at, for ſo much queſtions of this rule may be varied aboue 1000 waies: but I woulde haue you forget ſuche fantaſies, tyll a tyme of more leyſure. And now go forward with ſome variation of this queſtion.

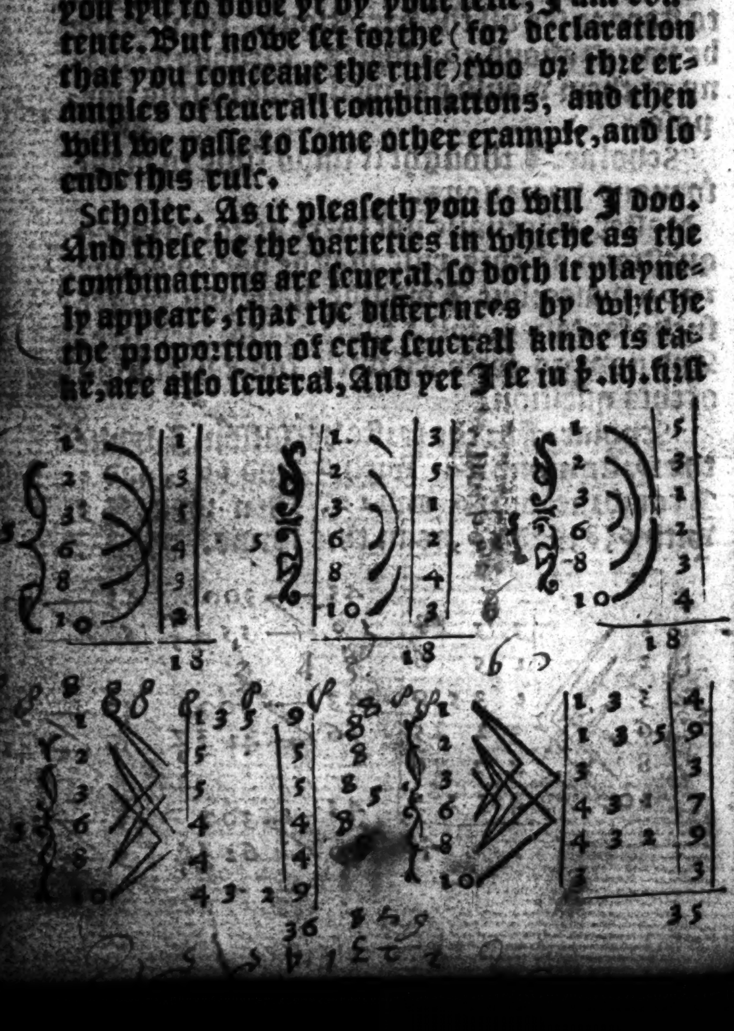
Scholar. For the fyrſte variation, I lynke the fyrſte numbre 1 with 8 and 10, and 2 I combine with 6 and 10, then ioyne I 3 with 6, 8, and 10, as in this forme.


5
1
2
3
6
8
10
3 5 | 8
1 5 | 6
1 3 5 | 9
3 2 | 5
4 2 | 6
4 3 2 | 9
43
43 300
8 55 35/43
43 300
5 34
43 300
6 41 37/43
43 300
6 41
43 300
9 62 34/43
43 300
9 62

And ſo dooth there appeare the portion of weyght for euerye kinde of drugge in thys myxture. Nowe for the triall.

Ma. Naye ſtaye there, you ſhall not nede to make triall in one example ſo often: or if you lyſt to dooe yt by your ſelfe, I am contente. But nowe ſet forthe (for declaration that you conceaue the rule) two or thre examples of ſeuerall combinations, and then will we paſſe to ſome other example, and ſo ende this rule.

Scholer. As it pleaſeth you ſo will I doo. And theſe be the varieties in whiche as the combinations are ſeueral, ſo doth it playnely appeare, that the differences by whiche the proportion of eche ſeuerall kinde is take, are alſo ſeueral, And yet I ſe in ye .iiij. firſt

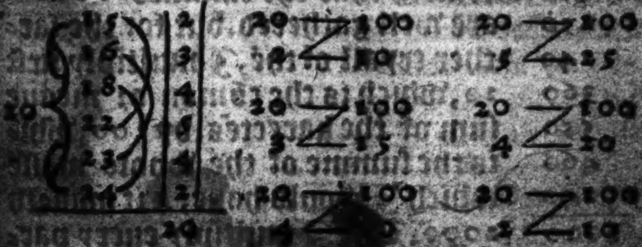


of theſe v. varieties, and in one other before, the totall ſumme of the differences to be one, that is to ſay 18, wherby I perceiue that the varietie of their mixture dooth depend of the varietie of their differences ſeuerall, and not of the varietie of their totall ſumme.

Ma. So is it. And ſeeing you conceaue it ſo well, I wil make an end of this rule, only exhibiting to you one queſtion or twoo of the mixture of metals, that by it you maye deuiſe other like, and exerciſe your ſelf therein alſo, bycauſe the vſe of it ſerueth often in buſyneſſe of charge, not ſo muche for goldſmythes, as for coynage in myntes. Firſt I demaund of you this queſtion. If a myntemaſter haue golde of 22 karectes, and ſome of 23 karectes, ſome of 24: Again ſome of 15, ſome of 16, and ſom of 18 karectes, and woulde myxe them ſo, that he myght haue 100 vnces of 20 karectes, how muche ſhall he take of euery ſorte?

A queſti- of myn- of Gold.

S. To knowe that, I ſet the numbres in ordre thus.



M. You haue wrought the queſtion well, but howe chaunced you made no doubte of that newe name Karecte.

Sc. Bicauſe I thought it oute of tyme to demaund ſuche queſtions nowe, ſeynge you make ſo muche haſte to ende: and againe in this caſe the proportion of the numbres is ſufficient for my pourpoſe in this worke, truſtynge that an other tyme you will inſtructe me as well of this, as of ſundrye other thynges, whiche as I haue heard you to talke of, ſo I haue a great deſire to know them.

M. Your anſwer is reaſonable: and your requeſt and truſt with gods help I intende to ſatiſfy. And now to go forward with this matter, let me ſee your examination of this laſte worke.

S. Fyrſt for the one part, I adde togyther all the particular ſumes as they appeare in the worke, and thei make 100, as here by their addition it dooth appeare.

10
15
20
25
20
10
100

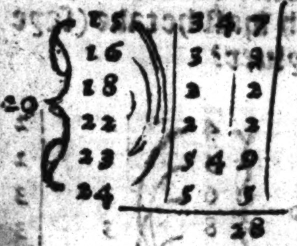
And ſo it ſeemeth that the ſummes are well gathered. but for the farther tryall of thē, I multiply firſt 20, which is the cōmon or meane ſum of the karectes, by 100, whiche is the ſumme of the whole maſſe which I would haue, & it maketh 2000. Then I multiply euery particular

150
240
360
550
460
240
2000

ticular ſumme by the korectes that it dooth contain, as 10 by 15, and that maketh 150. Likewiſe I multiply 15 by 16, & it yeldeth 240: ſo 20 by 18 maketh 360. And 15 by 22 yeldeth 550: like waies 20 by 23 bryngeth forth 460: and laſte of all, 10 multiplied by 24, yeldeth 240: which ſums al ioyned togither make 2000, that dooth agree with the like ſumme before. wherfore I may wel ſay that the worke is good. And now if it pleaſe you, I would ſet forth ſome varieties of this queſtion, to proue my witte.

Maſter. Go to, lette me ſee.

Scholar. Here be foure varieties.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18
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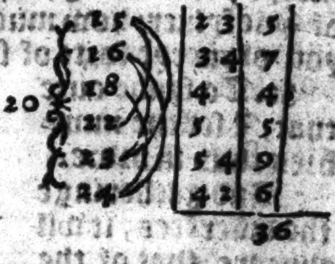
24
3 4 7
3 2
2 2
2 2
5 4 9
5 5
28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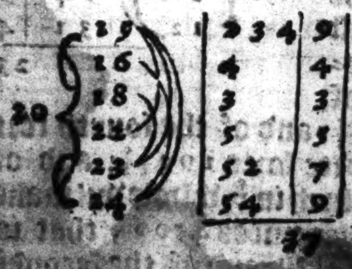
5 4 9
4 2 6
36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2 3 4 9
4 4
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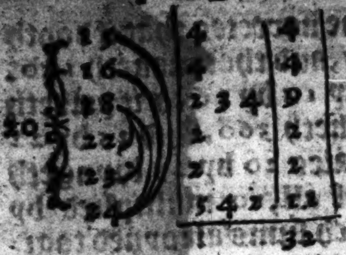
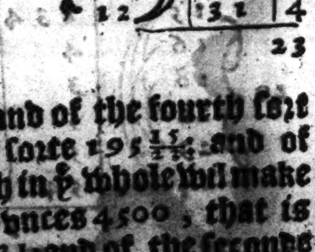
5 5
5 2 7
5 4 9
37

And more yet I could make, but not lyke to the noumbre that you ſpake of in the variation of the other queſtion.



Ma. That wyll I teach you at more leaſure, ſeeynge it is a thynge rather of pleaſure, then of anye neceſſitye. But now for your exerciſe in this rule one other queſtion I wyll propoſe. A mynte maiſter hath 6 ingottes of ſiluer of ſundrye fineneſſe, ſome of 4 ounces fyne, and ſome of 5 unces, ſome of 6, and other of 8; ſome of 11, & other of 12: and his deſyre is to myxe 500 pound weight, ſo that in the hole maſſe euerye pound waight ſhould beare 9 unces of fyne ſyluer: how much ſhall he take (ſaye you) of euerye ſorte of ſyluer?

estion ixing Siluer.

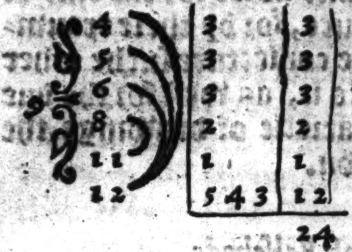


Sc. To fynde oute that, I ſet the numbres thus in ordre.

And gatheringe the differẽces, it wil appeare, that of the fyrſte ſort ther muſt bee 43 $\frac{11}{23}$: of the ſecond lyke muche: of the thyrd ſort, 65 $\frac{5}{23}$: and of the fourth ſort as muche. of the fyfte ſorte 195 $\frac{15}{23}$: and of yͤ ſyxt ſort 86 $\frac{22}{23}$. which in yͤ whole wil make 500 li waighte: and in unces 4500, that is of the fyrſte ſorte 173 $\frac{21}{23}$: and of the ſeconde ſorte ſort 217 $\frac{9}{23}$: of the third ſorte 391 $\frac{7}{23}$: of the fourth ſort 521 $\frac{17}{23}$: of the fyft ſort 2152 $\frac{4}{23}$, and of the ſyxt ſorte 1043 $\frac{11}{23}$: whiche all togither doo make 4500 vnces, agreeable to ye multiplication of 9 by 500.

Ma. This is well doone of you, therefore nowe make three or foure varieties, and ſo an ende of this rule.

S. Theſe 4 varieties I ſette for example.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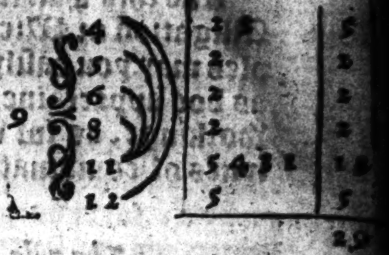
12
3 3
3 3
3 3
2 2
1 1
5 4 3 12
24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12
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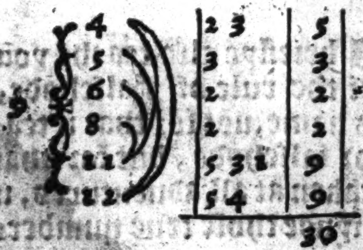
2 2
2 2
2 2
5 4 3 1 1
5 5
29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4
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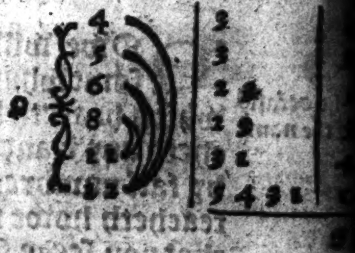
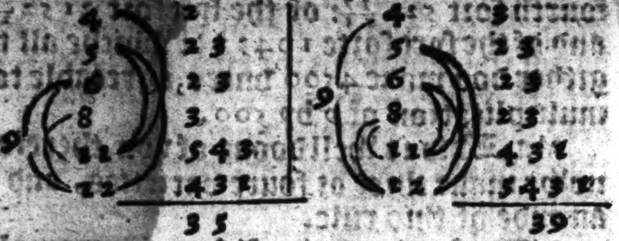
6
8
11
12
2 3 5
3 3
2 2
2 2
5 3 1 9
5 4 9
30


9
4
5
6
8
11
12
2
3
2 2
2 9
3 1
9 4 3 1

And by theſe it appeareth that you cã find out more, with which I wyll not now medle, ſaue only for to ſhewe you an eaſy helpe in drawing the lines of Combinatiõ, I will ſette forthe two varieties here.

And



And this ſhall ſuffiſe nowe for the rule of Alligation or Mixture, for by theſe exaumples may you eaſilye coniecture ſuche other as doo appertaine to it, as well for the due workinge, as for varietie of drawinge the lynes of combination.

The rule of Falſehode.

The occaſiõ of the name

NOwe will I brieflye alſo teache you ſomewhat of the rule of Falſehode, which beareth his name, not for that it teacheth any fraude or falſehode, but for that by falſ numbres taken at all aduentures, it teacheth howe to fynde thoſe true numbers that you ſeeke for.

S. So myght any other rule be called the rule of Falſhode, for they worke by wronge numbers, and by them find out the right nũbres: ſo doth the rule of Alligation, the rule of Fellowſhip, and the Golden rule partly.

M.

Maſter. In the Golden rule, the rule of Fellowſhip, and the rule of Alligation, althoughe the numbres that you woorke by, be not the true numbres that you ſeeke for, yet are they numbres in iuſt proportion, and are founde by ordrely woorke: where as in this rule the numbres are not taken in any proportion, nor founde by ordrelye woorke, but taken at all aduentures.

And therfore I ſometimes beyng merrye with my friendes, and talking of ſuche queſtiōs, haue cauſed them that proponed ſuche queſtions, to call vnto them ſuche chyldren or ydeotes, as happened to bee in the place, and to take their anſwere, declaring that I wolde make theym to ſolue thoſe queſtions that ſeemed ſo doubtfull.

And in deed I did anſwere to the queſtiō and woorke the tryall thereof alſo, by thoſe anſweres whiche they happened at all aduentures to make: whiche numbres ſeeyng they be taken as manifeſt falſe, therefore is this rule called the rule of Falſe poſitions, and for brieſneſſe, the rule of Falſhode: whiche rule for readyneſſe of remembraunce, I haue compriſed in theſe fewe verſes folowinge, in forme of an obſcure Ryddle.

Geſſe at this woorke as happe doth leade.
By chaunce to truthe you may procede.
And firſte woorke by the queſtion,
Although no truthe therein be don.
Suche falſehode is ſo good a grounde,
That truth by it will ſoone be founde.
From many bate to many mo,
From to fewe take to fewe alſo.
With to muche ioyne to fewe againe,
To to fewe adde to manye plaine.
In croſſewaies multiplye contrary kinde,
All truthe by falſehode for to fynde.

The ſence of theſe verſes, and the ſumme of this rule, is this.

When any queſtion is proponed appertayning to this rule, firſt imagine any numbre that you lyſte, which you ſhall name the Firſt poſition, and put it in ſteed of the true numbre, and then worke with it as the queſtion importeth: and if you haue miſſed, then is the laſte numbre of that woorke other to great or to little, that ſhall you note as herafter ſhall be taught you, and you ſhall call it the firſt errour.

Then begynne againe, and take an other numbre, which ſhalbe called the Second poſition, and worke by the queſtion: if you haue myſſed againe, note the exceſſe or default as it is, and cal that the Second errour. Then multiplye croſſe wayes the fyrſte poſition by the ſeconde errour, and againe the ſeconde poſition by the fyrſte errour, and note theyr totalles ſeuerally by the names of Totalles. Then marke whether the two errours were bothe lyke, that is to ſaye, bothe to muche, or both to lyttle: or whether they be vnlike, that is, the one to muche, and the other to lyttle. for if they be lyke, then ſhall you ſubtracte the one totall from the other (I meane the leſſer from yͤ greater) and the remayner ſhall bee your dyuidend. ſo muſte you abate the leſſer errour out of the greater, and the reſydue ſhall be the diuiſour. Nowe dyuide the diuidende by that diuiſour, and the quotiente will ſhewe you the true numbre that you ſeeke for. But and if the errors bee vnlyke, then muſte you adde bothe thoſe totals (whiche you noted) togither, and take that whole numbre for the diuidend: ſo ſhall you adde bothe the errours togither, and that whole numbre ſhalbe the diuiſour. And the quotient of that diuiſion ſhall giue you the true noumbre that the queſtion ſeeketh for. And this is the whole rule.

S. This rule ſemeth ſo vnlyke any other, that without ſome example I ſhall not eaſily vnderſtand it.

M. Therfore take this example: A maſon was bounde to buyld a wal in 40 daies, and

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it was couenaunted ſo with him, that euery daye that he wrought he ſhuld haue for his wages 2 ß, 1 penye, and euerye day that hee wrought not, he ſhoulde bee amerced 2 ſhyllynges, 6 pence. ſo that when the wall was made, and the reckening taken of the daies that he wrought, and of the other that hee wrought not, the maſon hadde cleerely but 5 ß, 5 pence for his worke. Nowe do I demaunde howe many daies dyd hee worke of thoſe 40, and how many dyd he not worke?

Sc. I praye you expreſſe the ordre of the worke, that I may partly by imitatiõ, and partly by comparing it with the rule, bee able againe to do the lyke.

M. This ordre ſhall you kepe in the worke of this rule. firſte take ſome numbre as you lyſte at aduenture: as for example, I ſay he plaied 12 dais, and wrought 28 dais. Now caſte you the wages of euery daye, and ſee whether it will agree with the ſumme of 5 ß 5 pence.

S. The 28 daies that he wroughte, after 25 pence by the day, yeldeth 700 pens. Thẽ the 12 daies that he wrought not, at 30 pẽs eche day, doth amount to 360 pence, which if I abate out of 700, ther reſteth 340: but you ſaye he had not ſo muche.

Maſt. He had but 65 pence, and by thys ſuppoſitiõ he ſhuld haue had 340: therfore is this ſumme to much by 275, whiche ſumme

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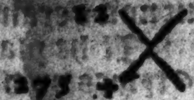
I muſt ſette downe after this ſorte as you ſee here, wherefyrſt I haue made a croſſe (commonly called ſainct Andrewes croſſe) & at the ouer corner on the lefte hande I haue ſette the firſt poſition 12, and at the other corner vnder it, I haue ſet 275 whiche is the firſte errour, wyth thys fygure +, whiche betokeneth to muche, as this lyne, — plaine without a croſſe lyne, betokeneth to lytle. On the righte hande of the croſſe I haue lefte two lyke roumes for the ſecond poſitiō and his errour. Therefore to proſecute the worke, I ſuppoſe hee playde 16 daies, and wrought 24.

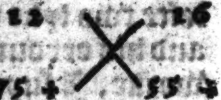


Sc. I was a while in doubte why you named the daies of his workyng, ſeeyng they be not ſet in the fygure: and I doubted how you knew them, or els whether that you did ſuppoſe them at all aduentures, as you dyd the daies that he played: but nowe I gather that ſeing 40 daies is the whole tyme lymited, then the daies that he played beyng ſuppoſed, the reſt of 40 muſt needs be the daies that he wrought, and therefore 28 foloweth 12 of neceſſitie, and 24 foloweth 16 alſo of neceſſitie: but yet I ſcarſe perceyue why you ſet not in the fygure as well 28 as 12.

Maſt. It forceth not whiche of theym I take, ſo that in the ſeconde poſition I take the

the numbre of the ſame nature, that is here both of woorking daies, or both of ydle dais. but nowe examine you this ſecond poſition.

S. If he playde 16 daies, then abatynge 16 tymes 30 đ, the ſum will be 480 đ. And for 24 daies that he wrought, euery day yeldinge 25 đ, the totall is 600 đ: ſo that abatyng 480 out of 600, there reſteth 120. and as you ſaye it ſhuld be but 65, therfore it is to much by 55, ꝧ muſt be ſet on the ryghte hande of the fygure at the nether part, and ouer it on the ſame ſide 16, whiche is the ſecond poſition, thus.



And as I gather by your woordes, it were all one if I did ſet 28 in ſteed of 12, & 24 in ſteed of 16.

M. So were it. But this ſhall you mark, that of what nature ſo euer the two poſitions be, of the ſame nature is the quotient. Therfore when the poſitions in this queſtiõ are 12 and 16, which bothe being numbres of the playing daies, the quotiente ſhall declare the true numbre of the playing daies: where as if the poſitions had beene 28 and 24, whiche are ſuppoſed to be the woorkinge daies, then woulde the quotiente declare the true numbre of the woorking dayes, and not of playing daies, as it will do now. And therfore to continue the woorke of this queſtiõ, and to fynde the true numbre of playinge

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daies, I muſt multiply croſſewaies the firſt poſition 12 by 55, that is the ſecond errour, and the totall will be 660, then I multiplye 275 by 16, and it yeldeth 4400. Nowe bycauſe the errours are lyke, that is to ſaye, bothe to muche, I muſt ſubtract 660 out of 4400, and ſo remaineth 3740, which is the diuidend. Again I muſt ſubtracte the leſſer errour 55 out of 275, that is the greater errour, and there will remaine 220, whyche ſhall be the diuiſoure. then dyuidinge 374, by 220, the quotient wyll be 17. Wherfore I ſaye nowe conſtantly, that 17 is the true numbre of daies that the maſon plaied: and then it foloweth that he wrought 23 daies. and ſo is the queſtion anſwered.

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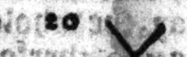
Now for the ordre of tryall of this worke there nedeth none other profe but only this, to worke with this numbre according to the queſtion, and if it agree, then appeareth the numbre to be it that you wold haue. As here now ſeeyng he wrought 23 daies, and muſt haue for euery day 25 pens, the whole ſum cometh to 575. Then again ſeyng he played 17 daies, and muſt abate 30 pence for euery day, the whole ſum of the abatement wyll be 510. therfore I ſubtract 510 out of 575, and there wyll remayne 65, whiche maketh 5 ſ. 5 d. the cleere wages of the maſon for his worke, accordyng to the queſtion.

S. Nowe I truſt I vnderſtand the worke,

and the rule ſo well (and the better by thy proofe) that I can bee able to doo the lyke. And for a proofe I take the ſame queſtion all ſaue the laſte numbre, where I will ſuppoſe that he had 10 ſ. for his wages cleere. And now to geſſe at the nũbre of the dais that he wrought. I ſuppoſe firſt that he wroght 20 dais: thẽ ſay I, if he wrought 20 daies, hys wages muſt be 500 d. thẽ did he play other 20 daies, for whiche muſt be abated 600 d. and then he leeſeth 100 d. And ſo am I at a ſtay, for it is not lyke to your former work.

M. You ſhould haue required of me ſom queſtion; and not haue taken a queſtion of your owne fantaſying, vntil you were more expert in this arte: for ſo might you as well happẽ on an impoſſible queſtiõ as on a poſſible. but now to go forwarde: conſidre that this nũbre is to little by 220, ſeing he ſhuld gain by your ſuppoſition 120 d. and in thys poſitiõ he leſeth 100, thoſe both make 220, which you ſhall ſet down for the firſt errour with this ſigne —, betokenyng to lyttle, as here in the forme folowyng dooth appeare. And now for the reſt go forward your ſelf ones again.



S. As my errour hath vttered my folly, ſo it hath procured me better vnderſtanding. Now therefore cõſidering this poſitiõ not to ſolue the queſtion, I take an other, ſuppoſing that he

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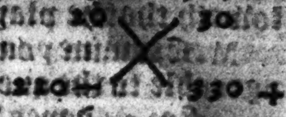
wrought 30 dais, thē for his wages he muſt be alowed 750 d, & for the 10 dais which he wrought not, hee muſt abate 300 d, & ſo remaineth cleere 450 d. but it ſhuld be ōly 120 d, therfore is it to much by 330, which I ſet down in the figure with the former poſition and his errour, & the fygure appeareth thus.

Now muſt I multiply in croſſe wais 220 by 30, and it will be 6600. Then againe I multiply 330 by 20, and it wyl be alſo 6660. Wherfore if I ſhall ſubtract the one out of the other, there will remaine nothing to be the diuidend.



M. In this you forget your ſelfe agayne: for in as muche as the ſignes of the errours be vnlyke, therfore muſt you worke by Addition, addyng togither thoſe two totals to make the dyuidend, and adding alſo the two errours to make the diuiſour. And bycauſe you ſhal no more forget this part of the rule take this brief remembrance.

Vnlike requyre Addition.
And lyke deſire Subtraction.

Sc. You meane that if the errours haue lyke ſignes, then muſt the diuidend and the diuiſor be made by ſubtraction, as is taught before; and if thoſe ſignes be vnlyke (as in this laſte example they bee) then muſt I by addition gather the diuidende and the diuiſour.

ſour. Wherfore muſte I adde 6600 to 6600, and it will be 13200, whiche ſhal be the diuidende. Then againe I adde 220 to 330, and it will be 550, whiche muſt be the diuiſour: Wherefore diuiding 13200 by 550, the quotient will be 24. Wherby I knowe that the maſon wrought 24 daies: and then it foloweth that he playd 16 daies.

M. Examine your worke whether it bee agreable to the queſtion or no.

S. For 24 dayes worke, the wages muſte be 600 d, and for 16 daies which the maſon wrought not, there muſt bee abated 480 d, and then remayneth clere to the maſon 120 pence, as the queſtion importeth: Wherefore it is euident, that 24 is the true nũbre of the daies that he wrought.

M. Although you ſeeme nowe to vnderſtand this worke, yet to acquaint your mind the better with the newe trade of this rule, I thynke it good to propone to you fyue or ſyxe examples more before I make an ende of it.

S. Syr I thanke you, that you do ſo conſidre my commoditie & profite in knowledg, for vndoubtedly it is practiſe & exerciſe that maketh men prompte and experte in euery kynde of knowledge.

Maſt. You ſaye well, ſo that they folowe ſome certaine preceptes to gouern and rule their practiſe by, els maye practiſe procure cuſtome of errour, and a repugnaunce to exactneſſe of knowledge, namely ſo longe as the errour is not playnly knowen to the vulgar ſorte. But to returne to our worke. Ther is a ſeruant that hath bought of veluet and damaſke for his maſter 40 yardes, the veluet at 20 s a yard, and the damaſke at 12 s: and when he commeth home, his maſter demaundeth of hym, howe much he hath boughte of eche ſorte; I can not tell (ſayth he) exactly, but this I know, that I payd for damaſke 48 s more then I payde for veluet. nowe muſte you geſſe howe manye yardes ther is of eche ſorte.

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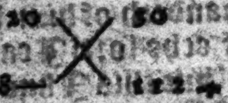
Sc. Although the geſſe ſeemeth difficulte, yet I will proove what I can do: for I remembre your ſaying, that it forceth not howe ſound or falſe the geſſe be, ſo it be ſomwhat to the queſtion, and not an anſwere of a contrary matter.

Therefore fyrſte I imagine that hee boughte 20 yardes of damaſke, for whiche hee ſhoulde paye after the former price 240 ſhillinges, then muſt he needes haue of veluet other 20 yards (to make vp the 40 yardes) and that wold coſt 400 s. So that the totall of the price of the damaſke is leſſe then the ſumme payde for veluet 160 s, and ſhuld be more by 48, therfore the fyrſte errour is 208 to lyttle. Then begynne I agayne and ſuppoſe hee bought of damaſke 30 yardes, that coſt 360 ſ. then hath he but 10 yardes of veluet, whiche coſte 200 ſ. and nowe the price of damaſke is greater then the price of the veluet by 160 ſhillinges, and ſhoulde bee but 48, therefore is the ſeconde errour 112 to muche, whiche I ſette in fourme of a figure as there doth appeare. Thenne doo I multiplye in croſſe waies 208 by 30, and the ſumme will bee 6240. Alſo I multiplye 112 by 20, and there wyll amount 2240. And in as muche as the ſignes of the errours bee vnlyke, I knowe I muſte worke by Addition. therfore adde I thoſe two totals togither, and thei make 8480, which is the diuidend: then adde I alſo the two errours togyther 208 and 112, and they make 320, whiche is the diuiſoure. Wherefore dyuidyng 8480 by 320, the quotient wyll bee 26½, whyche is the true ſumme of yardes of damaſke that he bought: and in veluet 13 yardes ½, and that appeareth by examination, thus: 26½ yardes of damaſke at 12 ſ. the yarde, maketh 318 ſ, then in veluet he had but 13 yardes and ½, that coſte 270 ſ, at 20 ſ. the yarde. Nowe ſubtracte 270 out of 318, and there will remayne 48, which is the numbre of ſhillinges that the damaſke did coſt more then the veluet.



Maſter. Nowe ſhall you haue a queſtion of an other kynde.

There are thre men that doo owe monye to me, and I haue forgotten what the total ſumme is, and what the particulars be.

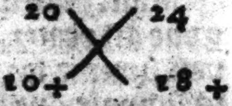
S. Why? then it is impoſſible to know yͤ debt.

M. Peace, ye are to haſty; there is more helpe in it then you yet ſee. I haue three ſeuerall notes, whereby it appeareth that I dyd confer their debtes togither, and founde the debt and the ſecond to amount to 47 li, the debte of the fyrſt man and the thyrd, did make 71 pound, and the ſecond mā his debt with the third, did riſe to 88 pounde. Now, can you tell what euerye man dyd owe, and what was the whole totall?

Sc. Nay in good fayth: but as I perceaue that it muſte be found by coniecture, ſo will I geſſe at it, ſuppoſynge that the fyrſte man dyd owe 20 pound, and the ſeconde man 30, and the thyrd

Maſt. Nay ſtay there, for you are to farre gone all ready. you may not ſuppoſe a ſeuerall ſumme for euery man, for it is inoughe to ſuppoſe one ſumme for the fyrſt man, and let the other riſe as the queſtion importeth. Therefore ſeyng you ſette the fyrſt man his debt to be 20 li, the ſecond man can not owe 30 li, for the declaration is that their debtes added togyther, dydde make 47 pounde, ſo muſte the ſeconde manne hys debte bee but 27 li. Nowe this ſeconde debte with the thyrde muſte make 88, therefore ſubtract 27 out of 88, and there will remaine 61, as the thyrde manne his debte. Then ſaythe the declaration that the fyrſt and thyrd mennes dettes doo make 71: but by this ſuppoſition they make 81, that is 10 to muche: whyche I muſt ſette for the fyrſt errour. Now work you the ſecond poſition.

5. I ſuppoſe the fyrſte mannes debt to be 24 pound, then muſt the ſeconde mans debte (by your declaratiō) be but 23 li. ſeyng both they do make but 47 li. Alſo the ſecond man his debte with the thyrd, do make 88 li, and the ſecond man oweth but 23, therefore the thyrd man muſt owe 65 li. Nowe the thyrde mans debt with the fyrſt, ſhuld make by the declaration 71 li. and they doo make 89 li. that is 18 li to much. and that is the ſecond errour, whiche I ſet downe with the fyrſte, and their poſitions in this forme: and then do I multiply in croſſewaies 20 by 18, and it is 360. Alſo 10 by 24 maketh 240. And



bycauſe the ſignes of the errours be like, I muſt worke by ſubtraction, therefore I ſubtract 240 out of 360, and there reſteth 120 whiche is the diuidend: then doo I ſubtract 10 out of 18 by the ſame reaſon, and ſo is the diuiſour 8, whiche is found 15 tymes in

120,

120, therefore I ſaye that the fyrſt man dyd owe 15 li, and then the ſeconde man muſte owe 32 li, for thoſe two do make 47 li, and the thirde manne his debt is 56, for ſo much remayneth if I bate 15 out of 71, or if I take 32 out of 88.

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Maſt. For the thirde exaumple take this eaſye queſtion for the varietie in woorke. Two menne hauynge ſeuerall ſums whiche I know not, do thus talke togither: the firſt ſayth to the ſecond, If you giue me 2 ß of your money, than ſhall I haue 3 tymes ſo muche money as you. the ſecond anſwereth: It were more reaſon, that our ſummes wer made equall, and ſo will it bee, if you gyue me 3 ſhillinges of your monye. Nowe geſſe what eche of them had.

S. I imagine that the fyrſte had 9 ß.

Note.

M. Conſydre euermore in your imaginations that you take a lykely ſum. as in this queſtion take ſuche a ſumme that hauynge 2 added vnto it, maye be dyuided into 3 partes euen.

S. Why? I remembre you ſayd before, it forced not, how fondly ſo euer I geſſed.

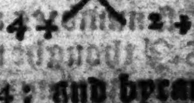
Ma. As for the poſſibilitie of the ſolution it is truthe, but for eaſyneſſe in woorke, the apteſt numbres are moſte conuenient.

S. I thought no leſſe, and therfore I toke 9 as an apte numbre to be parted into 3, but I perceaue I ſhuld haue conſidered the aptneſſe of that partition after the addition of 2 vnto it, and then 7 had been more meeter.

Ma. That is truthe, and then ſhoulde the ſeconde man his ſumme be 5: for although he haue nowe but the third part of 9, that is 3, yet you muſte remembre that hee lente the fyrſte manne 2, and ſo had he 5.

S. Then to go forward: if the ſecond man had 3 of the fyrſte man, then ſhould he haue 8, and the fyrſte man but 4, ſo hath hee double to the fyrſte man: yet he ſayd in the queſtion, they ſhoulde haue equall. Wherefore it appeareth that he hathe 4 to muche. Therefore I note that error with his ſuppoſitiō, and geſſe againe that he hath 10 ſhillynges, whereunto I adde two ſhyllinges borowed of the ſeconde manne, and then hathe he 12 ſhyllinges, ſo the ſecond man hathe remaynyng but fower, wherunto yf I adde the 2 that he lent to the fyrſt manne, ſo hadde hee but 6 ſ̄ at the beginnyng. Then take 3 ſhyllinges from the fyrſte man and gyue to the ſeconde, then hath the fyrſte man but 7 and the ſeconde hathe 9, which are not equall, but there are 2 to manny.

Wherefore I ſette downe bothe the poſitions wyth their errours, as here you ſee, and multiply a croſſe,



ſo commeth there 40 and 14: and bycauſe the ſygnes bee lyke, I take 14 out of 40, and

and ſo reſteth 26 to be the diuidende, then lykewaies I take 2 oute of 4, and there reſteth 2, by which I diuide 26, and the quotient wyll be 13, whiche is the ſumme that the fyrſte man hadde. And ſo appeareth that 2 beynge added therto, the ſumme wyll be 15, ſo hath the ſeconde man now but 5, and before he had 7, then take 2 from the fyrſte and put to his ſeuen, ſo haue eche of them tenne, and that is equalle, as the queſtion woulde.

M. For the fourth example take this queſtion. One man ſayde to an other: I thynke you had this yeare twoo thouſand lammes. So hadde I ſayde the other: but what with payinge the tythe of theym, and then the ſeuerall loſſes, they are muche abated, for at one tyme I loſte halfe as manye as I haue nowe lefte, and at an other tyme the thyrde parte of ſo manye, and the third tyme $\frac{1}{4}$ ſo many. Now geſſe you how manye are lefte.

The
examp

Sc. Bicauſe here is mention made of certaine partes, I muſte take a nombre that may haue all thoſe partes: that is to ſaye ½, ⅓ and $\frac{1}{4}$, whiche wyll bee 24, howe bee it 12 hathe the ſame partes. Therefore fyrſte I take 12 to be the nombre that do remain, ſo hath he loſt 6, 4 and 3, that is 13, and in the whole 25: but it ſhould be 2000.

M. Ye are deceiued yet ſtill, you haue forgotten goeth the 10 parte, whiche muſt bee defalked, ÿ is 200, ſo there remayneth but 1800: and nowe go on agayne.

Sch. Then to fynde the errour, I take 25 oute of 1800, and there remaineth 1775 to fewe, whiche I ſette for the fyrſte erroure. Thē for the ſecond poſitiō I take 24, whoſe halfe is 12, the thirde part 8, and the quarter 6, whereby riſeth 50, which is to lyttle by 1750; therefore I ſet down both the poſitions with their errours thus.

12 ╳ 24
1775— 1750—

And multiplye in croſſewaies 1775 by 24, whereof commeth 42600. And I multiplye 1750 by 12, and there riſeth 21000. And bicauſe the ſygnes are lyke, I do ſubtract the one from the other, and ſo remaineth the diuidend 21600. then do I ſubtracte 1750 out of 1775, and there reſteth 25: by which I dyuide 21600; and the quotient is 864, whereof the halfe is 432, & the thyrd parte is 288, the quarter is 216, which al being added together, wyll make 1800. And if you adde thereto the tenth which was abated before, then wil the whole ſumme be 2000. And nowe dooth there come a queſtion to my memorye which was demaunded of me, but I was not able to an-

$$\begin{array}{r} 864 \\ 432 \\ 288 \\ 216 \\ \hline 1800 \end{array}$$

anſwere to it, and now me thinketh I could ſolue it.

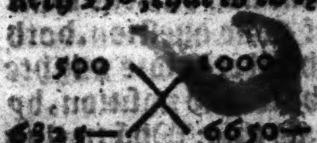
Maſter. Propone your queſtion.

A exa
of shee
tyllag

Sch. There is ſuppoſed a lawe made that (for furthering of tillage) euerye man that doth kepe ſheepe, ſhall for euerye 10 ſheepe eare and ſowe one acre of grounde; and for his allowance in ſheepe paſture, ther is appointed for euery 4 ſhepe 1 acre of paſture: Nowe is there a riche ſheepemaſter whyche hathe 7000 acres of grounde, and woulde gladlye keepe as manye ſheepe as he might by that ſtatute, I demaund how many ſheep ſhall he keepe?

M. Anſwere to the queſtion your ſelfe.

Sch. Fyrſt I ſuppoſe hee maye keepe 500 ſheepe, and for them he ſhall haue in paſture after the rate of 4 ſheepe to an acre, 125 acres, and in earable grounde 50 acres, that is 175 in all: but this errour is to lyttle by 6825. Therfore I geſſe agayne that he may keepe 1000 ſheepe, that is in paſture 250 akers: and in tyllage 100 acres, which maketh 350, that is to lyttle by 6650.



These bothe errours with their poſitions I ſet downe as you ſee, and multiply in croſſe 6825 by 1000, and it maketh 6825000. Then I multiplye 6650 by 500, and it doth amount to 3315000, whiche ſum I do ſubtract oute of the fyrſte, and there remaineth 3500000 as the diuidend. Alſo I doo ſubtract the leſſer errour out of the greater, and ſo remayneth 175, by which I dyuide the ſaide diuidend, and the quotient wil be 20000, ſo that I ſe that by this rate he that hathe 7000 acres of ground, may kepe 20000 ſhepe: and therby I coniecture that many menne may kepe ſo many ſhepe. for many men (as the cōmon talke is) haue ſo many acres of groūd.

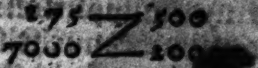
Maſter. That talke is not lykelye, for ſo much groūd is in cōpaſſe aboue 48 $\frac{3}{4}$ miles. but leaue thys talke, and retourne to your queſtions, leaſte your pointinge bee ſcarſe well taken.

S. In deed I do remembre that the Egyptians did grudg ſo much againſt ſhepherds, tyl at length the ſmarted for it, and yet they were but ſmall ſheepmaſters to ſome menn that be now, & the ſheep are waxen ſo fierce now and ſo myghty, that none can withſtād them but the Lyon.

other
ay of
orking.

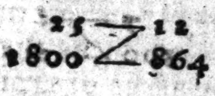
M. I perceaue you talke as you hear ſome other: but to ẏ work of your queſtion. both this laſt queſtion and the next before mighte be wrought without the ſecond poſition, by the rule of proportion, as this: Whā in this queſtion ye found in the fyrſt error, that for 300 ſheepe there muſte be 175 acres, then might you reduce it to the Goldē rule, thus

It



If 175 acres will admit in allowance 500 ſheepe, than 7000 wil haue 20000. And ſo by one poſition with the healpe of the Golden rule maye you aunſwere that queſtion.

Likewaies for the queſtion of lambes. whe you had foūd that 12 cam of 25, you might haue ſette the figure thus as you ſe, & haue



ſayd: If 25 do leaue but 12, what ſhall 1800 leaue? and it would appeare to be 864.

S. Syr I thank you for this ayde, for it doth much ſhorten the woorke of this rule.

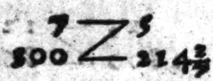
Ano
way

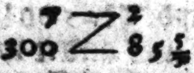
M. Yet againe will I ſhew you an other way to anſwer to this laſt queſtion wythout this rule of falſe poſition, & that by the rule of Felowſhipp. for it appeareth in the propoſing of the queſtion, that ten ſheep muſte haue in paſture 2 akers and $\frac{1}{2}$, and for theym muſte ther be eared but one aker: ſo it followeth, that for two acres eared, ther muſt be fyue ſette to paſture. And yf you putte them both into one ſumme, they wyll make 7. Therfore looke what proportion 7 beeynge this totall, dooth beare to 5 and to 2, ſuche proportion ſhal any totall in this queſtion bear to the paſture grounde and the eared ground.

Scholar. This ſerueth wonderous aptly.

There-

Therfore to proue it, I demaund this by the former ſuppoſition: If a man haue 300 acres, howe muche ſhall he leaue in paſture, and howe muche ſhall he tourne to tyllage? You ſaye that as 7 is to 5, ſo ſhall 300 bee to the acres of paſture. and as 7 is to 2, ſo is 300 to the acres of tyllage: wherof for both I haue ſet examples here folowinge, whereby appeareth that of paſture there ſhal be 214 2/7 akers, and of tyllage, 85 5/7, whiche bothe ſummes added togither doo make 300.

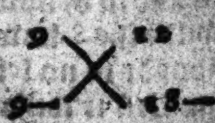




ther
tion.

M. Nowe take an other example: A man hathe three ſyluer cuppes with one couer, the couer wayeth 18 ounces, the ſecond cup wayeth euen halfe the waighte of the fyrſte and the thyrde. Nowe if the couer be put to the fyrſte cuppe, they waye iuſt as much as all the three cuppes do waye: and the couer be ioyned with the ſecond cuppe, they weye as muche as the ſecond twiſe and the third: and if the couer be put to the thyrd cup, thei will weye twiſe as much as the fyrſt and the ſeconde cuppe. Now trye you what was the iuſte waighte of euery cuppe.

S. I doo ſette the weight of the fyrſt cup to be 9 ounces, thenne in as muche as theſe two (that is to ſaye, the couer and the fyrſt cup) doo weye the waight of the thre cups, I ſe ỷ the thre cuppes muſt weye 27 vnces, for ſo muche is 18 and 9. Alſo bycauſe the fyrſte and the thyrd do wey double ſo much as the ſeconde, therfore is it the thyrd parte of that waight, that is 9, and then would it folowe, that the thyrd cuppe alſo ſhuld weye 9 ounces: but then the queſtion ſaythe, that the couer beyng ioyned to the ſecond cuppe, they weye as much as the ſecond twiſe and the third ones, that ſhould be 27: and ſo it dooth. then beyng ioyned with the thyrde cuppe they ſhuld weye twiſe as much as the fyrſte and the ſecond, that ſhould be 36, and they weye but 27, ſo is that errour 9 to litle. Then begyn I agayne, and ſay that the fyrſte cuppe doothe waye 12 ounces, whiche I ioyne with the couer, and they make 30 vnces: then ſeeyng the ſecond is ⅓ of that waight, it muſt needes weye 10 ounces, and the thyrde muſte weye 8 ounces, ſeeyng the fyrſte and thyrde muſte weye 20. Nowe put I the couer to the ſeconde cuppe, and they weye 28 ounces, whiche ſhould bee euen ſo: then ioyne I the couer with the thirde cup, and ſo ſhould it waye twiſe the fyrſt and the ſecond, that is 44 ounces, and they do wey but 26, that is 18 to lytle. thoſe errours with their poſitiõs I ſet down, & multiply in croſſewaies 9 by 12, wherof commeth



108

108. Alſo 9 by 18, and that yeldeth 162, and in as muche as the ſignes be lyke, I abate the leſſer out of the greater, and there doth remayn 54. Then do I alſo abate the leſſer errour from the greater, and ſo remayneth 9, by whiche I diuide 54, and the quotiente is 6. Whiche I take for the true weighte of the firſt cuppe: which being ioyned with the couer, muſt waye as much as the thre cuppes: ſo do they weye but 24 ounces. Then ſeyng the ſeconde cuppe is the thyrde parte of that waight, for the other two cups (you ſai) muſt wey double his weight. the weight of the ſecond cup is 8 vnces, & ſo the weight of the third muſt be 10 vnces. Now put the couer to the ſecond cup, and it will make 26 ounces: that muſt be the weighte of the ſecond twiſe, and the third ones, that is twiſe 8, and ones 10, & ſo is it. Againe put the couer to the third cuppe of 10 vnces, and they muſt weye twiſe as muche as the fyrſte and the ſecond, that is 28: and ſo is al agreable.

Maſter. Then anſwere to this queſtion.

queſtion
water.

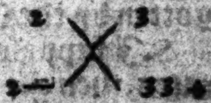
Ther is a ceſtern with iiij. cocks, conteinyng 72 barrels of water, And if the greateſt cocke be opened, the water will auoyde cleane in vj howers: at the ſeconde cocke it will aſke 8 howers: at the third cocke it wil auoyde in no leſſe then 9 houres: and at the ſmalleſt it wil requyre 12 howers: Now I demaund, in what ſpace will it auoyd all the corkes being ſet open?

Sco. Fyrſt I imagine that it will auoyd in 2 howers.

M. Then muſt there auoyd by the fyrſt cocke $\frac{1}{3}$ of the water, that is 24 barrels, and bi the ſecond cocke $\frac{1}{4}$, that is 18, and by the thyrde cocke $\frac{2}{9}$, that is 16 barrels, and by the ſmalleſt cocke $\frac{1}{6}$ that is 12 gallons, all whiche ſummes put togither, do make 70, as by theyr addition it dooth appeare. but it ſhuld be 72, therfore the errour is 2 to fewe.

24
18
16
12
70

S. Then I begin again by your fauor, bicauſe I think I vnderſtand the work, and I put three hours for the due time: ſo ſhall there run out at the greateſt cock $\frac{1}{2}$, that is 76 barrelles, and at the ſeconde hole $\frac{3}{8}$, that is 27, and at the thyrde cocke $\frac{1}{3}$, that is 24, and at the ſmalleſt hole $\frac{1}{4}$, that is 18 barrels, whiche all togither doo make 105. and ſhuld be but 72, ſo is it to much by 33. therfore wⁿ I ſet the errors in ordre of yͤ figure, wͭ their poſitions, & worke by multiplication in croſſe, ſaying: 2 tymes 3, is 6, and 2 tymes 33 maketh 66, and bicauſe the ſignes are vnlyke, I muſt adde thoſe two totalles togither, which make 72: alſo I adde the ij. errours, and they make 35, by which I dy-



uide 72, and the quotient riſeth 2 $\frac{2}{35}$, whereby I ſee, that all the cockes beyng ſet open, the water will auoide in 2 houres and $\frac{2}{35}$ of an houre.

M. This exerciſe maketh you to grow expert in the rule. Therefore I will inure you ſomwhat more with a queſtion or two.

ueſtion
artners

There were two men that had been partners, and had in accompte betwene them 300 duckets: whereof the one ſhoulde haue for his parte 180, and the other 120, but in the parting of them, they fel at variance, ſo that eche of them catched as manny as hee coulde, yet afterward beyng reconciled, thei agreed that he which had gotten moſt part of them, ſhould lay downe $\frac{3}{4}$ of them again, and he that had gotten leaſt, ſhuld lay doun $\frac{1}{3}$ of thoſe whiche hee had taken, and then parting them into two equall partes, eche man to haue halfe thereof, and ſo had they their iuſte portions as they ought. nowe I demaunde of you what eche of them hadde gotten by the ſcamblyng?

S. I ſuppoſe he that had leaſt, gat 108 ducates, then the other hadde 192, wherefore in layinge downe agayne of the 192, there was put downe $\frac{3}{4}$, that is 144. and ſo hadde he lefte but 48. Alſo of the 108 there was layde downe 36, that is $\frac{1}{3}$, and ſo hee hadde lefte 72. Then I put togather 144, and 36, and

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& it maketh 180. whiche I part into 2 partes euen, and so commeth 90 to be gyuen to eche of them: whiche summe put to 72, maketh 162: and ioyned to 48, it maketh 138. and now I doubt how I ſhal go forward.

M. You need not to take but one of them. whiche you lyſt, the greater or the ſmaller, for all commeth to one purpoſe: and ſo maye you compare it that you take to any of the other ſummes, remembryng that you make comparison to the ſame in the ſecond work. as for example of the fyrſt part: If you compare 138 with the leſſer ſumme due, that is 120, ſo is it 18 to muche: and if you compare it with the greater ſumme, than is it 42 to lyttle. Againe, if you compare 162 to the greater ſumme, the errour wil be 18, as it was in the other: but it will haue a contrary ſigne. and if you compare it with the leſſer ſumme, it will be 42 to muche: ſo that the errour bothe waies is either 18 or 42. and as for þ ſignes it little forceth, for in the is nothinge conſidered heere, but lykeneſſe and vnlykeneſſe, whiche in this caſe doothe neyther further nor hynder. But now go on with the worke.

S. If it be ſo, then am I out of my greateſt doubte. Then I ioyne that 90 (which I founde as the halfe of the latter partition) vnto 48, whiche is lefte with the one man, and ſo hath he 138, which (I may ſay) is 18 to manye, for the leaſt ſhuld be but 120; that errour do I note, and then make a new poſition, ſuppoſyng the one man to haue 204, and the other to haue 96, wherefore of the 204 there muſt be layde downe 153, and ſo remaineth with him 51. Alſo of the 96 ther muſt be layde downe $\frac{1}{3}$, that is 32, and ſo reſteth with that man 64. Nowe of the 153 and 32 I make one ſumme as 185, whiche I muſte diuide into 2 equall partes, and ſo eche mā ſhall haue 92 $\frac{1}{2}$, wherunto if I adde their former portions reſerued, then the one ſhall haue 156 $\frac{1}{2}$, and the other hathe 145 $\frac{1}{2}$. Wherefore I take the leſſer ſumme nowe againe as I dyd before, that is 143 $\frac{1}{2}$, & fynde that he hath to many by 23 $\frac{1}{2}$, for he ſhould haue but 120. ſo haue I for my two poſitions ij. errours, whiche I ſette downe as here maye be ſeene, eche errour vnder his poſition, and than by the rule I doo multiply in croſſe waies 108 by 23 $\frac{1}{2}$, and ther riſeth 2538 which I note, then again I multiply 96 by 18, and therof amounteth 1728. Now bycauſe the ſignes are both lyke, that is bothe to many, I muſt worke by ſubtraction, & ſo abatinge 1728, out of 2538, there wyll reſte for the diuidend 810. then for the diuiſour I ſubtract 18 out of 23 $\frac{1}{2}$, and ther remayneth 5 $\frac{1}{2}$, by which I dyuide 810, and the

$$\begin{array}{ccc} 108 & & 96 \\ & \times & \\ 18\,+ & & 23\frac{1}{2}\,+ \end{array}$$

the quotient will bee $147\frac{3}{11}$, whiche is the iuſte portion of him that had the leaſt ſum. And if I do ſubtract it out of 300 being the totall ſumme, then will there remaine $152\frac{8}{11}$ as the portion that the other dyd get.

M. For the profe of this worke, you may choſe whether you will examine thoſe numbres accordyng to the forme of the queſtion or els work by other 2 poſitions for to fynd the ſeconde neumbre, and if thoſe poſitions brynge the ſame numbres that dyd amount by the fyrſt 2 poſitions, then doth eche work confirme other.

Sch. By your pacience I will proue both waies, not only to ſee their agreement, but alſo to accuſtome my mynde to thoſe workes: for I perceaue it is exerciſe that muſt be the chieffe engrauer of theſe rules in my memorye.

M. You conſydre it well: then go to.

Scholar. Fyrſt I wyll by two other poſitions trye to finde the portion of him which had moſte.

M. Although you may do it with any poſitiōs, yet to ſe the agreement of your work the better, take the ſame poſitions that you dyd before, comparynge them nowe to the greater, as you dyd before vnto the leſſer.

Schol. Then I ſuppoſe that he that had moſte, had 192, ſo had the other 108. Now if I take $\frac{3}{4}$ of 192, that wyll be 144, and there will reſt to that man but 48. And frõ the ſecond, which had 108. if I take ⅓, that is 36, ther will remain to him 72: then ioynynge 144 with 36, it will make 180: the halfe wherof beyng 90, If I adde to eche of thoſe two mens portions remaynyng wyth them, the one ſhall haue 138, and the other 162. of whiche two I take the greater (that is 162) and ſee it to be 18 to few, for it ſhuld bee 180. that errour I note vnder his poſition. Then for the ſeconde poſition, I take (as I dyd before) 204 for the one, and ſo reſteth 96 for the other: then take I ¾ of 204, and it will be 153, and there reſteth to hym 51: Alſo of the 96 I take ⅓ that is 32, and there remaineth to him 64. Now put I that 32 vnto 153, and it yeldeth 185, which beynge parted in equall halues, maketh 92½, to be added to eche mannes remaynder, and ſo the one hathe 143½, and the other 156½. Wherfore I take the greateſt ſumme, and it is 23½ to lyttle, that do I note alſo, and ſet bothe theſe errours vnder their poſitions, as in this exaumple doth appeare. And then multiplying 192 by 23½, ther doothe ariſe 4512.

192 ╳ 204
18 — 23½ —

Againe, I multiplye 204 by 18, and it maketh 3672, which I do ſubtract out of 4512 bicauſe the ſignes be lyke, and there reſteth

840 for the diuidend: then ſubtractinge 18 out of 23 $\frac{1}{2}$, there will remayne 5 $\frac{1}{2}$, whych I muſt take for the diuiſor. And ſo dyuidynge 840 by 5 $\frac{1}{2}$, the quotiēt will be 152 $\frac{8}{11}$: wherby I haue found an agreable ſumme to that which I founde by the former poſitions, for him that had moſte. Whiche if I do ſubtract out of 300, that is the totall, there will reſt 147 $\frac{3}{11}$, which was the portion of hym that hadde the leaſte parte.

M. So by dyuers poſitions you ſee, that one doth confyrme the woorke of the other. Now examine thoſe twoo numbres by the forme of the queſtion, and ſo ſhall you proue your woorke good alſo.

S. If that he whiche gat moſt had 152 $\frac{8}{11}$, then muſte he laye downe $\frac{3}{4}$ of his ſumme, that is 114 $\frac{6}{11}$, & ſo ſhall remayne with hym but only 38 $\frac{2}{11}$. The other whiche had leaſt that is 147 $\frac{3}{11}$, muſt put downe of his ſum $\frac{1}{3}$, that is 49 $\frac{1}{11}$, & ſo doth ther remayne with him yet 98 $\frac{2}{11}$. Then doo I adde togyther 114 $\frac{6}{11}$ and 49 $\frac{1}{11}$. and it will make 163 $\frac{7}{11}$, which I muſt part into ii. equal partes, and that will be 81 $\frac{9}{11}$ to be giuen to eche of them, ſo puttinge 81 $\frac{9}{11}$, vnto 38 $\frac{2}{11}$, there doth amounte 120 iuſte, which is the true portion of him that ſhoulde haue the leſſer ſumme: and adding 81 $\frac{9}{11}$ to 98 $\frac{2}{11}$, the totall will bee 180, the true portion of the other. And ſo is the worke by this proofe alſo tried to bee good. And this I marke by the way, that in their ſcamblyng, hee gat moſte (as it chanceth oftē) that ought to haue had leaſt by iuſt partition.

M. Let your ſtudy be to learn truthe and iuſte arte of proportion, and to diſtribute ẏ part accordyng therevnto, as often as occaſiō ſhalbe miniſtred. And here wold I make an ende of this rule, ſaue that I remembre one pleaſaunt queſtion whiche I can not ouerpaſſe, whiche I will declare ſomewhat largelye, bycauſe you ſhall as well vnderſtande ſome reaſon in the pleaſaunte inuention, as apte proceedyng in the wittye workynge thereof.

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Hiero kynge of the Syracuſans in Sicilia hadde cauſed to bee made a crowne of golde of a wonderfull weight, to be offered for his good ſucceſſe in warres: in makynge wherof, the goldſmyth fraudulētly toke out a certayne portion of gold, and put in ſyluer for it, ſo that there was nothing abated of the full waight, although there was muche of the value dyminiſhed; whiche thynge at length beeing vttered, (as no euill can alwayes lye hydde) the king was ſore moued, and beyng deſirous to trye the truthe without breaking of the crowne, proponed the doubt to Archimedes, vnto whoſe witte no thyng ſemed vnpoſſible. Whiche althoughe

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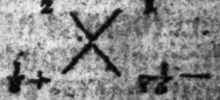
preſently he could not anſwere vnto, yet he had good hope to deuiſe ſome policy for that inuention. And ſo muſynge thereon, as he chaunced to entre into a bayne full of water to waſhe hym, he obſerued that as his body entred into ẏ bayne, the water dyde run ouer the tubbe: whereby his ready witte of ſuche ſmall effectes coniecturinge greatter workes, conceaued by and by a reaſon of ſolution to the kings queſtiō, and therfore reioycing exceedingly more then if he had gotten the crown it ſelf, forgat that he was naked, and ſo canne home, crying as he ranne εὕρηκα, εὕρηκα, I haue foūd, I haue found. And therupon cauſed ij. maſſy peces one of gold, and an other of ſyluer to be prepared, of the ſame weight that the ſaid crown was of: and conſidering that golde is heauyer of nature then ſyluer, and therfore gold of like weight with ſyluer, muſt needes occupy leſſe roume, by reaſon it is more compacte and ſound in ſubſtance, he was aſſured that puttinge the maſſe of golde into a veſſell brym full of water, there would not ſo much water runne ouer, as when he ſhould put in the ſyluer maſſe of the lyke waight. Wherfore he tryed bothe, and noted not only the quantities of the water at eche tyme, but alſo the difference or exceſſe of the one aboue the other: wherby he learned what proportion in quantitie is betwene golde and ſyluer of equall weyght. And then putting the croune it ſelfe into the veſſell of water brymfull (as before) marked howe muche water dyd run out then, and comparyng it wyth the water that ranne out when the golde was pute in, noted howe muche it dyd exceede that. & like wayes comparinge it to the water that ran oute for the ſyluer, marked howe muche it was leſſe then that, and by thoſe proportions founde out the iuſte quantitie of golde that was taken out of the croune, and how muche ſyluer was put in ſteed of it. But ſeyng Vitruuius whiche writeth this hiſtory doothe not declare the particular woorke of this tryall, it ſhall bee no inconuenience to ſuppoſe an exaumple for declarations ſake, wherin although the true and iuſt proportions be not expreſſed, yet the forme of tryall ſhall bee truely ſet foorth. And for an example I ſuppoſe the weight of the croune to be 8 li, and ſo of eche of the other two maſſes. And when the maſſe of golde was put into the water, I imagine that there ranne oute 2 pound of water: and when the maſ[illegible]uer was put in, I ſuppoſe there ranne oute 3 poũd $\frac{1}{2}$. Agayne when the croune was put in, there ran out 2 pound $\frac{1}{4}$. Nowe to knowe what quantitie of ſiluer was in the croune, worke by the rule of falſe poſition, and imagyn that ther was 2 pound of ſyluer: then muſt ther be 6 poũd of golde. Then ley thys by the rule of Proportiõ: If 8 poũd of gold do expel 2 li. of water, what ſhall 6 li. expel? and it will be 1 pound $\frac{1}{2}$. Againe for the ſiluer: If 8 pound of ſyluer expell 3 pounde $\frac{1}{2}$ of water, what ſhall 2 pound of ſyluer put out? it will be $\frac{7}{8}$. Nowe adde thoſe two weightes of water togither, and they will make two pound $\frac{3}{8}$, & it ſhuld be by the ſuppoſitiõ 2 li. $\frac{1}{4}$. ſo is it to muche by $\frac{1}{8}$.

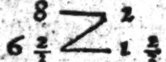
S Now do I vnderſtand the worke as I thinke, therfore I pray you let me work the reſt of the queſtion. And bicauſe this fyrſte ſuppoſition dyd erre, I note that poſition & his errour, and take a newe poſition, eſteeming the ſyluer to be but one pound, ſo muſt there be in gold 7 pound. Then ſay I: If 8 pounde of golde yelde 2 pounde of water, what ſhall 7 pounde yeld? and it will be 1 li. $\frac{3}{4}$. Again if 8 li. of ſiluer expell 3 li. $\frac{1}{2}$ of water, what ſhall 1 pound expell? and it will be $\frac{7}{16}$. Nowe muſte I adde thoſe twoo ſummes togither, and they make 2 li. $\frac{3}{16}$, and they ſhoulde make 2 li $\frac{1}{4}$, ſo is it to lyttle by $\frac{1}{16}$. Therfore I ſet the poſitions with their errours in order, as here foloweth. And thẽ I multiply in croſſe waies 2 by $\frac{1}{16}$, and it maketh $\frac{1}{8}$: lykewaies 1 multiplyed by $\frac{1}{8}$, maketh $\frac{1}{8}$. And bicauſe the ſignes be vnlyke, I muſt adde thoſe two ſums, which make $\frac{1}{4}$



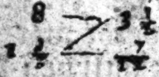
and that is the dyuidend. Again I muſt adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will be $\frac{3}{16}$, that is the diuiſor. Nowe I ſhall diuide $\frac{1}{4}$ by $\frac{3}{16}$, and the quotient it wyll be $\frac{16}{12}$, that is, 1 $\frac{1}{3}$, wherby I know that there was put 1 pounde and $\frac{1}{3}$ of ſyluer into the crowne, and ſo muche golde taken out for it.

M. Proue it nowe by examination accordyng to the queſtion.

S. If ther were 1 pound $\frac{1}{3}$ of ſyluer, then was there of golde 6 pound $\frac{2}{3}$. Nowe ſaye I by the rule of proportion: yf eight poũd of gold expel two pound of water what ſhal 6 poũd $\frac{2}{3}$ expel?



It wyll be one pound $\frac{2}{3}$. Againe if 8 li of ſyluer expell 3 li $\frac{1}{2}$ of water, what ſhall 1 $\frac{1}{3}$ expell? It will be $\frac{7}{12}$. Now muſt I adde togither 1 li $\frac{2}{3}$ and $\frac{7}{12}$, & they will make 2 li, $\frac{3}{12}$, that is 2 li $\frac{1}{4}$, accordyng to the ſuppoſition of the queſtiõ: wherby I perceyue the woorke to be well done. And as I can not but muche reioyce of this excellent inuention, ſo my deſire is kindled vehemẽtly to be perfectly inſtructed in euery part therof, & namely in this point, whether the proportion betwene water and gold bee ſuch, that for 8 li. of golde put into a veſſell full of water, there ſhall run out two pound of water. And for as muche ſyluer, whe-



ther 3 li. $\frac{1}{2}$ of water wolde auoyde?

M. I perceaue your meanyng, and coniecture your imagination to bee thus: that if you knewe the exacte proportion betweene golde and ſyluer and water bothe in theyr waight and in their quantities, then coulde you eaſilye fynde out the mixtures of them. Whiche thinge I haue reſerued for an other woorke that intreateth ſuche matters ſpecially. And at this tyme you muſte conſydre, that you learne Arithmetyke, which intreateth of the manner to ſolue doubtfull queſtions touching numbre, without regard what matter is ſignified bi that numbre, els wer it neceſſary in Arithmetike to teache al arts, ſeing in it mai be moued queſtiōs of al artes. But ſeing you ar ſo deſirous to know this thyng, I wyll tell it you in ſuch a ſort, that you ſhall practiſe youre arte in fynding it, and propounde it in forme of a queſtion. Golde beareth greater proportion to water then ſyluer doth, and their two proportions bee in proportion togither as $\frac{48}{25}$. But to helpe you ſomwhat in this riddle, you ſhall note that the proportion of quick ſiluer vnto water, is the iuſt myddle numbre proportionall in Progreſſion geometricall, betweene the proportiōs of gold & ſiluer vnto water. And his proportion is, as $\frac{292}{21}$. Nowe if you will know the iuſt numbres of theſe 3 proportions, then muſt you fynd out 3 numbres

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in progreſſion geometricall, wherof the middlemoſte muſt be $\frac{280}{27}$, and the fyrſt muſt bee vnto the laſte, as 25 to 48. And thus I wyl leaue you to fynd thoſe numbres when you be at leaſure.

S. Yet ſir I thāk you hartily for this much. for now I ſe the poſſibility to fynd the out. Howbeit bycauſe this queſtiō ſemeth ſtrāge, if it might pleaſe you to inſtructe me ſomewhat in the order of working for it, I ſhuld the more eaſyly fynd the true workinge.

Ma. You deſyre to muche eaſe if you wyll ſtudy for nothing: therfore to occaſiō you to ſtudye the better, I will leaue this doubte wholly to your own ſearch. But as tuching the generalty of the rule, Archimedes neded not to take two maſſes of gold and ſyluer equall in waight with the croun; for the proportion myght as wel be found in any other weight, yea although the maſſe of gold were of one waight, and the maſſe of ſyluer of an other. As for example. If the croun were of 8 pound weight, as I did ſuppoſe, & I haue not ſo muche other fyne golde, but only 1 li. and trying that by water, & fynding that it dothe expell but $\frac{3}{4}$ of an vnce of water, yet than by it may I infer, that 8 pound of gold wold expell 6 vnces of water. And likewiſe of the ſyluer: wherof if I had but 2 pound, and that it doth expel iij. vnces of water, then might I affirm that 8 li. wold expell 12 vnces. ẏ is 1 li. weight. And ſo is it as good as if the 3 maſſes wer al of one weight. And thus for this time I wyll make an ende of this other parte of Arithmetike.

S. Although I can not ſufficiently thanke you for this, yet your promiſe made me to loke for ẏ arte of Extractiō of roots, wherof hytherto I haue learned nothyng.

M. I will not breake my promiſe, but intende (god willyng) to perfourme it within theſe three or foure moneths, yf I perceaue this my pains to be well taken in the meane ſeaſon. And you ſhall not repent the taryeng for it, for it ſhalbe increaſed by the tarying. And in the meane time, you ſhall take this addition not for the ſeconde parte of Arithmetike which I promiſed, but for an augmentation of the fyrſte part. Vnto whiche I wold haue annexed ẏ Extractiō of Roots, Square & cubik, namely for Exāples of the ſtatut of Aſſiſe of wod, but that in ẏ Secōd part I muſt write of diuers other Roots, & therfore thought it beſt to reſerue thoſe Rootes alſo with their exāples to the ſame Second parte.

Sc. Syr although I can not recompence your goodneſſe, yet I ſhall alwaies do mine indeuour to occaſion you not to repent your benefite on me thus imployed.

Maſt. That recompence is ſufficient for your parte. FINIS.

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by Reginalde VVolfe.

ANNO DOMINI. M. D. LVIII.

1558

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When this book was first printed

Henri the 4th of France was aged 5

Michael Angelo 84

Cardinal Pole died same year

Shakspeare was born 6 years after

Sir Walter Raleigh was 6

Tasso 14

Stow the Antiquary was (both) 33

Speed = Historian (Tailors) 3

Sir Philip Sidney 4

Servetus burned 5 years before
to the eternal damnation of
Calvins (charity)

Cervantes was 9

Gama (Vasco De) was 32

8

87

1797

EW 29/8/87